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Some invariant solutions for non-conformal perfect fluid plates in 5-flat form in general relativity

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Abstract. A set of six invariant solutions for non-conformal perfect fluid plates in 5-flat form is obtained using one-parametric Lie group of transformations. Out of the six solutions so obtained, three are in implicit form while the remaining three could be expressed explicitly. Each solution describes an accelerating fluid distribution and is new as far as authors are aware.

Keywords. Einstein's field equations; perfect fluid distributions; embedding class one; Lie group of transformations.

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1. Introduction

The space V_n embedded in E_{n+p} (flat space) attracts much attention due to the idea proposed by Randall and Sundrum [1] where such spaces are related to Brane theory. Also such spaces are frequently used in extrinsic gravity, string and membranes, rigid particles and Zitterbewegung theory [2].

When an n-dimensional flat space V_n is embedded in $n + p$-dimensional flat space, where p is a minimum number, then V_n is said to be of embedding class p. Many physically important solutions, e.g. Friedmann Universe and Schwarzchild's interior solutions are of class one ($p = 1$), the well-known Schwarzchild's exterior solution [3] is of class two ($p = 2$) and Kerr metric [4] is of class five ($p = 5$). All the class-one perfect fluid distributions except non-conformal accelerating fluids barring Zeldovich fluids [5] are known. In the present article some non-conformal fluid plates are found by using invariance of the concerned Einstein's equations under the one-parametric Lie group of transformations, and the solutions so obtained are analysed physically. The plus point of all the solutions obtained is that every one is expressible in terms of the coordinates of the 5-flat embedding space.

2. Field equations

A 5-flat metric can be expressed as

$$\mathrm{d}s^2 = -(\mathrm{d}z^1)^2 - (\mathrm{d}z^2)^2 - (\mathrm{d}z^3)^2 + (\mathrm{d}z^4)^2 - (\mathrm{d}z^5)^2. \tag{2.1}$$

Plane symmetry can be imposed on (2.1) by means of the transformations

$$z^1 = t\theta\cos\phi, \quad z^2 = t\theta\sin\phi, \quad z^3 = \frac{\theta^2}{2}t + 2r - u,$$
$$z^4 = \left(\frac{\theta^2}{2} + 1\right)t + 2r - u, \quad z^5 = t - u,$$

where $u = u(r, t)$ and (2.1) transforms to

$$\mathrm{d}s^2 = -t^2(\mathrm{d}\theta^2 + \theta^2\,\mathrm{d}\phi^2) - u'^2\,\mathrm{d}r^2 - \dot{u}^2\,\mathrm{d}t^2 + 2(2 - \dot{u}u')\mathrm{d}r\,\mathrm{d}t \tag{2.2}$$

where $(1 - \dot{u}u') > 0$. The Einstein's field equations for perfect fluid distributions can be written as

$$-8\pi T^i_j = -8\pi[(\rho + p)v^i v_j - p\delta^i_j] = R^i_j - \frac{1}{2}R\delta^i_j. \tag{2.3}$$

On inserting (2.2) into eq. (2.3) we get

$$8\pi T^1_1 = \frac{4u'}{P^2 t}[\ddot{u}u'^2 + \dot{u}'(2 - \dot{u}u')] + \frac{u'^2}{Pt^2} = 8\pi[(\rho + p)v_1 v^1 - p], \tag{2.4}$$

$$8\pi T^2_2 = 8\pi T^3_3 = \frac{2}{P^2 t}[\dot{u}^2 u'' + 2t(\dot{u}'^2 - \ddot{u}u'') + u'^3\ddot{u} + 2u'\dot{u}'(2 - \dot{u}u')] = -8\pi p, \tag{2.5}$$

$$8\pi T^4_4 = \frac{4}{P^2 t}[\dot{u}^2 u'u'' + u'\dot{u}'(2 - \dot{u}u')] + \frac{u'^2}{Pt^2} = 8\pi[(\rho + p)v_4 v^4 - p], \tag{2.6}$$

where $P = 4(1 - \dot{u}u')$, while the component of the flow vector v^i are

$$v^2 = v_2 = v^3 = v_3 = 0 \quad \text{such that } v^1 v_1 + v^4 v_4 = 1. \tag{2.7}$$

The expressions for pressure (p) and density (ρ) in terms of u can easily be had from the relations (2.4)–(2.7) as

$$8\pi p = \frac{u'^2}{4t^2(1 - \dot{u}u')}, \tag{2.8}$$

$$8\pi\rho = \frac{(\ddot{u}u'' - \dot{u}'^2)}{2(1 - \dot{u}u')^2} + \frac{u'^2}{4t^2(1 - \dot{u}u')}. \tag{2.9}$$

Consistency of the Einstein's field equations (2.4)–(2.7) demands

$$(W + 8\pi T)W = 0 \ [6], \tag{2.10}$$

where $T = \rho - 3p$, while

$$W = \frac{2}{P^2 t}[-\dot{u}^2 u' u'' + 2t(\dot{u}'^2 - \ddot{u}u'') - u'^3\ddot{u} - 2u'\dot{u}'(2 - \dot{u}u')] + \frac{u'^2}{Pt^2}. \tag{2.11}$$

W can easily be proved to be an eigenvalue of the Weyl's conformal curvature tensor. Vanishing of W describes the conformally flat case while vanishing of the bracketed expression $(W + 8\pi T)$ corresponds to a non-conformally flat perfect fluid distribution of embedding class one given by the equation,

$$\dot{u}'^2 - \ddot{u}u'' + \frac{u'}{2t}[\dot{u}^2 u'' + u'^2\ddot{u} + 2\dot{u}'(2 - \dot{u}u')] + \frac{u'^2}{t^2}(1 - \dot{u}u') = 0. \tag{2.12}$$

In the present article our aim is to obtain the solutions of (2.12) by Lie group of transformations.

3. Invariant perfect fluid solutions

As eq. (2.12) is highly non-linear, we have decided to make use of its invariance under the one-parameter (say $\mathcal{E}$) Lie group of transformations:

$$\begin{aligned} u^* &= u + \mathcal{E}\eta(x,t,u) + O(\mathcal{E}^2),\\ x^* &= x + \mathcal{E}\xi(x,t,u) + O(\mathcal{E}^2),\\ t^* &= t + \mathcal{E}\tau(x,t,u) + O(\mathcal{E}^2), \end{aligned} \tag{3.1}$$

where (η, ξ, τ) are infinitesimals and can be furnished as below [7] for eq. (2.12) as

$$\eta = (a_1 u + a_2), \quad \xi = (2a_1 - a_3)r + a_4 t^3 + a_5, \quad \tau = a_3 t, \tag{3.2}$$

where a_1, a_2, a_3, a_4 and a_5 are five arbitrary parameters. Hence the infinitesimal generators corresponding to a five-parameter Lie group of non-trivial point transformation acting at (x, t, u)-space can be written as

$$\begin{aligned} X_1 &= 2r\frac{\partial}{\partial r} + u\frac{\partial}{\partial u}\\ X_2 &= \frac{\partial}{\partial u}\\ X_3 &= -r\frac{\partial}{\partial r} + t\frac{\partial}{\partial t}\\ X_4 &= t^3\frac{\partial}{\partial r}\\ X_5 &= \frac{\partial}{\partial r} \end{aligned} \tag{3.3}$$

Table 1.

	X_1	X_2	X_3	X_4	X_5
X_1	0	$-X_2$	0	$-2X_4$	$-2X_5$
X_2	X_2	0	0	0	0
X_3	0	0	0	$4X_4$	X_5
X_4	$+2X_4$	0	$-4X_4$	0	0
X_5	$2X_5$	0	$-X_5$	0	0

The commutator table for the Lie algebra arising from the infinitesimal generators (3.3) is given in table 1.

It implies that all the five generators are linearly independent. Now each generator may be used in surface invariant conditions

$$\frac{\mathrm{d}r}{\xi} = \frac{\mathrm{d}t}{\tau} = \frac{\mathrm{d}u}{\eta}, \tag{3.4}$$

to obtain the form of solution of eq. (2.12). It can easily be observed that only X_1 and X_3 can provide forms for the solutions, but those forms are not sufficiently general to obtain as many solutions as possible. Therefore, we plan to obtain the solutions by starting with the generalized generators involving all the parameters. Moreover, particular values will be assigned to these parameters to get possible solutions. Also parameters can be adjusted to accommodate reality conditions like $\rho > p > 0$.

Case I. Equations (3.4) with (3.2) read as

$$\frac{\mathrm{d}r}{[(2a_1 - a_3)r + a_4\, t^3 + a_5]} = \frac{\mathrm{d}t}{a_3 t} = \frac{\mathrm{d}u}{(a_1 u + a_2)}. \tag{3.5}$$

The similarity form of the solution of eq. (3.5) can be written as

$$u = A + t^{\alpha} F(t^{(1-2\alpha)}(r + Bt^3 + C)), \tag{3.6}$$

where

$$A = -\frac{a_2}{a_1}, \quad B = \frac{a_4}{2a_3(\alpha - 2)}, \quad C = \frac{a_5}{a_3(2\alpha - 1)},$$
$$\alpha = \left(\frac{a_1}{a_3}\right) \neq 2, \frac{1}{2}, \quad a_1 \neq 0, \quad a_3 \neq 0. \tag{3.7}$$

Case II. For $a_1 = 2a_3 \neq 0$ or $\alpha = 2$, eq. (3.5) gives

$$\frac{\mathrm{d}r}{(3a_3 r + a_4\, t^3 + a_5)} = \frac{\mathrm{d}t}{a_3 t} = \frac{\mathrm{d}u}{(a_1 u + a_2)}. \tag{3.8}$$

Hence the solution of eq. (3.8) can be written as

$$u = A + t^2 F(t^{-3}(r + Bt^3 \log t + C)), \tag{3.9}$$

where

$$A = -\frac{a_2}{2a_3}, \quad B = -\frac{a_4}{a_3}, \quad C = \frac{a_5}{3a_3}.$$

In order to remove the singularity at $t = 0$, we set $B = a_4 = 0$.

Case III. For $2a_1 = a_3 \neq 0$ or $\alpha = \frac{1}{2}$, the solution of (3.5) is given by

$$u = A + t^{1/2} F(r + Bt^3 + C \log t), \tag{3.10}$$

where

$$A = -\frac{2a_2}{a_3}, \quad B = -\frac{a_4}{3a_3}, \quad C = \frac{a_5}{a_3}.$$

To remove the singularity at $t = 0$, we set $C = a_5 = 0$.

Case IV. For $a_3 \neq 0$, $a_1 = 0$, the solution of (3.5) assumes the form as

$$u = A \log t + F(t(r + Bt^3 + C)), \tag{3.11}$$

where

$$A = \frac{a_2}{a_3}, \quad B = -\frac{a_4}{4a_3}, \quad C = -\frac{a_5}{a_3}.$$

By the way, solution corresponding to X_3 can be had by setting $A = B = C = 0$ in (3.11). To have u regular at $t = 0$, we write $A = a_2 = 0$.

Case V. For $a_1 = 0$, $a_3 = 0$, the solution of eq. (3.5) is furnished as

$$u = \frac{r}{A + Bt^3} + F(t), \tag{3.12}$$

where

$$A = \frac{a_5}{a_2}, \quad B = \frac{a_4}{a_2} (a_2 \neq 0).$$

Case VI. For $a_1 \neq 0$, $a_3 = 0$, we get the solution of (3.5) as

$$u = A + \sqrt{(Br + Ct^3 + D)} F(t), \tag{3.13}$$

where

$$A = -\frac{a_2}{a_1}, \quad B = 2a_1, \quad C = a_4, \quad D = a_5.$$

Solution corresponding to X_1 can be had by taking $A = D = C = 0$ in (3.13).

Therefore, the method of Lie group of transformations offer six forms of u which on inserting in eq. (2.12) provide six ordinary differential equations for the similarity function F whose solutions can be obtained in the following manners:

Case (I). On inserting the value of u from (3.6) in eq. (2.12), we get the second-order ordinary differential equation in F as

$$4(2-\alpha)(1-2\alpha)x\bar{F}\bar{\bar{F}} + 2\alpha(1-\alpha)F\bar{\bar{F}} + \alpha^2F^2\bar{F}\bar{\bar{F}} + 2(2-\alpha)^2\bar{F}^2 + \alpha(3\alpha-5)F\bar{F}^3 + 2(\alpha-2)(1-2\alpha)x\bar{F}^4 = 0, \tag{3.14}$$

where $x = t^{(1-2\alpha)}(r + Bt^3 + C)$ and bar stands for differentiation with respect to x.

As a consequence, the expressions for pressure (p) and density (ρ) can be had from (2.8) and (2.9) as

$$8\pi p = \frac{t^{-2\alpha}\bar{F}^2}{4[1-\alpha F\bar{F} - \{(1-2\alpha)x + 3Bt^{(4-2\alpha)}\}\bar{F}^2]},$$

$$8\pi\rho = -\frac{[\alpha(\alpha-1)t^{-2\alpha}F\bar{\bar{F}} + \{6Bt^{(4-4\alpha)} - 2(1-\alpha)(1-2\alpha)xt^{-2\alpha}\} \times \bar{F}\bar{\bar{F}} - (1-\alpha)^2t^{-2\alpha}\bar{F}^2]}{2[1-\alpha F\bar{F} - \{(1-2\alpha)x + 3Bt^{(4-2\alpha)}\}\bar{F}^2]^2} + \frac{t^{-2\alpha}\bar{F}^2}{4[1-\alpha F\bar{F} - \{(1-2\alpha)x + 3Bt^{(4-2\alpha)}\}\bar{F}^2]}, \tag{3.15}$$

where F is given by eq. (3.14).

A particular solution where pressure is equal to density can be obtained by setting arbitrary constants $\alpha = 1$ and $B = 0$. The expressions for pressure and density read as

$$8\pi\rho = 8\pi p = \frac{t^{-2}\bar{F}^2}{4[1 - F\bar{F} + x\bar{F}^2]}, \tag{3.16}$$

where F can be obtained from the following equation:

$$F^2\bar{\bar{F}} - 4x\bar{\bar{F}} + 2\bar{F} - 2F\bar{F}^2 + 2x\bar{F}^3 = 0, \tag{3.17}$$

which is a Zeldovich fluid and belongs to the set of the solutions obtained by Gupta *et al* [5]. In the present case the solution is expressed in 5-flat form which is a new outcome.

Again a particular solution of eq. (3.14) can be furnished by taking

$$F = \pm 2\sqrt{x} \tag{3.18}$$

and the expressions for pressure and density are given as

$$8\pi\rho = 3(8\pi p) = -\frac{1}{4Bt^4}. \tag{3.19}$$

Positive pressure and density conditions require $B < 0$. The case implies a conformally flat fluid.

Case (II). On substituting the regular value of u from (3.9) into the eq. (2.12), we get

$$F\bar{F}^3 + 2F^2\bar{F}\bar{\bar{F}} - 2F\bar{\bar{F}} = 0, \tag{3.20}$$

where

$$x = t^{-3}(r + C). \tag{3.21}$$

On integration (3.20) gives

$$F \mp \frac{2(1 + C_1F)^{3/2}}{3C_1} = -C_1x + C_2, \quad C_1 \neq 0 \tag{3.22}$$

or

$$\begin{aligned} &4(C_1F)^3 + 3(C_1F)^2 + 6C_1F\{2 + 3(C_1x - C_2)\} \\ &\quad - 9C_1^2(C_1x - C_2)^2 + 4 = 0, \end{aligned} \tag{3.23}$$

where C_1 and C_2 are the arbitrary constants of integration.

Equation (3.23) is a cubic equation in F and the three roots of this equation can be had by using the standard results related to cubic equation in algebra as

$$F = \frac{\{4(y + z) - 1\}}{4C_1}, \quad \frac{\{4(y\omega^2 + z\omega) - 1\}}{4C_1}, \quad \frac{\{4(y\omega + z\omega^2) - 1\}}{4C_1} \tag{3.24}$$

where

$$y = \left(\frac{9}{64}(8\beta^2 + 4\beta - 1) + \frac{9}{8}\sqrt{\left(\beta^4 + 25\beta^3 + 45\beta^2 + 28\beta + \frac{47}{8}\right)}\right)^{1/3},$$

$$z = \left(\frac{9}{64}(8\beta^2 + 4\beta - 1) - \frac{9}{8}\sqrt{\left(\beta^4 + 25\beta^3 + 45\beta^2 + 28\beta + \frac{47}{8}\right)}\right)^{1/3},$$

$\beta = C_1(C_1x - C_2)$ and ω is the cube root of unity.

Therefore, the expressions of pressure and density can be written as

$$8\pi p = \frac{(1 \pm \sqrt{(1 + C_1F)})^2}{4t^4[3x(1 \pm \sqrt{(1 + C_1F)})^2 - (1 \pm 2\sqrt{(1 + C_1F)})F^2]}, \tag{3.25}$$

$$8\pi\rho = \frac{\left(1 \pm \sqrt{(1 + C_1F)}\right)(6 + C_1F) \pm (6 + 4C_1F)}{4t^4\left(1 \pm \sqrt{(1 + C_1F)}\right)\left[3x\left(1 \pm \sqrt{(1 + C_1F)}\right)^2 - \left(1 \pm 2\sqrt{(1 + C_1F)}\right)F^2\right]}, \tag{3.26}$$

where F is given by eq. (3.24). The fluid distribution is accelerating and quite new as far as authors are aware.

On the other hand if $C_1 = 0$, we get

$$F = \pm 2\sqrt{x + C_3}, \tag{3.27}$$

where C_3 is a constant.

The expressions for pressure and density can be obtained on replacing B by C_3 in the eq. (3.19).

Case (III). On inserting regular u from (3.10) into (2.12) the equation reduces to

$$18\bar{F}^2 + F^2\bar{F}\bar{\bar{F}} - 7F\bar{F}^3 + 2F\bar{\bar{F}} = 0. \tag{3.28}$$

$$x = r + Bt^3. \tag{3.29}$$

On integration (3.28) gives

$$\bar{F} = \frac{A_1}{4}F^7 + \frac{2}{F} \pm \frac{F^3}{4}\sqrt{A_1^2F^8 + 16A_1}, \quad A_1 \text{ being a constant.} \tag{3.30}$$

The expressions for pressure and density are given by

$$8\pi p = \frac{64}{t[(A_1F^4 - z_1)F^6 - 768Bt^3]}, \tag{3.31}$$

$$8\pi(\rho - p) = \frac{64(A_1F^8 - z_1F^4 + 8)^2[3Bt^3(7z_2 - 16)(z_2 + 8) - 4z_2F^2]}{t(z_2 + 16)[(z_1 - A_1F^4)F^6 + 768Bt^3]^2}, \tag{3.32}$$

where $z_1 = \pm\sqrt{(A_1^2F^8 + 16A_1)}$, $z_2 = z_1F^4 + A_1F^8$, while F is given by eq. (3.30).

The particular solution of eq. (3.29) can also be furnished as

$$F = \pm 2\sqrt{x + c_1} \quad \text{where } x = (r + Bt^3),\ c_1 \text{ being a constant.}$$

The expressions for pressure and density are the same as in Case (I).

Case (IV). On inserting regular u from (3.11) into the eq. (2.12) we get

$$2\bar{F}^2 + 2x\bar{F}\bar{\bar{F}} - x\bar{F}^4 = 0, \tag{3.33}$$

where

$$x = rt + Bt^4 + Ct. \tag{3.34}$$

First integration of (3.33) is

$$\bar{F} = \frac{1}{\sqrt{x(1 + c_1x)}}, \quad c_1 \text{ being a constant.} \tag{3.35}$$

The corresponding expressions for pressure and density are given as

$$8\pi p = \frac{1}{4(c_1x^2 - 3Bt^4)}, \tag{3.36}$$

$$8\pi\rho = \frac{3}{4}\left[\frac{c_1x^2 - 3Bt^4 - 4Bc_1xt^4}{(c_1x^2 - 3Bt^4)^2}\right] \tag{3.37}$$

which provides $8\pi(\rho - 3p) = -[3Bc_1xt^4/4(c_1x^2 - 3Bt^4)^2]$. Therefore the strong energy condition ($\rho > 3p$) can be satisfied by taking $Bc_1 < 0$. Also the positivity of p and ρ requires $c_1 > 0$, $B < 0$. The fluid distribution is accelerating and the solution is new as far as authors are aware.

Case (V). On substituting u from (3.12) into (2.12) we get

$$\bar{\bar{F}} + \left(6Bt^2Z - \frac{2}{t}\right)\bar{F} + \left(18B^2t^5Z - 12Bt^2 + \frac{2}{Zt}\right) = 0,$$
$$Z = \frac{1}{A + Bt^3}. \tag{3.38}$$

On integrating (3.38), we get

$$\bar{F} = \frac{(6A^2Bt^3 - (8/7)B^3t^9 + A^3 + c_1t^2)}{(A + Bt^3)^2}. \tag{3.39}$$

Here an overhead bar stands for derivative with respect to t.

The expressions for pressure and density are given by

$$8\pi p = \frac{(A + Bt^3)}{4t^4[3Br - 3A^2Bt + 3AB^2t^4 + (15/7)B^3t^7 - c_1]}, \tag{3.40}$$

$$8\pi\rho = \frac{(A + Bt^3)(3Br - 3A^2Bt + 3AB^2t^4 + (15/7)B^3t^7 - c_1) - 18B^2t^4(A + Bt^3)^2}{4t^4(3Br - 3A^2Bt + 3AB^2t^4 + (15/7)B^3t^7 - c_1)^2}. \tag{3.41}$$

It is very clear from (3.40) and (3.41) that $8\pi(\rho - p) = -72B^2t^8(8\pi p)^2$ which implies violation of energy condition $\rho \geq p$. However, when $B = 0$, we get physically valid Zeldovich fluid.

Case (VI). On substituting u from (3.13) into (2.12), we get

$$4F^2 + 8tF\bar{F} + 4t^2(\bar{F}^2 + F\bar{\bar{F}}) + B(tF^3\bar{\bar{F}} - 3tF^2\bar{F}^2 - 2F^3\bar{F}) = 0, \tag{3.42}$$

where

$$x = Br + Ct^3 + D.$$

A particular solution of the above equation can be had by taking $\bar{F}^2 + F\bar{\bar{F}} = 0$ which gives

$$F = \pm\frac{2}{\sqrt{B}}\sqrt{t+b_1}, \quad b_1 \text{ being an arbitrary constant of integration.} \tag{3.43}$$

The expressions for pressure and density can be furnished as

$$8\pi p = -\frac{B}{12Ct^4}, \tag{3.44}$$

$$8\pi\rho = -\frac{B\,(3t+b_1)}{12Ct^4\,(t+b_1)}. \tag{3.45}$$

For positive pressure, the constants B and C should be of opposite sign. Moreover, for the positivity of density $(t + b_1)$ should be greater than zero. Initially ρ and p are quite large. However $(p/\rho) \to 1$ as $t \to 0$. The solution satisfies the strong energy condition $\rho \geq 3p$ for $b_1 > 0$. The fluid distribution is seen to possess zero acceleration [8,9] but here in this study the solution is appearing in 5-flat form.

4. Conclusion

In the process of obtaining solutions of the Einstein's perfect fluid equations by using their invariance under the Lie group of transformations method, we come across very complicated non-linear second-order ordinary differential equations. However, some particular solutions are obtained.

Case (I). Corresponding to $\alpha \neq 2, \frac{1}{2}$ is the most general case under investigation. The most general solution could not be found. Therefore, the expressions for pressure and density furnished along with a second-order ordinary differential equation remains to be satisfied. A trivial solution leads to Tolman-disordered radiations and is conformally flat ($F = \pm 2\sqrt{x}$) for all α. A Zeldovich fluid is also obtained by taking $\alpha = 1$ and $B = 0$.

Case (II). A set of three solutions is obtained explicitly which are new. An acceleration-free case has also been obtained by taking ($F = \pm 2\sqrt{x+c}$).

Case (III). One solution is obtained by taking $C = 0$ with the first-order ordinary differential equation unsolved.

Case (IV). A particular solution is explicitly displayed with all reality conditions satisfied, i.e. $\rho \geq 3p > 0$. The solution is accelerating and the present authors claim this to be new.

Case (V). The most general explicit solution is obtained. Unfortunately in this case $\rho \leq p$, and hence reality conditions are violated, but $B = 0$ provides valid Zeldovich fluid.

Case (VI). Yields a reasonable particular solution with zero acceleration.

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