PRAMANA — journal of physics | © Indian Academy of Sciences | Vol. 74, No. 4 April 2010 pp. 633–641

On the excited state wave functions of Dirac fermions in the random gauge potential

H MILANI MOGHADDAM
Department of Physics, Faculty of Basic Sciences, Mazandaran University, 47415 Babolsar, Iran
E-mail: milani@umz.ac.ir; hossainmilani@yahoo.com

MS received 20 October 2008; revised 29 October 2009; accepted 1 December 2009

Abstract. In the last decade, it was shown that the Liouville field theory is an effective theory of Dirac fermions in the random gauge potential (FRGP). We show that the Dirac wave functions in FRGP can be written in terms of descendents of the Liouville vertex operator. In the quasiclassical approximation of the Liouville theory, our model predicts that the localization length ξ scales with the energy E as $\xi \sim E^{-b^2/(1+b^2)^2}$, where b is the strength of the disorder. The self-duality of the theory under the transformation $b \to 1/b$ is discussed. We also calculate the distribution functions of $t_0 = |\psi_0(x)|^2$, (i.e. $p(t_0)$; $\psi_0(x)$ is the ground state wave function), which behaves as the log-normal distribution function. It is also shown that in small t_0, $p(t_0)$ behaves as a chi-square distribution.

Keywords. Disordered systems; localization; Liouville field theory.

PACS Nos 71.23.An; 05.45.Mt; 03.65.Sq

1. Introduction

Localization of a particle by a random potential has been extensively investigated for the past several decades [1–12]. It is known that, for the strong disorder, single-particle wave functions are confined and have exponentially decaying tails beyond the scale of the localization length ξ [4,13,14]. But for the weak disorder, the localization length can be very large in one- and two-dimensional conductors and infinite in three dimensions (3D). Now a natural question arises: what is the behaviour of the wave functions at distances smaller than the localization length? Despite its importance, the problem of the structure of quantum states of weakly disordered conductors for scales below the length ξ has started to attract interest only in the preceding decade [8,15–17]. This problem is well understood only for extended states, i.e., in the limit of small wave function amplitudes $t = |\psi(x)|^2$ [15]. Since extended states explore the entire sample, one can neglect their spatial variations and treat the Hamiltonian as a Wigner–Dyson random matrix theory

(RMT) [18]. In the RMT approach, the distribution functions for the wave functions' amplitude (i.e. $p(t)$) are derived by means of RMT. It depends only on the global symmetry of the ensemble and has a chi-square form. The asymptotic form of $p(t)$ in 2D samples for $L \ll \xi$ was found using the renormalization group and replica techniques [15]. The nonperturbative approach is based on the supersymmetric σ-model [19]. It was shown in ref. [19] that in order to describe the exceptional events most affected by the disorder, one could look for the saddle point of the supersymmetric σ-model. Fal'ko and Efetov [19] have used this idea to study the properties of a single quantum state in the discrete spectrum of a confined system. It is also shown that, although the distribution of relatively small amplitudes can be approximated by the chi-square distribution (Porter–Thomas distribution; see for example [20]) the asymptotic statistics of large t's is strongly modified by localization effect. They have shown that the distribution function of the largest amplitude fluctuations of the wave functions in 2D and 3D conductors are logarithmically normal (log-normal) asymptotic. Examples for which models of the random Dirac fermions are appropriate, include degenerate semiconductors [21], two-dimensional graphite sheets [22,23], tight-binding Hamiltonians of the flux phase [24], the d-wave superconductors [25–27], the Ising model [28] etc.

In this paper, we use the supersymmetric quantum mechanics to construct the excited state wave functions of the FRGP model. Using the results of ref. [8], we demonstrate that in the quasiclassical approximation of the Liouville field theory the localization length ξ scales with the energy E as $\xi \sim E^{-b^2/(1+b^2)^2}$, where b is the strength of the disorder. The dependence of the exponents on b is consistent with the results of ref. [29]. The self-duality of the theory is demonstrated as a consequence of the self-duality of the Liouville theory.

It is also shown that the distribution function $p(t_0)$ is the log-normal distribution function which depends on localization length and strength of the disorder. We observe that for small t_0, $p(t_0)$ behaves as a chi-square distribution.

2. The wave functions of the FRGP model

We recall the Dirac operator with a static magnetic field normal to the plane [30,31]

$$H[A] = c\sum_{i=1}^{2}\sigma_i(P_i - A_i) + \sigma_3 mc^2, \tag{1}$$

where σ_i $(i = 1,2,3)$ and m are the Pauli matrices and the electron mass, respectively. The standard representation of $H[A]$ has an abstract supersymmetric form

$$H[A] = \begin{pmatrix} mc^2 & cD^* \\ cD & -mc^2 \end{pmatrix}, \tag{2}$$

where the operator D is given by $D = (P_1 - A_1) - i(P_2 - A_2)$.

Note that $D \neq D^*$ in two dimensions. It is well-known that the spectrum of $H[A]$ is symmetric with respect to zero except possibly at $\pm mc^2$. The open interval $(-mc^2, mc^2)$ does not belong to the spectrum.

At first, we are only considering the Dirac Hamiltonian in the homogeneous magnetic field. Supersymmetry essentially determines the spectrum of $H[A]$. In the case of a homogeneous magnetic field $B(x) = (0, 0, B_0)$, it is sufficient to consider the two-dimensional Dirac Hamiltonian $H[A]$, with the following vector potential:

$$A(x) = \frac{B_0}{2}(-x_2, x_1) \tag{3}$$

and $D = (i\partial_1 - A_1) + (\partial_2 - iA_2)$. One knows how to find the following equation:

$$DD^* = D^*D + 2B_0. \tag{4}$$

Hence the DD^* spectrum equals the D^*D spectrum shifted by $2B_0$. It is easy to show that D^*D and DD^* have the following spectra [30]:

$$\sigma(D^*D) \subset \{0, 2B_0, 4B_0, \ldots\}, \tag{5}$$

$$\sigma(DD^*) \subset \{2B_0, 4B_0, \ldots\}. \tag{6}$$

Hence the following result is attained:

$$\sigma(H[A]) \subset \{(2nB_0 + m^2c^4)^{1/2}, -(2(n+1)B_0 + m^2c^4)^{1/2}, n = 0, 1, \ldots\}. \tag{7}$$

In order to obtain eigenfunctions it is proceeded as follows: assume $D^*D\psi_0 = 0$ or equivalently $D\psi_0 = 0$ for some $\psi_0 \in L^2(R^2)$. At the same time ψ_0 is an eigenvector of DD^* because from eq. (4), it can be written as $DD^*\psi_0 = 2B_0\psi_0$. This shows that $2B_0 \subset \sigma(DD^*)$ provided $0 \subset \sigma(D^*D)$. Supersymmetry implies that $\psi_1 = D^*\psi_0$ is an eigenvector of D^*D belonging to the same eigenvalue $2B_0$. (By applying eq. (4) again it is observed that

$$DD^*\psi_1 = 4B_0\psi_1. \tag{8}$$

If we proceed in this way, we obtain a sequence of eigenvectors $\psi_n = (D^*)^n\psi_0, n = 0, 1, 2, \ldots$ satisfying

$$D^*D\psi_n = 2nB_0\psi_0, \tag{9}$$

$$DD^*\psi_n = 2(n+1)B_0\psi_0. \tag{10}$$

The corresponding eigenvectors of the Dirac equation can be found by an inverse Foldy–Wouthuysen transformation.

$$H[A]U_{\mathrm{FW}}^{-1}\begin{pmatrix}\psi_n\\0\end{pmatrix} = (2nB_0 + m^2c^4)^{1/2}U_{\mathrm{FW}}^{-1}\begin{pmatrix}\psi_n\\0\end{pmatrix}, \tag{11}$$

$$H[A]U_{\rm FW}^{-1}\begin{pmatrix}0\\ \psi_n\end{pmatrix} = -(2(n+1)B_0 + m^2c^4)^{1/2}U_{\rm FW}^{-1}\begin{pmatrix}0\\ \psi_n\end{pmatrix}. \tag{12}$$

Hence everything depends on whether one could find ψ_0 with $D\psi_0 = 0$ or equivalently a solution of the Dirac equation with energy mc^2.

We now develop the above discussion and should try to obtain the excited state wave functions of the FRGP model with the random magnetic field normal to the plane. We consider the case where the only disorder present, is the vector potential or, correspondingly, the random scalar potential.

The two-component eigenfunction in a fixed realization of disorder, with $E = 0$ and $m = 0$, has been constructed in ref. [32]. Let us write the Hamiltonian of the FRGP model in complex plane

$$\hat{H}_{\rm Dirac} = \begin{pmatrix} mc^2 & 2i(-\partial_z + \partial_z\varphi) \\ 2i(-\partial_{\bar z} - \partial_{\bar z}\varphi) & -mc^2\end{pmatrix}, \tag{13}$$

where we have used the Coulomb gauge (i.e. $\partial_i A_i = 0$). Thus one can express the vector potential in terms of the scalar field $\varphi(x)$ so that $A_i = \varepsilon_{ij}\partial_j\varphi$ and $z = x + iy, \bar z = x - iy$.

Comparing with eq. (2), D^* is given by

$$D^* = 2i(-\partial_z + \partial_z\varphi), \tag{14}$$

and

$$D = 2i(-\partial_{\bar z} - \partial_{\bar z}\varphi). \tag{15}$$

Direct calculations show that

$$DD^* = D^*D + 2B(z,\bar z), \tag{16}$$

in which $B(z,\bar z) = 4\partial_{\bar z}\partial_z\varphi = \nabla^2\varphi$. We could demonstrate that D in the presence of the random magnetic field is a lowering operator (Appendix A). ψ_0 (in the case of random field) can be derived using the equation

$$D\psi_0 = 0, \tag{17}$$

or

$$-2i(\partial_{\bar z} + \partial_{\bar z}\varphi)\psi_0 = 0, \tag{18}$$

which, solving for ψ_0, yields

$$\psi_0 = \frac{{\rm e}^{-\varphi}}{[\int {\rm d}^2x{\rm e}^{-2\varphi}]^{1/2}}. \tag{19}$$

This expression for ψ_0 has been found by direct calculation in ref. [8]. It is demonstrated that the excited state wave functions of the FRGP model, $\psi_n, n = 0, 1, 2, 3, ...$, are in terms of ψ_0 as

$$\psi_n = \frac{1}{2^n}(D^* - g)^n \psi_0, \tag{20}$$

where $g = i\mathrm{e}^{-\varphi} \int \mathrm{d}\bar{z}(\tilde{B} - 1)\mathrm{e}^{\varphi}$ (Appendix A).

According to ref. [8], the effective action describing the correlation functions of the FRGP model can now be described. We could derive the effective action by means of correlation functions of ψ_0. The reason is that using eq. (20), one can write the correlation functions of ψ_i's in terms of correlation functions of only ψ_0's.

According to ref. [8], we consider $\nabla\varphi$ as a random variable with the Gaussian distribution

$$P[\varphi] = \frac{1}{Z_0} \mathrm{e}^{-(1/4\pi b^2) \int (\nabla\varphi)^2 \mathrm{d}^2 x}. \tag{21}$$

It is expressed as the moments of the first component of ψ_0 which is defined by

$$G(1, \ldots, N) = \int D\varphi P[\varphi] |\psi_0(x_1)|^2 \cdots |\psi_0(x_N)|^2. \tag{22}$$

Inserting eqs (19) and (21) into the above relation, the N-point moment of normalizable wave function squares can be written as follows:

$$G(1, \ldots, N) = \int_0^\infty \mathrm{d}\mu \frac{\mu^{N-1}}{(N-1)!} \int D\varphi \prod_{i=1}^{N} \mathrm{e}^{-2\varphi(x_i)} \mathrm{e}^{-S_\mu}, \tag{23}$$

where the action S_μ is given by

$$S_\mu = \frac{1}{4\pi b^2} \int [(\partial\varphi)^2 + 4\pi b^2 \mu \mathrm{e}^{-2\varphi}] \mathrm{d}^2 x. \tag{24}$$

Thus, the multipoint moment (22) is now expressed in terms of the reducible multipoint correlation function of the so-called Liouville field theory. The latter theory has been introduced by Polyakov [33] in the context of string theory and studied extensively.

At present, we consider the action of the Liouville model, S_μ. We rescale $\varphi = -b\phi$. It is conventional to add another term to the Liouville Lagrangian density, $\frac{Q}{4\pi} R\sqrt{g}\phi$, where R is the scalar curvature of background metric $g_{\mu\nu}$, and parameter Q is adjusted to ensure that all physical quantities are independent of a particular choice of this background. However, it is possible to choose a specific background which is flat everywhere except for a few selected points [34]. This term translates then into appropriate boundary terms as follows [34]:

$$S_\mu = \frac{1}{4\pi} \int_\Gamma [(\partial\phi)^2 + 4\pi\mu \mathrm{e}^{2b\phi}] \mathrm{d}^2 x + \frac{Q}{\pi R} \int_{\partial\Gamma} \phi \, \mathrm{d}l + 2Q^2 \log(R), \tag{25}$$

where R is the size of the sample. The last term is introduced to make the action finite at $R \to \infty$. This type of boundary condition is conventionally called the background charge Q at infinity. Q parametrizes the central charge c of the Liouville theory as

$$c = 1 + 6Q^2. \tag{26}$$

It is well-known that the exponential Liouville operators

$$V_\alpha(x) = \mathrm{e}^{2\alpha\phi(x)}, \tag{27}$$

are the spinless primary conformal fields of dimensions

$$\Delta_\alpha = \alpha(Q - \alpha). \tag{28}$$

The two-, three- and four-point correlation functions of Liouville field theory for a given α are calculated exactly in [34,35]. The factor Q is fixed in terms of b in such a way that the term $\mathrm{e}^{2b\phi}$ in the action can be a conformal field with dimensions (1,1) and the property of being microscopic operators. Therefore we obtain the following equation:

$$Q = b + \frac{1}{b}, \tag{29}$$

and the central charge can be written in terms of b as

$$c = 1 + 6\left(b + \frac{1}{b}\right)^2. \tag{30}$$

Now according to ref. [34], we impose the following boundary condition on the Liouville field:

$$\phi(x) = -Q\log|x|^2 \quad \text{at } |x| \to \infty, \tag{31}$$

where Q is given by eq. (29). This boundary condition ensures that all the wave functions tend to zero in large distance or decay at infinity.

In addition, it could be found that the scale μ dependence of any correlation function in Liouville theory [36–38] is as in the following equation:

$$\left\langle \prod_{i=1}^{N} \mathrm{e}^{2\alpha_i\phi(x_i)} \right\rangle_Q = (\pi\mu)^{(Q-\sum_{i=1}^{N}\alpha_i)/b} F_{\alpha_1\ldots\alpha_N}(x_1,\ldots,x_N). \tag{32}$$

Here $F_{\alpha_1\ldots\alpha_N}(x_1,\ldots,x_N)$ is independent of μ.

To determine the μ dependence of $\bar{B} = \langle B\rangle$, we consider the field equation of the Liouville theory as follows:

$$\bar{B} = \langle B\rangle = 4\pi b\mu\langle \mathrm{e}^{2b\phi}\rangle. \tag{33}$$

According to eq. (32), $\bar{B}$ rescales with μ as

$$\bar{B} \sim \mu^{1+\frac{1}{b^2}}. \tag{34}$$

Using the fact that μ has dimension $\mu \sim [\text{length}]^{-(2+2b^2)}$ we find

$$\bar{B} = \langle B\rangle \approx \xi^{-2\frac{(1+b^2)^2}{b^2}}, \tag{35}$$

where ξ is the localization length. We could conclude that the localization length scales with the energy as

$$\xi \sim E^{-1/Q^2}, \tag{36}$$

where Q is given in eq. (29).

It is noted that the exponent of E is invariant when $b \to 1/b$, which means that the theory is self-dual. This duality is the reflection of duality of Liouville theory when $b \to 1/b$. To see this, we recall the partition function of the Liouville theory. According to ref. [35], the partition function has the following form:

$$Z = -\frac{\mu}{\sqrt{2}\pi^2(b+1/b)} \left(\frac{\pi\mu\Gamma(b^2)}{\Gamma(1-b^2)}\right)^{1/b^2} \frac{\Gamma(-1/b^2)}{\Gamma(1/b^2-1)}, \tag{37}$$

under the following transformation of b and μ:

$$b \to \frac{1}{b}, \quad \mu \to \bar{\mu} = \frac{1}{\pi\gamma(1/b^2)}(\pi\mu\gamma(b^2))^{1/b^2}, \tag{38}$$

where $\gamma(x)$ (x is $1/b^2$ or b^2) is given by $\gamma(x) = \Gamma(x)/\Gamma(1-x)$. It is easy to show that the partition function transforms as

$$Z(b,\mu) \Rightarrow Z\left(\frac{1}{b},\bar{\mu}\right) = -\frac{1}{b^2}Z(b,\mu). \tag{39}$$

This duality transformation was observed first by Zamolodchikov *et al* in the exact expression for the three-point functions of the Liouville exponential fields [34]. From eq. (37), we observe that there exists the following sequence of critical values for b, so that the partition function becomes singular:

$$b_N^2 = \frac{b_c^2}{N}, \tag{40}$$

where $b_c = 1$. The singularity of the partition function for b_N is the signal of special type of phase transition. We recall that when $b = 1$ the total fluxes through the system reaches the value 2 which corresponds to the first change in the ground state degeneracy [8].

Let us now calculate the distribution function $p(t_0)$ for t_0's such that

$$\langle t_0^n \rangle = \int_0^\infty t_0^n p(t_0)\mathrm{d}t_0, \tag{41}$$

where t_0 is given by $t_0 = |\psi_0|^2$. One can show that $p(t_0)$ has the following form:

$$p(t_0) = f(t_0)\mathrm{e}^{-\frac{1}{\log\xi^a}\log^2(t_0\xi^d)}, \tag{42}$$

in which $f(t_0)$ is a smooth function of $\log(t_0)$ and $a = 16b^6, d = 2b(1+4b^2)$. This distribution is the famous log-normal distribution which is considered as a characteristic feature of disorder system. It appears that for $t_0 \sim (1/\xi^{2b(1+4b^2)})$ the chi-square distribution functions with its variance controlled by ξ are obtained.

3. Summary

Summarizing, we use the supersymmetric quantum mechanics to construct the excited state wave functions of the FRGP model. We show that in the quasiclassical approximation of the Liouville field theory the localization length ξ scales with the energy E as $\xi \sim E^{-b^2/(1+b^2)^2}$, where b is the strength of the disorder. The dependence of the exponents on b is consistent with the results of ref. [29]. The self-duality of the theory is demonstrated as a consequence of the self-duality of the Liouville theory. It is shown that the distribution function $p(t_0)$ is the log-normal distribution function which depends on the localization length and strength of disorder. We find that in small t_0, $p(t_0)$ behaves as a chi-square distribution.

Appendix A

Let us try to find the raising and lowering operators of the FRGP model as D and D^*. D and D^* scale to $B_0^{1/2}$. $B(z,\bar{z})$ scales to B_0 that B_0 is the typical magnetic field in the localization length. In our calculations, $\tilde{B}$ is defined as $\tilde{B} = B(z,\bar{z})/B_0$ that in the case of homogeneous magnetic field, $\tilde{B} = 1$. The ladder operators of the FRGP model are as follows:

$$\begin{aligned}(D^* - g_n)\psi_n &= \beta_n\psi_{n+1},\\ (D - f_n)\psi_n &= \alpha_n\psi_{n-1},\end{aligned} \tag{A1}$$

in which f_n and g_n are in terms of z and $\bar{z}$.

f_n, g_n, β_n and α_n are determined in the limit of the homogeneous magnetic field, i.e. $\tilde{B} \to 1$. Also the ψ_0 wave function must be such that $D\psi_0 = 0$. Then we can write

$$g_n(z,\bar{z}) = g(z,\bar{z}), \quad f_n(z,\bar{z}) = 0, \quad \alpha_n = n, \quad \beta_n = 2. \tag{A2}$$

g satisfies the following relation:

$$Dg + 2 = 2\tilde{B}. \tag{A3}$$

By solving the above relation, g is equal to

$$g = i\mathrm{e}^{-\varphi}\int \mathrm{d}\bar{z}(\tilde{B} - 1)\mathrm{e}^{\varphi}. \tag{A4}$$

in which $B(z,\bar{z}) = 4\partial_{\bar{z}}\partial_z\varphi = \nabla^2\varphi$ in the term $\tilde{B} = B(z,\bar{z})/B_0$.

We find that D in the presence of the random magnetic field is a lowering operator and the raising operator shifts by g. We could show that

$$(D^* - g)D\psi_n = 2n\psi_n, \tag{A5}$$

$$D(D^* - g)\psi_n = 2(n+1)\psi_n, \tag{A6}$$

where $n = 0, 1, 2, 3, \ldots$ and ψ_n, in terms of ψ_0, is as follows:

$$\psi_n = \frac{1}{2^n}(D^* - g)^n\psi_0. \tag{A7}$$

References

[1] P W Anderson, *Phys. Rev.* **109**, 1492 (1958)
[2] D Thouless, *Phys. Rev. Lett.* **39**, 1792 (1972)
[3] M Kaveh and N Mott, *J. Phys.* **C15**, L697 (1982)
[4] B Kramer and A MacKinnon, *Rep. Prog. Phys.* **56**, 1469 (1993)
[5] M Turek and W John, *Physica* **E18**, 530 (2003)
[6] Rudolf A Romer *et al*, *Physica* **E9**, 397 (2001)
[7] Klaus Frahm *et al*, *Z. Phys.* **B102**, 261 (1997)
[8] Ian I Kogan, C Mudry and A M Tsvelik, *Phys. Rev. Lett.* **77**, 707 (1996)
[9] A Eilmes and R A Romer, *Phys. Status Solidi* **B241**, 2079 (2004)
[10] J C Flores, *Phys. Rev.* **B62**, 33 (2000)
[11] M Groß, J Bosse and H Gobriel, *Ann. der Phys.* **505**, 547 (2006)
[12] M B Hastings and S L Sondhi, *Phys. Rev.* **B64**, 094204 (2001)
[13] K B Efetov, *Adv. Phys.* **32**, 53 (1983)
[14] S Chakravarty and A Schmid, *Phys. Rep.* **140**, 193 (1986)
[15] B L Altshuler, V E Kravtsov and I V Lerner, in: *Mesoscopic phenomena in solids* (Elsevier, Amesterdam, 1991)
[16] B A Muzykantskii and D E Khmelnitskii, *Phys. Rev.* **B51**, 5480 (1995)
[17] H E Castillo, C C Chamon, E Fradkin, P M Goldbart and C Mudry, *Phys. Rev.* **B56**, 10668 (1997)
[18] R A Jalabert, A D Stone and Y Alhassid, *Phys. Rev. Lett.* **68**, 3468 (1992)
[19] V I Fal'ko and K B Efetov, *Europhys. Lett.* **32**, 627 (1995); *Phys. Rev.* **B52**, 17413 (1995)
[20] T A Broody *et al*, *Rev. Mod. Phys.* **53**, 385 (1981)
[21] E Fradkin, *Phys. Rev.* **B33**, 3257 (1986)
[22] G W Semenoff, *Phys. Rev. Lett.* **53**, 2449 (1984)
[23] K S Novoselov *et al*, *Nature (London)* **438**, 197 (2005)
[24] M P A Fisher and E Fradkin, *Nucl. Phys.* **B251**, 457 (1985)
[25] A A Nersesyan, A M Tsvelik and F Wegner, *Nucl. Phys.* **B438**, 561 (1995)
[26] D V Khveshchenko, A G Yashenkin and I V Gornyi, *Phys. Rev. Lett.* **86**, 4668 (2001)
[27] I Ichinose, *Mod. Phys. Lett.* **A17**, 1355 (2002)
[28] R Shankar, *Phys. Rev. Lett.* **58**, 2466 (1987)
[29] Y Morita and Y Hatsugai, *Phys. Rev. Lett.* **79**, 3728 (1997)
[30] B Thaller, in: *The Dirac equation* (Springer-Verlag, 1992)
[31] C Murdry, C Chamon and X-G Wen, *Nucl. Phys.* **B466**, 383 (1996)
[32] A W W Ludwig, M P A Fisher, R Shankar and G Grinstein, *Phys. Rev.* **B50**, 7526 (1994)
[33] A M Polyakov, *Phys. Lett.* **B103**, 207 (1981)
A M Polyakov, *Gauge fields and strings* (Harwood Academic, Chur, Switzerland, 1987)
[34] A B Zamolodchikov and Al B Zamolodchikov, *Nucl. Phys.* **B477**, 577 (1996)
[35] H Dorn and H J Otto, *Phys. Lett.* **B291**, 39 (1992); *Nucl. Phys.* **B429**, 375 (1994)
[36] V Knizhnic, A Polyakov and A Zamolodchikov, *Mod. Phys. Lett.* **A3**, 819 (1988)
[37] F David, *Mod. Phys. Lett.* **A3**, 1651 (1988)
[38] J Distler and H Kawai, *Nucl. Phys.* **B321**, 509 (1989)