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Einstein–Rosen inflationary Universe in general relativity

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Abstract. Einstein–Rosen inflationary Universe is investigated in the presence of massless scalar field with a flat potential. To get an inflationary Universe, we have considered a flat region in which the potential V is constant. Some physical properties of the model are discussed.

Keywords. Einstein–Rosen metric; inflationary Universe; general relativity.

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1. Introduction

In recent years, there has been a lot of interest in cosmological models of the Universe which are important in understanding the mysteries of the early stages of its evolution. In particular, inflationary models play an important role in solving a number of outstanding problems in cosmology like the homogeneity, the isotropy and the flatness of the observed Universe. The standard explanation for the flatness of the Universe is that it has undergone at an early stage of the evolution a period of exponential expansion named as inflation.

It is well-known that self-interacting scalar fields play a vital role in the study of inflationary cosmology. Guth [1], Linde [2] and La and Steinhardt [3] are some of the authors who have investigated several aspects of inflationary Universe in general relativity. Burd and Barrow [4], Wald [5], Barrow [6], Ellis and Medsen [7] and Heusler [8] studied different aspects of scalar fields in the evolution of the Universe. The role of self-interacting scalar fields in inflationary cosmology has been investigated by Bhattacharjee and Baruah [9], Bali and Jain [10] and Rahaman *et al* [11].

Reddy *et al* [12], Reddy and Naidu [13] have discussed inflationary Universe in general relativity in four and five dimensions. Recently, Reddy *et al* [14] have investigated a plane symmetric Bianchi type-I inflationary Universe in general relativity. Very recently, Reddy [15] has discussed Bianchi type-V inflationary Universe in general relativity. Also Reddy *et al* [16], Katore and Rane [17] have studied the Kantowski–Sachs inflationary Universe in general relativity. In this paper, we have investigated Einstein–Rosen inflationary Universe in general relativity in the presence of massless scalar field with a flat potential. To get determinate solution, we have considered a flat region in which the potential is constant.

2. Metric and field equations

We have considered the cylindrically symmetric Einstein–Rosen metric in the form

$$\mathrm{d}s^2 = \mathrm{e}^{2\alpha-2\beta}(\mathrm{d}t^2 - \mathrm{d}\rho^2) - \rho^2\mathrm{e}^{-2\beta}\mathrm{d}\psi^2 - \mathrm{e}^{2\beta}\mathrm{d}z^2, \tag{1}$$

where α and β are functions of cosmic time t only and $x^1 = \rho, x^2 = \psi, x^3 = z, x^4 = t$.

The non-vanishing components of the Einstein tensor for the metric (1) are

$$G_1^1 = -\mathrm{e}^{-2\alpha+2\beta}\beta_4^2,$$
$$G_2^2 = -\mathrm{e}^{-2\alpha+2\beta}(\alpha_{44} + \beta_4^2),$$
$$G_3^3 = \mathrm{e}^{-2\alpha+2\beta}(2\beta_{44} - \alpha_{44} - \beta_4^2),$$

and

$$G_4^4 = \mathrm{e}^{-2\alpha+2\beta}\beta_4^2. \tag{2}$$

Here the subscript 4 denotes the differentiation with respect to t.

In the case of gravity minimally coupled to a scalar field $V(\phi)$ the Lagrangian is

$$L = \int \left[R - \frac{1}{2}g^{ij}\phi_{,i}\phi_{,j} - V(\phi)\right]\sqrt{-g}\mathrm{d}^4x, \tag{3}$$

which by varying L with respect to dynamical fields leads to Einstein field equations

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} \tag{4}$$

with

$$T_{ij} = \phi_{,i}\phi_{,j} - \left[\frac{1}{2}\phi_{,k}\phi^{,k} + V(\phi)\right]g_{ij}, \tag{5}$$

$$\phi^i_{;i} = -\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi}, \tag{6}$$

where comma (,) and semicolon (;) indicate ordinary and covariant differentiation respectively. Other symbols have their usual meanings and units are taken so that

$$8\pi G = c = 1.$$

Now the Einstein field equations (4) for the metric (1) with the help of eq. (5) are given by

$$e^{-2\alpha+2\beta}\left(-\beta_4^2-\frac{1}{2}\phi_4^2\right)=V(\phi), \tag{7}$$

$$e^{-2\alpha+2\beta}\left(-\alpha_{44}-\beta_4^2-\frac{1}{2}\phi_4^2\right)=V(\phi), \tag{8}$$

$$e^{-2\alpha+2\beta}\left(2\beta_{44}-\alpha_{44}-\beta_4^2-\frac{1}{2}\phi_4^2\right)=V(\phi), \tag{9}$$

$$e^{-2\alpha+2\beta}\left(\beta_4^2+\frac{1}{2}\phi_4^2\right)=V(\phi) \tag{10}$$

and eq. (6) for the scalar field takes the form

$$e^{-2\alpha+2\beta}\phi_{44}=-\frac{dV}{d\phi}. \tag{11}$$

Here the subscript 4 denotes differentiation with respect to t.

3. Solutions of the field equations and the model

Here, we are interested in inflationary solutions of the field equations (7)–(11). Stein-Schabes [18] has shown that Higg's field ϕ with potential $V(\phi)$ has a flat region and the field evolves slowly but the Universe expands in an exponential way due to vacuum field energy. It is assumed that the scalar field will take sufficient time to cross the flat region so that the Universe expands sufficiently to become homogeneous and isotropic on the scale of the order of the horizon size. Thus we are interested, here, in inflationary solutions of the field equations, the flat region is considered where the potential is constant, i.e.

$$V(\phi)=\text{ constant }=V_0 \text{ (say)}. \tag{12}$$

Using eq. (12), the field equations (7)–(11) admit the exact solutions

$$e^{2\alpha}=e^{2(a_1t+a_2)}, \tag{13}$$

$$e^{2\beta}=e^{2(a_3t+a_4)} \tag{14}$$

and

$$\phi=a_5t+a_6, \tag{15}$$

where a_1, a_2, a_3, a_4, a_5 and a_6 are constants of integration.

After suitable choice of coordinates and constants of integration, the Einstein–Rosen inflationary cosmological model can be written as

$$ds^2=(dT^2-d\rho^2)-\rho^2e^{-2T}d\psi^2-e^{2T}dz^2. \tag{16}$$

It is interesting to note that the model (16) is isotropic and has no initial singularity.

4. Physical properties of the model

The model (16) represents an exact Einstein–Rosen inflationary cosmological model in general relativity, when the scalar field ϕ is minimally coupled to the gravitational field. The model has no initial singularity at $T = 0$.

The physical parameters for the model (16) have the following expressions:

Expansion scalar: $\theta = \frac{2}{3}(a_3 - a_5)$.

Shear scalar: $\sigma^2 = \frac{2}{27}(a_3 - a_5)^2$.

Deceleration parameter: $q = -1 < 0$.

We observed that the expansion scalar (θ) and shear scalar (σ) are constants. The role of the deceleration parameter (q) seems to specify the expansion of the Universe. The positive value of the deceleration parameter q indicates that the model decelerates in the standard way. But in the present observation the model inflates because the deceleration parameter q is negative.

5. Conclusions

In this paper, we have obtained cylindrically symmetric Einstein–Rosen inflationary Universe in the presence of massless scalar field with flat potential in general relativity. It is observed that the model is non-singular and expanding. The inflationary model obtained here has considerable astrophysical significance. For example, classical scalar fields are essential in the study of the present day cosmological models. In view of the fact that there is an increasing interest, in recent years, in scalar fields in general relativity and alternative theories of gravitation in the context of inflationary Universe and they help us to describe the early stages of evolution of the Universe. We hope that our model obtained here will be useful for a better understanding of inflationary cosmology in Einstein–Rosen space-time.

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