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Absolute parametric instability of low-frequency waves in a 2D nonuniform anisotropic warm plasma

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Abstract. Using the separation method, absolute parametric instability (API) of electrostatic waves in a magnetized pumped warm plasma is investigated. In this case the effect of static strong magnetic field is considered. The problem of strong magnetic field is solved in two-dimensional (2D) nonuniform plane plasma. Equations which describe the spatial part of the electric potential are obtained. Also, the growth rates and conditions of the parametric instability for periodic and aperiodic cases are obtained. It is found that the spatial nonuniformity of the plasma exerts a stabilizing effect on the API. It is shown that the growth rates of periodic and aperiodic API in warm plasma are less when compared to that in cold plasma.

Keywords. Absolute parametric instability; anisotropic warm plasma; separation method.

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1. Introduction

The method developed in refs [1,2] is best suited for investigating the parametric effects under high-amplitude pump wave, $W = n_0 T_e$. This method was modified [3] for investigating the API in a magnetoactive nonuniform plasma by a monochromatic HF electric field of an arbitrary amplitude.

It has been shown [4] that dispersion equation describing the parametric excitation of surface waves at the boundary of isotropic plasma (vacuum) to within the eigenfrequency renormalization coincide with the equations that determine the parametric excitation of volumetric waves in uniform unbounded plasma. Following this conclusion the method for investigating the parametric interaction of external HF electrical field with electrostatic oscillations in an isotropic bounded nonuniform plasma has been proposed [5–7]. The method makes it possible to separate the problem into two parts. The 'dynamical' part describes the parametric build-up of oscillations and the corresponding equations within the renormalization of eigenfrequencies coincide with equations for the parametrically unstable waves in an infinite uniform plasma. Natural frequencies of oscillations and spatial distribution of the amplitude of the self-consistent electrical field are determined from the solution of a boundary-value problem ('space' part) taking into account specific spatial distribution of the plasma density. The proposed approach ('separation method') [6,7] is significantly simpler than the method ordinarily employed in the theory of parametric resonance in a nonuniform plasma [4,8,9]. Therefore, it is of special interest to apply the separation method to solve different problems involving parametric excitation of electrostatic waves in a bounded nonuniform plasma.

Demchenko *et al* [10] have reported an analysis of the effect of spatial plasma nonuniformity on parametric instability of electrostatic waves in magnetized cylindrical waveguides subjected to an intense HF electric field.

The absolute parametric instability (API) is absolute instability as it is known (see, e.g. chapter V in [11]) that the spatial nonuniformity of the plasma density leads to (1) an increase of the threshold value of the pump wave amplitude above which parametric amplification occurs and (2) the localization of unstable waves in the finite region of a plasma. This suggests that instability has assumed an absolute character. The absolute parametric instability due to the decay of the pump wave into two plasmons is one of the best-understood case [12–16]. It should be emphasized that, from an experimental point of view, it is quite important to know whether a given parametric instability is absolute or connective (see, e.g. Chapter III in [17]). This is so essential because the nature of the parametric instability determines the mechanism of their saturation. The connective instability reaches saturation at a comparatively low level, due to the convection of energy of the decay product (secondary waves) away from the three-wave resonance region. The absolute instability saturates at a higher level under the action of various nonlinear effects. From this point of view, absolute parametric instability (API) plays a crucial role in the process of energy transfer from the electromagnetic radiation to the plasma and may have important consequences for experiments on RF plasma heating in tokamaks and for laser fusion [16,18].

A method by Demchenko and Omelchenko [19] is applied in this paper which permits reducing the problem of API excited by a monochromatic pumping field of arbitrary amplitude in a nonuniform magnetoactive plasma to the problem of parametric excitation of spatial oscillations in a uniform isotropic plasma. They have discussed the parametric excitation of low-frequency waves whose dispersion is completely determined by a high-frequency field, in a strong magnetic field, when the cyclotron frequency of ions significantly exceeds the frequency of the excited oscillations.

Using the separation method, we investigate the API in a bounded nonuniform warm plasma under the pump field and static magnetic field. The pump field $\vec{E}_{\rm p} = \vec{E}_0 \sin(\omega_0 t)$ and the static magnetic field $\vec{B}_0$ are both directed along the z-axis. Assuming the intensity of the magnetic field to be strong enough ($\omega_{{\rm c}_\alpha} \gg \omega_{{\rm p}_\alpha}$), the motion of plasma particles is considered to be confined along z-axis only. We are going to study API in 2D nonuniform bounded warm plasma.

2. Mathematical model

2.1 *Separation method in API in a 2D nonuniform warm bounded plasma*

Following [19], let us consider nonuniform plasma with equilibrium density $n_{\alpha_0} = n_{\alpha_0}(x,y)$. The pump field $\vec{E}_{\rm p} = \vec{E}_0 \sin(\omega_0 t)$ and the static magnetic field $\vec{B}_0$ are directed along the z-axis. Assuming the intensity of the magnetic field to be strong enough, ($\omega_{{\rm c}_\alpha} \gg \omega_{{\rm p}_\alpha}$ where $\omega_{{\rm c}_\alpha}$ and $\omega_{{\rm p}_\alpha}$ are respectively the cyclotron and plasma frequencies for a particle of species α and $\alpha = (e,i)$).

We can consider the motion of plasma particles only along z-axis. The plasma is unperturbed at $t = 0$, so that $t > 0; n_\alpha = n_{\alpha_0}(x) + \delta n_\alpha; \vec{V}_\alpha = u_{\alpha_0} + \delta\vec{V}_\alpha$, where the perturbations of velocity $\delta\vec{V}_\alpha(0,0,\delta V_\alpha)$, density δn_α and electrical potential Φ could be represented in the form $(\delta\vec{V}_\alpha, \delta n_\alpha, \Phi) \sim \exp(ik_z z)$, $k_z \equiv k$.

The initial system of equations consists of two fluid equations in combination with the Poisson equation:

$$\frac{\partial \vec{V}_\alpha}{\partial t} + (\vec{V}_\alpha \cdot \vec{\nabla})\vec{V}_\alpha = \frac{e_\alpha}{m_\alpha}\left(\vec{E}_{\rm p} + \frac{1}{c}[\vec{V}_\alpha \vec{B}_0] - \vec{\nabla}\Phi\right) - \frac{1}{n_\alpha m_\alpha}\vec{\nabla}P, \qquad (1)$$

$$\frac{\partial n_\alpha}{\partial t} + \mathrm{div}(n_\alpha \vec{V}_\alpha) = 0, \qquad (2)$$

$$\Delta\Phi = -4\pi \sum_\alpha e_\alpha n_\alpha, \qquad (3)$$

where n_α and $\vec{V}_\alpha$ are the density and velocity of particles of species α and Φ is the potential self-consistent electric field, and $\alpha \equiv (e,i)$.

In the equilibrium state, the particle velocity $\vec{u}_\alpha(0,0,u_\alpha)$ is determined by the following expression:

$$\vec{u}_\alpha = -\frac{e_\alpha \vec{E}_0}{m_\alpha \omega_0}\cos(\omega_0 t). \qquad (4)$$

Linearizing the system of equations of two-fluid hydrodynamics (1) and (2) supplemented by Poisson's equation (3), we obtain for the quantities $\nu_\alpha = e_\alpha \delta n_\alpha \exp(-iA_\alpha)$ and the electric field potential Φ:

$$\frac{\partial^2 \nu_\alpha}{\partial t^2} + \eta_\alpha^2 \nu_\alpha = -\frac{e_\alpha^2}{m_\alpha} \mathrm{e}^{-iA_\alpha(t)} \hat{L}_2 \Phi, \qquad (5)$$

$$\hat{L}_1 \Phi = 4\pi \sum_\alpha \nu_\alpha \mathrm{e}^{+iA_\alpha(t)}, \quad \hat{L}_1 \equiv -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + k_z^2, \qquad (6)$$

where

$$\hat{L}_2 = -k_z^2 n_{0\alpha}, \quad A_\alpha = -a_\alpha \sin(\omega_0 t), \quad a_\alpha = \frac{e_\alpha k E_0}{m_\alpha \omega_0^2} \approx a_\mathrm{e},$$

and

$$\eta_\alpha^2 = V_{\mathrm{th}_\alpha}^2 k^2.$$

In this paper, plasma electrons are considered to have a thermal velocity (i.e., at $\eta_\alpha^2 = \eta_\mathrm{e}^2 = \eta^2 = V_\mathrm{th}^2 k^2$).

Assuming $\nu_\alpha(x,y,t) = \nu_{\alpha_1}(t)\nu_{\alpha_2}(x,y)$, $\Phi(x,y,t) = \Phi_1(t)\Phi_2(x,y)$, and separating the variables in eqs (5) and (6) we find the final form of equations describing 'dynamical' (parametric) part and the space part of our problem:

$$\frac{\mathrm{d}^2\nu_{\mathrm{e}1}}{\mathrm{d}t^2} + \eta^2\nu_{\mathrm{e}1} + p^2(\nu_{\mathrm{e}1} + \omega_{\mathrm{i}1}\mathrm{e}^{-i(A_\mathrm{e}-A_\mathrm{i})}) = 0, \tag{7}$$

$$\frac{\mathrm{d}^2\omega_{\mathrm{i}1}}{\mathrm{d}t^2} + \frac{m_\mathrm{e}}{m_\mathrm{i}}p^2(\omega_{\mathrm{i}1} + \nu_{\mathrm{e}1}\mathrm{e}^{i(A_\mathrm{e}-A_i)}) = 0, \tag{8}$$

$$\Delta_\perp\Phi_2(x,y) - k_z^2\varepsilon(x,y,p)\Phi_2(x,y) = 0, \tag{9}$$

where

$$\Delta_\perp \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \varepsilon(x,y,p) = 1 - \frac{\omega_{p_\mathrm{e}}^2(x,y)}{p^2},$$

$$w_\mathrm{i} = \frac{m_\mathrm{e}}{m_\mathrm{i}}\frac{\alpha_\mathrm{i}}{\alpha_\mathrm{e}}\nu_{\mathrm{i}_1}, \quad \alpha_\mathrm{s}\alpha_\mathrm{i} = -\frac{m_\mathrm{i}}{m_\mathrm{e}}\frac{4\pi}{p^2}$$

and p is a separation constant in 2D plasma nonuniformity. From eq. (9) one finds that necessary condition of wave propagation is $\varepsilon(x,y,p) < 0$. It means that in eq. (9) one will have $|\varepsilon(x,y,p)|$.

Having solved eq. (9) with specific boundary conditions taken into account, we find the possible values of the separation constant p. The system of eqs (5) coincides (within the redefinition of $\omega_{p_\mathrm{e}}^2 \to p^2$, $\omega_{p_\mathrm{i}}^2 \to (m_\mathrm{e}/m_\mathrm{i})p^2$, $V_\mathrm{th} = 0$ in cold plasma) with the system describing HF suppression of Buneman instability in a uniform unbounded plasma [4].

For a cylindrical geometry [7], parametric part of the problem is identical to eqs (7) and (8) for cold plane plasma (at $\eta^2 = V_\mathrm{th}^2 k^2 = 0$). It means that parametric part does not depend explicitly on the geometry (only through p).

2.2 *Solution of the 'Temporal' (time-dependent) eqs* (7) *and* (8)

Following the procedure developed in refs [20,21], from eq. (5) we can derive the dispersion equation of low-frequency oscillations $l_y \to \infty$. Under the parametric resonance condition ($n\omega_0 \approx s$, n – integer), we get

$$\omega^4 - \frac{\Delta_n^2 s^2}{4}\omega^2 - \frac{m_\mathrm{e}}{2m_\mathrm{i}}p^4\Delta_n J_n^2(a) = 0, \tag{10}$$

where $\Delta_n = (s/n\omega_0)^2 - 1$, $s^2 = p^2 + \eta^2$, $\eta^2 = v_{\rm th}^2 k_z^2$ and we suppose here that the resonance 'mismatch' Δ_n satisfies the inequalities $(m_{\rm e}/m_{\rm i}) \cdot (p/s)^4 \ll |\Delta_n| \ll 1$. From eq. (10) we find the frequencies of parametrically excited plasma oscillations:

$$\omega^2 = \frac{\Delta_n^2 s^2}{8}\left(1 \pm \left[1 + \frac{32}{\Delta_n^3} J_n^2(a)\frac{m_{\rm e}}{m_{\rm i}} \cdot \frac{p^4}{s^4}\right]^{1/2}\right), \tag{11}$$

where $J_n(a)$ is the Bessel function. Expression (11) yields an unstable solution in two cases.

2.2.1 *Periodic instability* ($\Delta_n < 0$)

In this case $\gamma_{\rm per} = {\rm Im}\,\omega > 0$, i.e., small perturbations in the plasma will grow exponentially in time, if the following condition is satisfied:

$$0 > \Delta_n > -2\left(4J_n^2(a)\frac{m_{\rm e}}{m_{\rm i}} \cdot \frac{p^4}{s^4}\right)^{1/3}. \tag{12}$$

The growth rate of instability is determined by the expression

$$\gamma_{\rm per} = s\frac{|\Delta_n|}{4}\left[-1 + \left(\frac{32J_n^2(a)}{|\Delta_n|^3}\frac{m_{\rm e}}{m_{\rm i}} \cdot \frac{p^4}{s^4}\right)^{1/2}\right]^{1/2}. \tag{13}$$

The maximum value of the growth rate $\gamma_{\rm per}$ is reached at

$$(n\omega_0)_{\rm max} = s\left[1 + \left(\frac{1}{4}J_n^2(a)\frac{m_{\rm e}}{m_{\rm i}}\left(\frac{p}{s}\right)^4\right)^{1/3}\right]. \tag{14}$$

Substituting expression (14) into (13) we can find

$$\gamma_{\rm per}^{\rm max} = s\left(\frac{\sqrt{27}}{32}J_n^2(a)\frac{m_{\rm e}}{m_{\rm i}} \cdot \frac{p^4}{s^4}\right)^{1/3}. \tag{15}$$

2.2.2 *Aperiodic instability* ($\Delta_n > 0$)

In this case, expression (11) describes the growth of oscillations when the minus sign is taken. We have then the following expression for the growth rate:

$$\gamma_{\rm aper} = \frac{\Delta_n}{2\sqrt{2}}s\left(\left[1 + \frac{32J_n^2(a)}{\Delta_n^3}\frac{m_{\rm e}}{m_{\rm i}}\frac{p^4}{s^4}\right]^{1/2} - 1\right)^{1/2}. \tag{16}$$

The maximum of the growth rate:

$$\gamma_{\text{aper}}^{\max} = s\left(\frac{1}{2}J_n^2(a)\frac{m_{\text{e}}}{m_{\text{i}}}\frac{p^4}{s^4}\right)^{1/3} \tag{17}$$

is attained under the condition

$$(n\omega_0)_{\text{aper}}^{\max} = s\left[1-\left(\frac{1}{2}J_n^2(a)\frac{m_{\text{e}}}{m_{\text{i}}}\frac{p^4}{s^4}\right)^{1/3}\right]. \tag{18}$$

The main feature of expressions (13)–(18) consists of the existence of a separation constant p which enables us to account for the plasma nonuniformity.

At $\eta^2 = V_{\text{th}}^2 k^2 = 0$, eqs (15) and (17) are in agreement with the corresponding equations for cold plasma [17–20] where

$$\gamma_{\text{per}}^{\max} = p_{\text{e}}\left(\frac{\sqrt{27}}{32}J_n^2(a)\frac{p_{\text{i}}^2}{p_{\text{e}}^2}\right)^{1/3} \tag{19}$$

$$\gamma_{\text{aper}}^{\max} = p_{\text{e}}\left(\frac{1}{2}J_n^2(a)\frac{p_{\text{i}}^2}{p_{\text{e}}^2}\right)^{1/3}. \tag{20}$$

On comparing the maximum growth rates of the periodic and aperiodic API in cold (eqs (19) and (20)) and warm plasma (eqs (15) and (17)), we conclude that the growth rates of periodic and aperiodic API decrease in warm plasma than in cold plasma.

2.3 *Solution of the spatial* (*space-dependent eq.* (9))

Suppose now that the distribution of the equilibrium plasma density has the form [22]

$$n_0(x,y) = N\left(1-\frac{x^2}{l_x^2}-\frac{y^2}{l_y^2}\right). \tag{21}$$

Accordingly, let us separate variables in eq. (9) as follows: $(\Phi_2(x,y) = f_1(x)f_2(y))$. We get the following two equations:

$$\frac{\mathrm{d}^2 f_1}{\mathrm{d}x^2} + (A_1 - B_1 x^2) f_1 = 0 \tag{22}$$

$$\frac{\mathrm{d}^2 f_2}{\mathrm{d}y^2} + (A_2 - B_2 y^2) f_2 = 0, \tag{23}$$

where

$$A_1 = k^2|\varepsilon_0| - k_1^2 > 0, \quad B_1 = \frac{k^2\omega_{\text{P}_0}^2}{p^2 l_x^2}, \quad A_2 = k_1^2, \quad B_2 = \frac{k^2\omega_{\text{P}_0}^2}{p^2 l_y^2}$$

which replace the corresponding equation in 1D plasma density nonuniformity

$$\frac{d^2\Phi_2}{dx^2} + (A - Bx^2)\Phi_2 = 0,$$

where

$$A = k^2|\varepsilon_0| > 0, \quad B = \frac{k^2\omega_{P_0}^2}{p_l^2 l_x^2}, \quad \omega_{P0}^2 = \frac{4\pi e^2 N_0}{m}$$

and p_l is the separation constant in 1D geometry. The new separation constant k_1 coincides with the wave vector k_y as one passes from 2D to 1D plasma density nonuniformity ($l_y \to \infty$).

Let us introduce new variables:

$$\Psi_i(\xi_i) = e^{\xi_i/2} f_i, \quad \xi_i = B_i^{1/2} \begin{pmatrix} x \\ y \end{pmatrix}, \quad i = 1, 2. \tag{24}$$

Equations (22) and (23) in such case reduce to

$$\frac{d^2\Psi_i}{d\xi_i^2} - 2\xi_i \frac{d\Psi_i}{d\xi} + 2l_i\Psi_i = 0, \tag{25}$$

where

$$2l_1 + 1 = \frac{|A_1|}{B_1^{1/2}} = \frac{k^2|\varepsilon_0| - k_1^2}{k\omega_{P_0}} pl_x, \tag{26}$$

$$2l_2 + 1 = \frac{|A_2|}{B_2^{1/2}} = \frac{k_1^2}{k\omega_{P_0}} pl_y. \tag{27}$$

As in ref. [19], eq. (9) in 1D geometry has the form

$$\psi''_{\xi\xi} - 2\xi\psi' + 2n\psi = 0 \tag{28}$$

for the function $\psi(\xi)$, where

$$\xi = \frac{x}{b}, \quad h = (k_z b)\frac{\omega_{P_0}}{p}, \quad a = \frac{p}{\omega_{P_0}}|\varepsilon_0|^{1/2}.$$

Solution of eq. (25) can be expressed through the Hermite polynomials $H_n(\xi)$. These polynomials describe localized oscillations only under the assumption that quantities l_i are positive integers. From eqs (26) and (27), we get the separation constant p and k_1 as

$$p_l = \frac{\omega_{P_0}}{2}[(Q_l^2 + 4)^{1/2} - Q_l], \tag{29}$$

$$k_1^2 = k^2 \frac{2l_y + 1}{2kl_y}[(Q_l^2 + 4)^{1/2} + Q_l], \tag{30}$$

where

$$Q_l = \frac{2l_1+1}{kl_x} + \frac{2l_2+1}{kl_y}.$$

At $l_y \to \infty$ and $Q_n \ll 1$ expression (29) transforms into the relation: $p_n \approx \omega_{\mathrm{P}_0}(1 - \frac{Q_n}{2})$ in the case of 1D nonuniformity in plane plasma [19–22]. According to eqs (29) and (30), in the case of a 2D plasma nonuniformity, the main conclusion of 1D nonuniformity in plane plasma [19,22] concerning stabilization effect of a density inhomogeneity on the parametric instability remains valid. The growth rate of unstable oscillations are determined by expressions (15) and (17) where separation constant p_l must be replaced by p from (29).

3. Discussions and conclusions

The problem of API in warm bounded plasma is investigated. In this case the effect of static strong magnetic field $\vec{B}_0 = \vec{e}_z B_0$ is considered ($\omega_{\mathrm{c}_\alpha} \gg \omega_{\mathrm{p}_\alpha}$). The problem of strong magnetic field is solved in 2D nonuniform plane plasma. The equation which describes the spatial part of the electric potential Φ_2 is obtained (eq. (25) in the case of a 2D plasma nonuniformity) and compared with the corresponding equation (eq. (28)) for 1D nonuniformity in plane and cold plasmas (at $\eta^2 = V_{\mathrm{th}}^2 k^2 = 0$).

We obtained the frequencies of parametric instability (11), which yield two mechanisms for such instability: periodic instability with negative resonance mismatch at $\delta\varepsilon_\alpha \sim p_\alpha^2/(n\omega_0)^2 \ll 1$ and aperiodic instability at opposite condition. The frequencies and maximum growth rates in both cases are obtained (relations (14) and (15) for the first case, and relations (18) and (17) for the second case).

It is found that nonuniformity of the plasma density results in a decrease of the growth rate of absolutely unstable oscillations and an increase in the threshold value of the HF pump wave amplitude in comparison to the case of a uniform plasma waveguide. Numerical results depicting spatial and temporal growth rates of the instability by employing suitable parameters of the plasma, temperature, magnetic field and other cases are subjects for a future paper.

Also, by comparing the maximum growth rates of the periodic and aperiodic API in cold plasma (eqs (19) and (20)) and warm plasma (eqs (15) and (17)), we conclude that the growth rates of periodic and aperiodic API reduce in warm plasma than in cold plasma. A physical explanation of the effect of temperature on the growth rates of the periodic and aperiodic API is as follows: When the plasma has inhomogeneity in density and temperature, radial flows of heat and particles must develop which shorten the lifetime of a confined plasma configuration [23]. The temperature gradient in a plasma changes the electric field in the plasma owing to the temperature dependence of conductivity. The instabilities are due to finite conductivity. The perturbations with finite conductivity are large in scale but have relatively low rise increments. The plasma conductivity increases with temperature.

It should be noted that the method ('separation method') and approach used here is significantly simpler than the method ordinarily employed in the theory of a parametric excitation of waves in a nonuniform plasma [8]. Therefore, it is of practical interest to use this method to solve different problems on parametric resonance in nonuniform plasma taking into account finite plasma temperature (thermal motion of particles) and nonuniformities of the HF (pump) electrical field and static magnetic field.

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