PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 74, No. 5
May 2010
pp. 705–718

Robust chaos synchronization using input-to-state stable control

CHOON KI AHN
Faculty of the Division of Electronics and Control Engineering, Wonkwang University, 344-2 Shinyong-dong, Iksan 570-749, Korea
E-mail: hironaka@wonkwang.ac.kr

MS received 1 September 2009; revised 6 December 2009; accepted 22 December 2009

Abstract. In this paper, we propose a new input-to-state stable (ISS) synchronization method for a general class of chaotic systems with disturbances. Based on Lyapunov theory and linear matrix inequality (LMI) approach, for the first time, the ISS synchronization controller is presented not only to guarantee the asymptotic synchronization but also to achieve the bounded synchronization error for any bounded disturbance. The proposed controller can be obtained by solving a convex optimization problem represented by the LMI. Simulation studies are presented to demonstrate the effectiveness of the proposed ISS synchronization scheme.

Keywords. Input-to-state stable (ISS) synchronization; chaotic systems; linear matrix inequality (LMI); Lyapunov theory.

PACS Nos 05.45.Gg; 05.45.Xt; 05.45.-a

1. Introduction

In the last two decades, synchronization in chaotic dynamic systems has attracted a great deal of interest among scientists from various research fields since Pecora and Carroll [1] introduced a method to synchronize two identical chaotic systems with different initial conditions. It has been widely explored in a variety of fields including physical, chemical and ecological systems [2]. In the literature, various synchronization schemes, such as variable structure control [3], OGY method [4], parameters adaptive control [5,6], observer-based control [7], active control [8,9], fuzzy system approach [10], backstepping design technique [11,12], passivity-based approach [13,14], and so on, have been successfully applied to the chaos synchronization. However, most research results in the chaos synchronization were restricted to chaotic systems without disturbances.

It is well-known that real physical systems are often affected by disturbances, such as perturbations in control or errors in observation. Thus, for real physical systems, control systems are required not only for stability, but also for having the property of input-to-state stability (ISS). When a control system is called ISS, it means that no matter what the initial state is, if the inputs are uniformly small then the state of the control system must eventually be small. ISS is an interesting concept first introduced in [15] to nonlinear control systems. It has been widely accepted as an important concept in control engineering and many research results have been reported in recent years [16–23]. To the best of our knowledge, however, for the ISS-based synchronization of chaotic systems with disturbances, there is no result in the literature so far, which still remains open and challenging.

In this paper, a new controller for the ISS synchronization of chaotic systems in the presence of disturbances is proposed. This controller is a new contribution to the topic of chaos synchronization. The proposed controller consists of two parts: one is the linear state feedback controller and the other is the nonlinear feedback controller. By the proposed control scheme, the closed-loop error system is asymptotically synchronized and remains bounded for any bounded disturbance. Based on Lyapunov method and linear matrix inequality (LMI) approach, an existence criterion for the proposed controller is represented in terms of an LMI. The LMI problem can be solved efficiently by using recently developed convex optimization algorithms [24].

This paper is organized as follows. In §2, the basic concept of ISS is introduced. In §3, we formulate the problem. In §4, an LMI problem for the ISS synchronization of chaotic systems is proposed. In §5, numerical examples are given, and finally, conclusions are presented in §6.

2. Basic concept of input-to-state stability

Consider the following differential equation:

$$\dot{X}(t) = F(X(t), U(t)), \tag{1}$$

where $X(t) \in R^n$ is the state variable, $U(t) \in R^m$ is the external input. F: $R^n \times R^m \rightarrow R^n$ is continuously differentiable and satisfies $F(0, 0) = 0$. Throughout this paper, we will use the following definitions:

DEFINITION 1
A function γ: $R_{\geq 0} \rightarrow R_{\geq 0}$ is a $\mathcal{K}$ function if it is continuous, strictly increasing and $\gamma(0) = 0$.

DEFINITION 2
A function γ: $R_{\geq 0} \rightarrow R_{\geq 0}$ is a $\mathcal{K}_\infty$ function if it is a $\mathcal{K}$ function and also $\gamma(s) \rightarrow \infty$ as $s \rightarrow \infty$.

DEFINITION 3
A function β: $R_{\geq 0} \times R_{\geq 0} \rightarrow R_{\geq 0}$ is a $\mathcal{KL}$ function if, for each fixed $t \geq 0$, the function $\beta(\cdot, t)$ is a $\mathcal{K}$ function, and for each fixed $s \geq 0$, the function $\beta(s, \cdot)$ is decreasing and $\beta(s, t) \rightarrow 0$ as $t \rightarrow \infty$.

The notion of ISS can be described as follows:

DEFINITION 4
The system (1) is said to be input-to-state stable if there exist a $\mathcal{K}$ function $\gamma(s)$ and a $\mathcal{KL}$ function $\beta(s,t)$, such that, for each input $U(t) \in L_\infty^m$ ($\sup_{t\geq 0}\|U(t)\| < \infty$) and each initial state $X(0) \in R^n$, it holds that

$$\|X(t)\| \leq \beta(\|X(0)\|, t) + \gamma(\|U(t)\|) \tag{2}$$

for each $t \geq 0$.

It is noted that, if a system is input-to-state stable, the behaviour of the system should remain bounded when its inputs are bounded. Now we introduce a useful result that is employed for obtaining the ISS synchronization controller.

Lemma 1 [18]. *A continuous function* $V(\cdot)$: $R^n \to R_{\geq 0}$ *is called an ISS-Lyapunov function for the system* (1), *if there exist* $\mathcal{K}_\infty$ *functions* α_1, α_2, α_3, *and* α_4 *such that*

$$\alpha_1(\|X(t)\|) \leq V(X(t)) \leq \alpha_2(\|X(t)\|) \tag{3}$$

for any $X(t) \in R^n$ and

$$\dot{V}(X(t)) \leq -\alpha_3(\|X(t)\|) + \alpha_4(\|U(t)\|) \tag{4}$$

for any $X(t) \in R^n$ *and any* $U(t) \in L_\infty^m$. *Then the system* (1) *is input-to-state stable if and only if it admits an ISS-Lyapunov function.* ∎

3. Problem formulation

Consider a class of chaotic systems described by the following nonlinear differential equation:

$$\dot{x}(t) = Ax(t) + Bf(x(t)) + Hg(x(t)), \tag{5}$$

where $x(t) \in R^n$ is the state vector, $A \in R^{n\times n}$, $B \in R^{n\times p}$, and $H \in R^{n\times q}$ are the known constant matrices, $f(x(t)) \in R^p$ is the nonlinear function vector that does not satisfy the global Lipschitz condition, and $g(x(t)) \in R^q$ is the nonlinear function vector satisfying the global Lipschitz condition with Lipschitz constant $L_g > 0$. The system (5) is considered as a drive system.

The synchronization problem of system (5) is considered by using the drive-response configuration. According to the drive-response concept, the controlled response system is given by

$$\dot{z}(t) = Az(t) + Bf(z(t)) + Hg(z(t)) + Cu_l(t) + u_n(t) + Gd(t), \tag{6}$$

where $z(t) \in R^n$ is the state vector of the response system, and $C \in R^{n\times m}$ and $G \in R^{n\times k}$ are known constant matrices of the controlled response system. $u_l(t) \in R^m$, $u_n(t) \in R^n$, and $d(t) \in L_\infty^k$ are the linear control input, the nonlinear control input, and the disturbance input, respectively. Define the synchronization error $e(t) = z(t) - x(t)$. Then we obtain the synchronization error system

$$\dot{e}(t) = Ae(t) + B(f(z(t)) - f(x(t))) + H(g(z(t)) - g(x(t))) + Cu_l(t) + u_n(t) + Gd(t). \quad (7)$$

DEFINITION 5 (Asymptotic synchronization)
The error system (7) is asymptotically synchronized if the synchronization error $e(t)$ satisfies

$$\lim_{t\to\infty} e(t) = 0. \quad (8)$$

DEFINITION 6 (ISS synchronization)
The error system (7) is ISS synchronized if there exist a $\mathcal{K}$ function $\gamma(s)$ and a $\mathcal{KL}$ function $\beta(s,t)$, such that, for each disturbance input $d(t) \in L_\infty^k$ and each initial synchronization error $e(0) \in R^n$, it holds that

$$\|e(t)\| \le \beta(\|e(0)\|, t) + \gamma(\|d(t)\|) \quad (9)$$

for each $t \ge 0$.

The purpose of this paper is to design the controllers $u_l(t)$ and $u_n(t)$ guaranteeing the ISS synchronization if there exists the disturbance input $d(t)$. In addition, these controllers $u_l(t)$ and $u_n(t)$ will be shown to guarantee the asymptotic synchronization when the disturbance input $d(t)$ disappears.

4. Main results

The LMI problem for achieving the ISS synchronization is presented in the following theorem.

Theorem 1. *For a given $Q = Q^T > 0$, if there exist $X = X^T > 0$ and Y such that*

$$\begin{bmatrix} XA^T + AX + CY + Y^TC^T & H & X & X & G \\ H^T & -I & 0 & 0 & 0 \\ X & 0 & -\frac{1}{L_g^2}I & 0 & 0 \\ X & 0 & 0 & -Q^{-1} & 0 \\ G^T & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad (10)$$

then the ISS synchronization is achieved and the controllers are given by $u_l(t) = YX^{-1}(z(t) - x(t))$ and $u_n(t) = -B(f(z(t)) - f(x(t)))$.

Proof. The closed-loop error system with the linear control input $u_l(t) = Ke(t)$ can be written as

$$\dot{e}(t) = (A + CK)e(t) + B(f(z(t)) - f(x(t))) + H(g(z(t)) - g(x(t))) + Gd(t) + u_n(t). \quad (11)$$

If we select the nonlinear control input $u_n(t) = -B(f(z(t)) - f(x(t)))$, we have

$$\dot{e}(t) = (A + CK)e(t) + H(g(z(t)) - g(x(t))) + Gd(t). \quad (12)$$

Consider a Lyapunov function

$$V(e(t)) = e^T(t)Pe(t), \tag{13}$$

where $P = P^T > 0$. Note that $V(e(t))$ satisfies the following Rayleigh inequality [25]:

$$\lambda_{\min}(P)\|e(t)\|^2 \le V(e(t)) \le \lambda_{\max}(P)\|e(t)\|^2, \tag{14}$$

where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the maximum and minimum eigenvalues of the matrix. The time derivative of $V(e(t))$ along the trajectory of (12) is

$$\begin{aligned}\dot{V}(e(t)) &= \dot{e}(t)^T Pe(t) + e^T(t)P\dot{e}(t)\\ &= e^T(t)[A^TP + PA + PCK + K^TC^TP]e(t)\\ &\quad + e^T(t)PH(g(z(t)) - g(x(t))) + (g(z(t))\\ &\quad - g(x(t)))^T H^T Pe(t) + e(t)^T PGd(t) + d^T(t)G^T Pe(t).\end{aligned}$$

If we use the inequality $X^TY + Y^TX \le X^T\Lambda X + Y^T\Lambda^{-1}Y$, which is valid for any matrices $X \in R^{n\times m}$, $Y \in R^{n\times m}$, $\Lambda = \Lambda^T > 0$, $\Lambda \in R^{n\times n}$, we have

$$\begin{aligned}&e^T(t)PH(g(z(t)) - g(x(t))) + (g(z(t)) - g(x(t)))^T H^T Pe(t)\\ &\quad \le (g(z(t)) - g(x(t)))^T(g(z(t)) - g(x(t))) + e^T(t)PHH^TPe(t)\\ &\quad \le L_g^2(z(t) - x(t))^T(z(t) - x(t)) + e^T(t)PHH^TPe(t)\\ &\quad = e^T(t)[L_g^2 I + PHH^TP]e(t)\end{aligned} \tag{15}$$

and

$$e(t)^T PGd(t) + d^T(t)G^T Pe(t) \le d^T(t)d(t) + e(t)^T PGG^T Pe(t). \tag{16}$$

Using (15) and (16), we obtain

$$\begin{aligned}\dot{V}(e(t)) \le{}& e^T(t)[A^TP + PA + PCK + K^TC^TP + L_g^2 I\\ &+ PHH^TP + PGG^TP]e(t) + d^T(t)d(t).\end{aligned}$$

If the following matrix inequality

$$A^TP + PA + PCK + K^TC^TP + L_g^2 I + PHH^TP + Q + PGG^TP < 0, \tag{17}$$

is satisfied, we have

$$\dot{V}(e(t)) < -e^T(t)Qe(t) + d^T(t)d(t) \tag{18}$$

$$\le -\lambda_{\min}(Q)\|e(t)\|^2 + \|d(t)\|^2. \tag{19}$$

Define functions $\alpha_1(r)$, $\alpha_2(r)$, $\alpha_3(r)$, and $\alpha_4(r)$ as

$$\alpha_1(r) \triangleq \lambda_{\min}(P)r^2, \tag{20}$$

$$\alpha_2(r) \triangleq \lambda_{\max}(P)r^2, \tag{21}$$

$$\alpha_3(r) \triangleq \lambda_{\min}(Q)r^2, \tag{22}$$

$$\alpha_4(r) \triangleq r^2. \tag{23}$$

Note that $\alpha_1(r)$, $\alpha_2(r)$, $\alpha_3(r)$, and $\alpha_4(r)$ are $\mathcal{K}_\infty$ functions. From (14) and (19), we obtain

$$\alpha_1(\|e(t)\|) \leq V(e(t)) \leq \alpha_2(\|e(t)\|), \tag{24}$$

$$\dot{V}(e(t)) \leq -\alpha_3(\|e(t)\|) + \alpha_4(\|d(t)\|). \tag{25}$$

According to Lemma 1, we can conclude that $V(e(t))$ is an ISS-Lyapunov function and the ISS synchronization is achieved. From Schur complement, the matrix inequality (17) is equivalent to

$$\begin{bmatrix} A^TP + PA + PCK + K^TC^TP & PH & I & I & PG \\ H^TP & -I & 0 & 0 & 0 \\ I & 0 & -\frac{1}{L_g^2}I & 0 & 0 \\ I & 0 & 0 & -Q^{-1} & 0 \\ G^TP & 0 & 0 & 0 & -I \end{bmatrix} < 0. \tag{26}$$

Pre- and post-multiplying (26) by $\mathrm{diag}(P^{-1}, I, I, I, I)$ and introducing change of variables such as $X = P^{-1}$ and $Y = KP^{-1}$, (26) is equivalently changed into the LMI (10). Then the gain matrix of the linear control input $u_l(t)$ is given by $K = YX^{-1}$. This completes the proof. ∎

COROLLARY 1

Without the disturbance input, if we use the control inputs $u_l(t)$ and $u_n(t)$ proposed in Theorem 1, the asymptotic synchronization is obtained.

Proof. When $d(t) = 0$, we obtain

$$\dot{V}(e(t)) < -e^T(t)Qe(t) \leq 0 \tag{27}$$

from (18). This guarantees

$$\lim_{t\to\infty} e(t) = 0 \tag{28}$$

from Lyapunov theory. This completes the proof. ∎

Remark 1. The LMI problem given in Theorem 1 is to determine whether the solution exists or not. It is called the feasibility problem. The LMI problem can be solved efficiently by using recently developed convex optimization algorithms [24]. In this paper, in order to solve the LMI problem, we utilize MATLAB LMI Control Toolbox [26], which implements state-of-the-art interior-point algorithms.

5. Numerical example

In this section, to verify and demonstrate the effectiveness of the proposed method, we discuss the simulation results for synchronizing Lorenz system, Chua's circuit, and hyperchaotic Rössler system.

5.1 *Application to Lorenz system*

Consider the following Lorenz system:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -8/3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -x_1(t)x_3(t) \\ x_1(t)x_2(t) \end{bmatrix}. \tag{29}$$

For the numerical simulation, we use the following parameters:

$$C = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{30}$$

Figure 1 shows state trajectories for drive and response systems when the initial conditions are given by

$$\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} -15.8 \\ -17.48 \\ 15.64 \end{bmatrix}, \quad \begin{bmatrix} z_1(0) \\ z_2(0) \\ z_3(0) \end{bmatrix} = \begin{bmatrix} 10.8 \\ 12 \\ 12 \end{bmatrix}, \tag{31}$$

and the disturbance input $d(t)$ is given by

$$d(t) = \begin{cases} w(t), & 0 \le t \le 3, \\ 0, & \text{otherwise}, \end{cases} \tag{32}$$

where $w(t)$ means a Gaussian noise with mean 0 and variance 100. From figure 1, it can be seen that drive and response systems are indeed achieving chaos synchronization. Figure 2 shows, by the proposed ISS synchronization method, that the synchronization error $e(t)$ is bounded on the interval where the disturbance input $d(t)$ exists. In addition, it is shown that the synchronization error $e(t)$ goes to zero after the disturbance input $d(t)$ disappears.

5.2 *Application to Chua's circuit*

The dynamics of Chua's circuit can be described in a dimensionless form by a third-order state equation:

$$\begin{aligned} \dot{x}_1(t) &= \alpha_c(x_2(t) - x_1(t) + f_c(x_1(t))), \\ \dot{x}_2(t) &= x_1(t) - x_2(t) + x_3(t), \\ \dot{x}_3(t) &= -\beta_c x_2(t), \end{aligned} \tag{33}$$

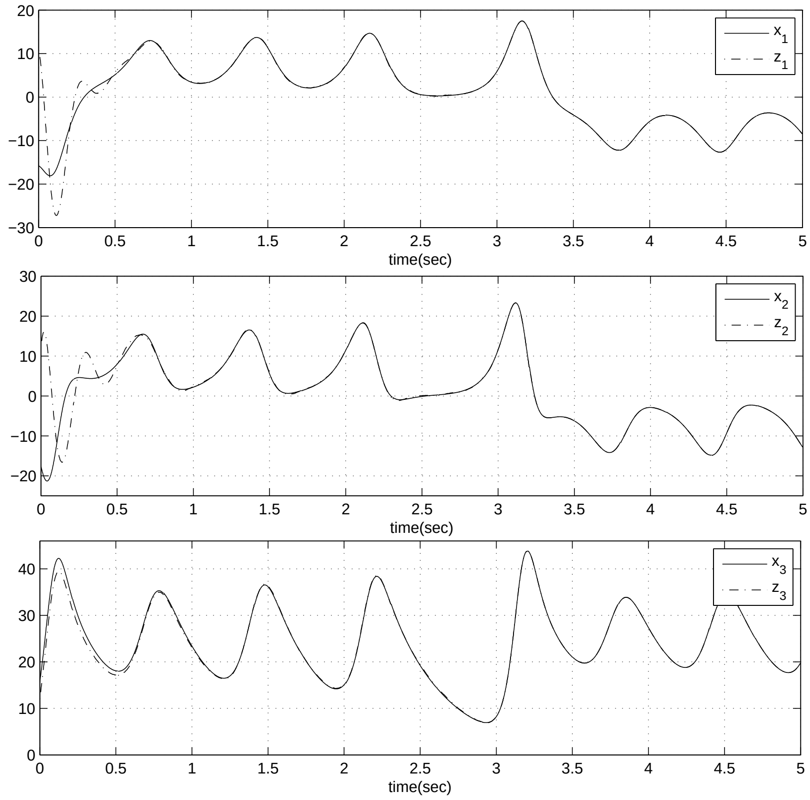

Figure 1. State trajectories for Lorenz system.

where α_c and β_c are real numbers and $f_c(x_1(t))$ is defined as the following piecewise-linear function:

$$f_c(x_1(t)) = m_1 x_1(t) + \frac{1}{2}(m_0 - m_1)(|x_1(t)+1| - |x_1(t)-1|), \tag{34}$$

where m_0 and m_1 denote the slope of the inner and outer segments of the piecewise-linear function $f_c(x_1(t))$. Let $\alpha_c = -1$, $\beta_c = 15$, $m_0 = -1.28$, and $m_1 = -0.69$. Then the Chua's circuit (33) becomes

$$\begin{aligned}\dot{x}_1(t) &= -10x_1(t) + 10x_2(t)\\ &\quad + \left[-0.69x_1(t) - \frac{0.59}{2}(|x_1(t)+1| - |x_1(t)-1|)\right],\\ \dot{x}_2(t) &= x_1(t) - x_2(t) + x_3(t),\\ \dot{x}_3(t) &= -15x_2(t).\end{aligned} \tag{35}$$

Since the function $\frac{1}{2}(|x_1(t)+1| - |x_1(t)-1|)$ in (35) satisfies the global Lipschitz condition with Lipschitz constant 1, the Chua's circuit (35) is rewritten as

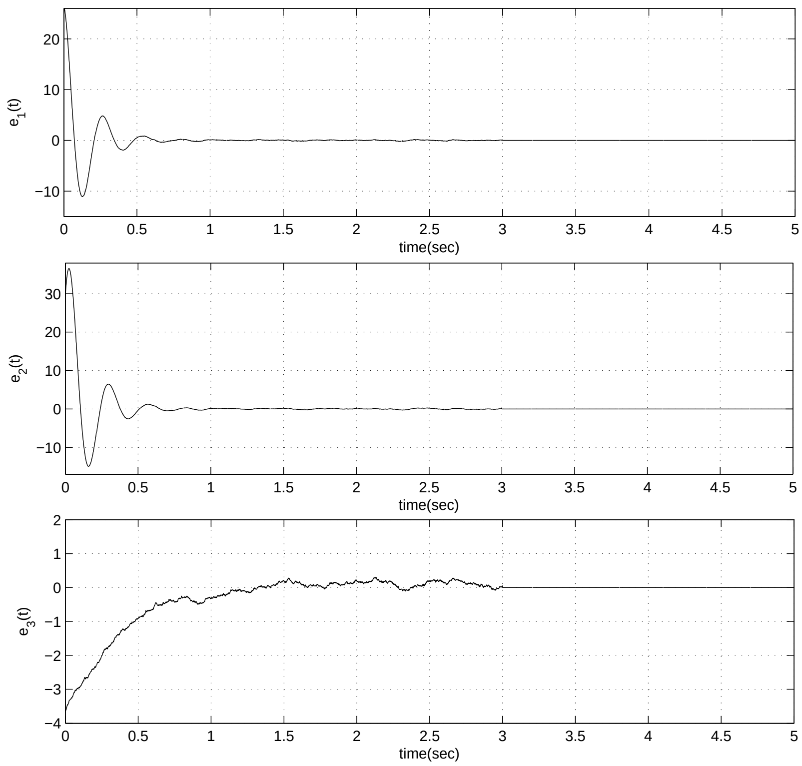

Figure 2. Synchronization errors for Lorenz system.

$$\dot{x}(t) = Ax(t) + Bf(x(t)) + Hg(x(t)), \tag{36}$$

where

$$A = \begin{bmatrix} -10.69 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -15 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad H = \begin{bmatrix} -0.59 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$f(x(t)) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad g(x(t)) = \begin{bmatrix} \frac{1}{2}(|x_1(t)+1| - |x_1(t)-1|) \\ 0 \\ 0 \end{bmatrix}. \tag{37}$$

For the numerical simulation, we use the parameters in (30). Applying Theorem 1 to the Chua's circuit (36) yields

$$X = \begin{bmatrix} 1.0500 & 0.4169 & 0.2033 \\ 0.4169 & 0.6361 & 0.3028 \\ 0.2033 & 0.3028 & 1.1312 \end{bmatrix}, \quad Y = \begin{bmatrix} 1.1793 & -5.0754 & 5.0019 \end{bmatrix}.$$

Figure 3 shows state trajectories when the initial conditions are given by $(x_1(0), x_2(0), x_3(0)) = (-1.8, -1.48, 1.64)$, $(z_1(0), z_2(0), z_3(0)) = (1.8, 1, -1)$, and

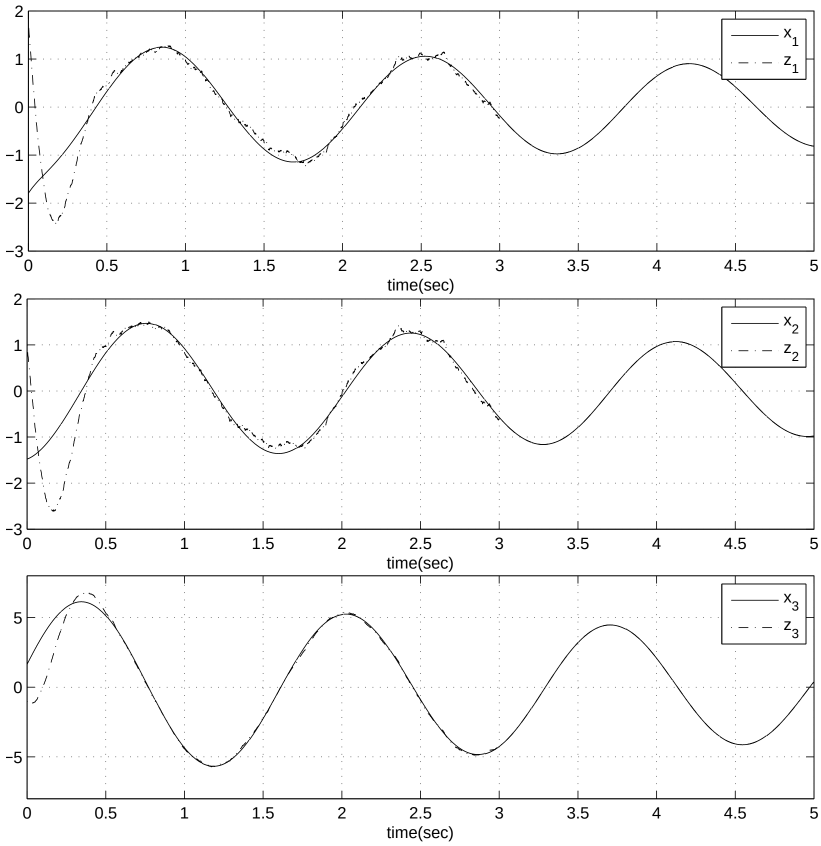

Figure 3. State trajectories for Chua's circuit.

the disturbance input $d(t)$ is given by (32). Figure 4 shows, by the proposed ISS synchronization method, that the synchronization error $e(t)$ is bounded on the interval where the disturbance input $d(t)$ exists. In addition, it is shown that the synchronization error $e(t)$ goes to zero after the disturbance input $d(t)$ disappears.

5.3 *Application to hyperchaotic Rössler system*

Consider the following hyperchaotic Rössler system [27,28]:

$$\begin{aligned}
\dot{x}_1(t) &= -x_2(t) - x_3(t),\\
\dot{x}_2(t) &= x_1(t) + 0.25x_2(t) + x_4(t),\\
\dot{x}_3(t) &= 3 + x_1(t)x_3(t),\\
\dot{x}_4(t) &= -0.5x_3(t) + 0.05x_4(t).
\end{aligned} \tag{38}$$

Since the function $3 + x_1(t)x_3(t)$ in (38) does not satisfy the global Lipschitz condition, the hyperchaotic Rössler system (38) is rewritten as

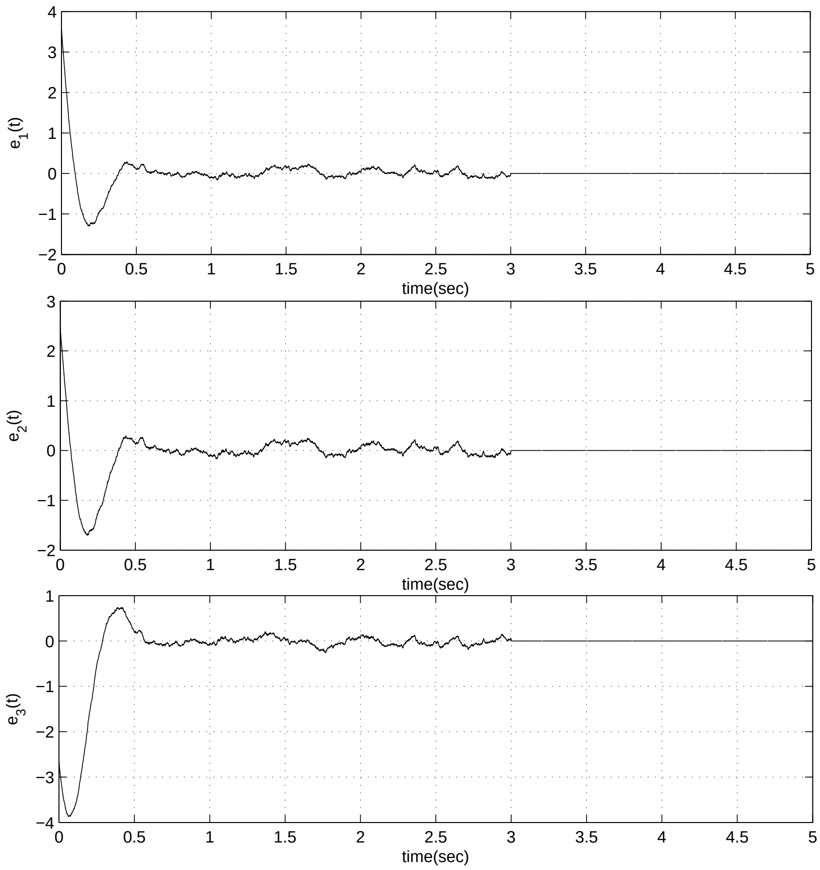

Figure 4. Synchronization errors for Chua's circuit.

$$\dot{x}(t) = Ax(t) + Bf(x(t)) + Hg(x(t)), \tag{39}$$

where

$$A = \begin{bmatrix} 0 & -1 & -1 & 0 \\ 1 & 0.25 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0.05 \end{bmatrix}, \quad B = H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$f(x(t)) = \begin{bmatrix} 0 \\ 0 \\ 3 + x_1(t)x_3(t) \\ 0 \end{bmatrix}, \quad g(x(t)) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{40}$$

In the numerical simulation, the following parameters are used:

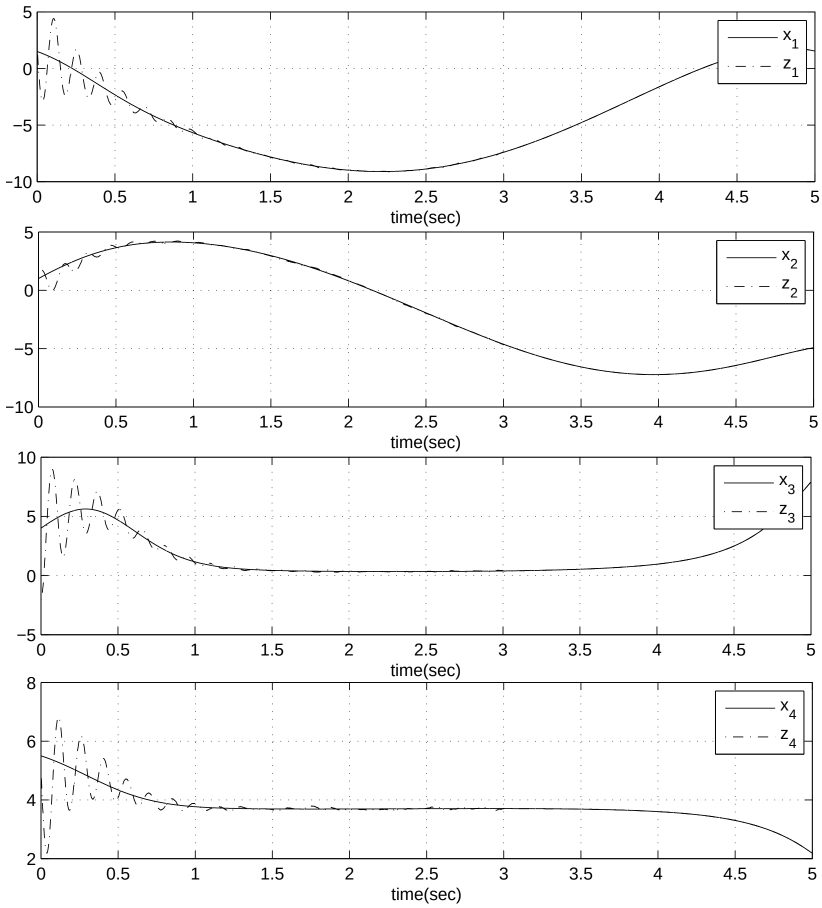

Figure 5. State trajectories for hyperchaotic Rössler system.

$$C = Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

Applying Theorem 1 to the hyperchaotic Rössler system (38) yields

$$X = \begin{bmatrix} 1.1295 & 0.0066 & 0.0066 & 0.0066 \\ 0.0066 & 1.1295 & 0.0066 & 0.0066 \\ 0.0066 & 0.0066 & 1.1295 & 0.0066 \\ 0.0066 & 0.0066 & 0.0066 & 1.1295 \end{bmatrix},$$

$$Y = \begin{bmatrix} -4.9935 & -1.7977 & 42.2344 & -3.3775 \\ 1.2681 & -5.3024 & -7.7008 & 8.3872 \\ -41.6261 & 7.1580 & -5.0068 & -21.7319 \\ 2.8659 & -10.0499 & 21.7684 & -5.0599 \end{bmatrix}.$$

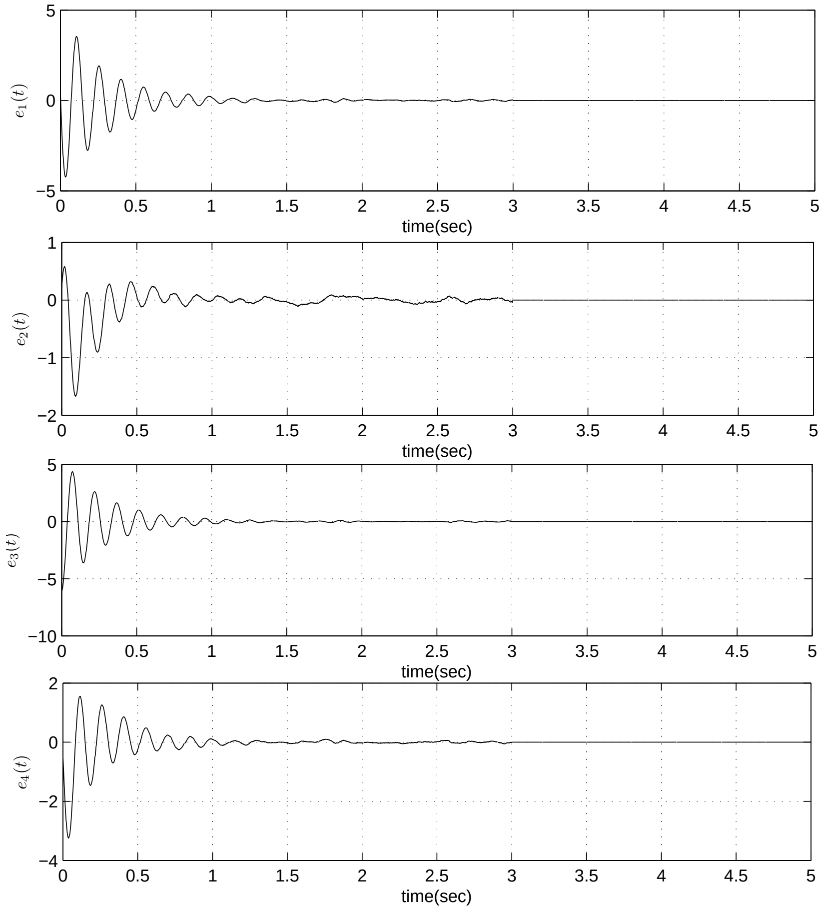

Figure 6. Synchronization errors for hyperchaotic Rössler system.

Figure 5 shows state trajectories when the initial conditions are given by $(x_1(0), x_2(0), x_3(0), x_4(0)) = (1.5, 1, 4, 5.5)$, $(z_1(0), z_2(0), z_3(0), z_4(0)) = (1.9, 1.2, -2.1, 5)$, and the disturbance input $d(t)$ is given by (32), where $w(t)$ is changed to a Gaussian noise with mean 0 and variance 9. Figure 6 shows the synchronization error $e(t)$. Figure 6 illustrates that the ISS synchronization method reduces the effect of the disturbance input $d(t)$ on the synchronization error $e(t)$. Simulation results reveal that the response system controlled using the proposed ISS method performs well. The effectiveness and accuracy of the proposed method is demonstrated.

6. Conclusion

In this paper, we propose a new ISS synchronization scheme for a general class of chaotic systems with disturbances. Based on Lyapunov theory and LMI approach, the proposed ISS method guarantees the asymptotic synchronization between the drive and response systems and the bounded synchronization error for any bounded disturbance input. Furthermore, the synchronization problems for the Lorenz system, the Chua's circuit, and the hyperchaotic Rössler system are given to illustrate the effectiveness of the proposed ISS scheme.

References

[1] L M Pecora and T L Carroll, *Phys. Rev. Lett.* **64**, 821 (1996)
[2] G Chen and X Dong, *From chaos to order* (World Scientific, Singapore, 1998)
[3] C C Wang and J P Su, *Chaos, Solitons and Fractals* **20**, 967 (2004)
[4] E Ott, C Grebogi and J A Yorke, *Phys. Rev. Lett.* **64**, 1196 (1990)
[5] J H Park, *Int. J. Nonlinear Sci. Numer. Simul.* **6**, 201 (2005)
[6] Y Wang, Z H Guan and H O Wang, *Phys. Lett.* **A312**, 34 (2003)
[7] X S Yang and G Chen, *Chaos, Solitons and Fractals* **13**, 1303 (2002)
[8] E Bai and K Lonngen, *Phys. Rev.* **E8**, 51 (1997)
[9] E W Bai and K E Lonngren, *Chaos, Solitons and Fractals* **11**, 1041 (2000)
[10] C K Ahn, *Nonlinear Analysis: Hybrid Systems* **4**, 16 (2010)
[11] X Wu and J Lu, *Chaos, Solitons and Fractals* **18**, 721 (2003)
[12] J Hu, S Chen and L Chen, *Phys. Lett.* **A339**, 455 (2005)
[13] C K Ahn, *Mod. Phys. Lett.* **B23**, 3531 (2009)
[14] C K Ahn, *Chin. Phys. Lett.* **27**, 010503 (2010)
[15] E D Sontag, *IEEE Trans. Autom. Contr.* **34**, 435 (1989)
[16] E D Sontag, *IEEE Trans. Autom. Contr.* **35**, 473 (1990)
[17] Z P Jiang, A Teel and L Praly, *Math. Contr. Signal Syst.* **7**, 95 (1994)
[18] E D Sontag and Y Wang, *Syst. Contr. Lett.* **24**, 351 (1995)
[19] P D Christofides and A R Teel, *IEEE Trans. Autom. Contr.* **41**, 1645 (1996)
[20] E D Sontag, *Syst. Contr. Lett.* **34**, 93 (1998)
[21] A R Teel, *IEEE Trans. Autom. Contr.* **43**, 960 (1998)
[22] W Xie, C Wen and Z Li, *IEEE Trans. Autom. Contr.* **46**, 1111 (2001)
[23] D Angeli and D Nesic, *IEEE Trans. Autom. Contr.* **46**, 1298 (2001)
[24] S Boyd, L E Ghaoui, E Feron and V Balakrishnan, *Linear matrix inequalities in systems and control theory* (SIAM, Philadelphia, 1994)
[25] G Strang, *Introduction to applied mathematics* (Wellesley Cambridge Press, Cambridge, 1986)
[26] P Gahinet, A Nemirovski, A J Laub and M Chilali, *LMI control toolbox* (The Mathworks Inc., Natick, 1995)
[27] O E Rössler, *Phys. Rev.* **E71**, 155 (1979)
[28] A Tamaševičius and A Čenys, *Phys. Rev.* **E55**, 297 (1997)