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Kadomstev–Petviashvili (KP) equation in warm dusty plasma with variable dust charge, two-temperature ion and nonthermal electron

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Abstract. In this work, the propagation of nonlinear waves in warm dusty plasmas with variable dust charge, two-temperature ion and nonthermal electron is studied. By using the reductive perturbation theory, the Kadomstev–Petviashvili (KP) equation is derived. The energy of the soliton and the linear dispersion relation are obtained. The effects of variable dust charge on the energy of soliton and the angular frequency of linear wave are also discussed.

Keywords. Dust; soliton; nonthermal.

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1. Introduction

The study of dusty plasmas represents one of the most rapidly growing fields of plasma physics. Usually, the dust grains are of micrometer or sub-micrometer size and their masses are very large. Rao *et al* [1] have theoretically reported the existence of dust acoustic waves (DAW) of low frequency in unmagnetized dusty plasma. Experimental observations have confirmed the linear and nonlinear features of both dust acoustic waves (DAW) and dust ion acoustic waves (DIAW) [2–4]. Barkan *et al* [5] have done laboratory observations of dust acoustic waves and compared them with available theories. Shukla and Silin [6] have shown the existence of DIAW at higher frequency. Mamun *et al* [7] have also derived rarefactive solitary waves in low-temperature dusty plasmas such as those in laboratory and astrophysical environments. Tagare [8] has studied the small-amplitude dust acoustic solitary waves in the dusty plasma consisting of cold dust particles and two-temperature isothermal ions. The charging process of dust particles is an important effect which has been investigated in [9,10]. The existence of dust particles with variable charge in plasma changes the behaviour of linear and nonlinear waves [11]. The small and large amplitudes of DAWs in a dusty plasma with variable dust charge and two-temperature ions has been investigated in [12]. Recently, the propagation of nonlinear DAWs in the dusty plasma consisting of a mixture of variation charged dust particles, isothermal electrons and two-temperature isothermal ions was studied by El-Labany *et al* [13]. They examined a necessary condition that must be satisfied to achieve the validity of the assumption of two-temperature ions for Saturn's F-ring, which is considered as an application of their model. Moreover, the study of the dusty plasma media are very attractive because of their theoretical features and also their applications which have been observed in the Earth's magnetosphere, cometary tail, planetary rings and so on [14–16]. In most investigations, reductive perturbation method has been used for deriving the Korteweg–de Vries (KdV), Zakharov–Kuznetsov (ZK) and Kadomstev–Petviashvili (KP) equations. The KP equation has been derived for dust acoustic wave in hot dusty plasma by Duan [17]. The KP, the modified KP and the coupled KP equations for dusty plasma with two ions have been obtained in [18]. Also, Gill *et al* [19] have derived KP equation for dusty plasma with variable dust charge and two-temperature ions. In most practical dusty plasmas, a gas flow which is usually introduced can charge quickly, while keeping relatively low temperature. Recently, Cairns *et al* [20] have considered a nonthermal plasma model and showed that the presence of a nonthermal distribution of electrons may change the nature of ion acoustic solitary structures and allow the existence of structures very similar to those observed by the Freja and Viking satellites [21]. So it is so much important to examine the validity of two-temperature ions and nonthermal electrons assumption in charge varying dusty plasmas. In this paper, the warm dusty plasma with the variable dust charge, two-temperature ions and nonthermal electrons has been considered. Our investigations can be useful in understanding the behaviour of DAW in space and astrophysical plasma environments that clearly indicate the presence of nonthermal electron population and two-temperature ions. In §2, the basic set of equations is introduced and in §3 by using the reductive perturbation method (RPM) the KP equation has been derived. Section 4 contains discussion on the energy of soliton. The linear dispersion relation and effects of variable dust charge on this relation has also been discussed in this section. Conclusions and remarks are given in §5.

2. Basic equations

We consider the propagation of dust acoustic waves in collisionless, unmagnetized warm dusty plasma consisting of electrons, two-temperature ions and high negatively charged dust grains. Total charge neutrality at equilibrium requires that

$$n_{0\mathrm{e}} + n_{0\mathrm{d}} Z_{0\mathrm{d}} = n_{0\mathrm{il}} + n_{0\mathrm{ih}}, \tag{1}$$

where $n_{0\mathrm{e}}$, $n_{0\mathrm{d}}$, $n_{0\mathrm{il}}$ and $n_{0\mathrm{ih}}$ are the equilibrium values of electrons, dust, lower temperature ions and higher temperature ion number densities respectively. $Z_{0\mathrm{d}}$ is the unperturbed number of charges on the dust particles. The following set of normalized two-dimensional equations of continuity, motion for the adiabatic dust and Poisson describe the dynamics of a dust acoustic wave in such a plasma:

$$\frac{\partial n_\mathrm{d}}{\partial t} + \frac{\partial}{\partial x}(n_\mathrm{d} u_\mathrm{d}) + \frac{\partial}{\partial y}(n_\mathrm{d} \nu_\mathrm{d}) = 0$$

$$\frac{\partial u_{\rm d}}{\partial t}+u_{\rm d}\frac{\partial u_{\rm d}}{\partial x}+\nu_{\rm d}\frac{\partial u_{\rm d}}{\partial y}+\frac{\sigma_{\rm d}}{n_{\rm d}}\frac{\partial P_{\rm d}}{\partial x}=Z_{\rm d}\frac{\partial \phi}{\partial x}$$
$$\frac{\partial v_{\rm d}}{\partial t}+u_{\rm d}\frac{\partial \nu_{\rm d}}{\partial x}+\nu_{\rm d}\frac{\partial \nu_{\rm d}}{\partial y}+\frac{\sigma_{\rm d}}{n_{\rm d}}\frac{\partial P_{\rm d}}{\partial y}=Z_{\rm d}\frac{\partial \phi}{\partial y}$$
$$\frac{\partial p_{\rm d}}{\partial t}+u_{\rm d}\frac{\partial p_{\rm d}}{\partial x}+\nu_{\rm d}\frac{\partial P_{\rm d}}{\partial y}+3P_{\rm d}\left(\frac{\partial u_{\rm d}}{\partial x}+\frac{\partial v_{\rm d}}{\partial y}\right)=0$$
$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}=Z_{\rm d}n_{\rm d}+n_e-n_{\rm il}-n_{\rm ih}. \tag{2}$$

$u_{\rm d}$ and $\nu_{\rm d}$ are velocity components of the dust particles in x- and y-directions and are normalized by the effective dust acoustic speed $C_{\rm d}=\sqrt{Z_{\rm 0d}T_{\rm eff}/m_{\rm d}}$. $P_{\rm d}$ and ϕ are the pressure of the dust particles and electrostatic potential respectively and they are normalized by $Z_{\rm d}n_{\rm 0d}T_{\rm d}$ and $T_{\rm eff}/e$, respectively. Here, $T_{\rm eff}=\left[\frac{1}{Z_{\rm 0d}n_{\rm 0d}}\left(\frac{n_{\rm 0e}}{T_{\rm e}}+\frac{n_{\rm 0il}}{T_{\rm il}}+\frac{n_{\rm 0ih}}{T_{\rm ih}}\right)\right]^{-1}$ is the effective temperature where $T_{\rm d}, T_{\rm e}, T_{\rm il}, T_{\rm ih}$ are the temperature of dust, electron and low-temperature and high-temperature ions. $n_{\rm d}$ and $Z_{\rm d}$ are the dust number density and the variable charge number of dust grains and they are normalized by $n_{\rm 0d}$ and $Z_{\rm 0d}$, respectively. The time and space variables are normalized by the dust plasma period $\omega_{\rm pd}^{-1}=\sqrt{m_{\rm d}/4\pi n_{\rm 0d}Z_{\rm 0d}^2e^2}$ and the Debye length $\lambda_{\rm D}=\sqrt{T_{\rm eff}/4\pi Z_{\rm 0d}n_{\rm 0d}e^2}$, respectively. Electrons and ions are assumed to be distributed with nonthermal and Maxwell–Boltzmann distribution functions, respectively. So the related dimensionless number densities for electrons ($n_{\rm e}$), low-temperature ions ($n_{\rm il}$) and high-temperature ions ($n_{\rm ih}$) are

$$n_{\rm e}=\frac{1}{\delta_1+\delta_2-1}\left[1-\frac{4\alpha}{1+3\alpha}\beta_1 s\phi+\frac{4\alpha}{1+3\alpha}(\beta_1 s\phi)^2\right]\exp(\beta_1 s\phi),$$
$$n_{\rm il}=\frac{\delta_1}{\delta_1+\delta_2-1}\exp(-s\phi),$$
$$n_{\rm ih}=\frac{\delta_2}{\delta_1+\delta_2-1}\exp(-\beta s\phi), \tag{3}$$

where

$$\beta_1=\frac{T_{\rm il}}{T_{\rm e}},\quad \beta_2=\frac{T_{\rm ih}}{T_{\rm e}},\quad \beta=\frac{\beta_1}{\beta_2}=\frac{T_{\rm il}}{T_{\rm ih}},\quad s=\frac{T_{\rm eff}}{T_{\rm il}}=\frac{\delta_1+\delta_2-1}{\delta_1+\delta_2\beta+\beta_1},$$
$$\delta_1=\frac{n_{\rm 0il}}{n_{\rm 0e}},\quad \delta_2=\frac{n_{\rm 0ih}}{n_{\rm 0e}},\quad \sigma=\frac{T_{\rm d}}{T_{\rm eff}}, \tag{4}$$

where α is the nonthermal parameter which determines the number of fast (nonthermal) electrons. From (1) it follows that

$$\delta_1+\delta_2-1\geq 0. \tag{5}$$

The dust charge variable $Q_{\rm d}$ is obtained from the charge-current balance equation [22]

$$\left(\frac{\partial}{\partial t}+\vec{V}\cdot\vec{\nabla}\right)Q_{\rm d}=I_{\rm e}+I_{\rm il}+I_{\rm ih}, \tag{6}$$

where $\vec{V} = (u_{\rm d}, v_{\rm d})$ and $I_{\rm e}$, $I_{\rm il}$ and $I_{\rm ih}$ are the electron and ion (low and high temperature) currents. Notice that the characteristic time for dust motion is around 10^{-3} s [23] while the dust charging time is typically about 10^{-9} s [24]. Therefore, the dust charge reaches their equilibrium position quickly. Thus $({\rm d}Q_{\rm d}/{\rm d}t) \ll I_{\rm e}, I_{\rm il}, I_{\rm ih}$ and charge-current balance equation (6) reads [25] as

$$I_{\rm e} + I_{\rm il} + I_{\rm ih} \approx 0. \tag{7}$$

The electron and ion currents are [26]

$$\begin{aligned} I_{\rm e} &= -e\pi r^2 \left(\frac{8T_{\rm e}}{\pi m_e}\right)^{1/2} n_{\rm e} \exp\left(\frac{e\Phi}{T_{\rm e}}\right), \\ I_{\rm il} &= e\pi r^2 \left(\frac{8T_{\rm il}}{\pi m_{\rm i}}\right)^{1/2} n_{\rm il} \left(1 - \frac{e\Phi}{T_{\rm il}}\right), \\ I_{\rm ih} &= e\pi r^2 \left(\frac{8T_{\rm ih}}{\pi m_{\rm i}}\right)^{1/2} n_{\rm ih} \left(1 - \frac{e\Phi}{T_{\rm ih}}\right), \end{aligned} \tag{8}$$

where Φ denotes the dust grain surface potential relative to the plasma potential ϕ. If the thermal velocities of electrons and ions are larger than their streaming velocities then we can write eq. (6) at equilibrium as

$$\begin{aligned} &\sqrt{\frac{\beta_1}{\mu_{\rm i}}}\delta_1(1 - s\psi)\exp(-s_\phi) + \sqrt{\frac{\beta_2}{\mu_{\rm i}}}\delta_2(1 - \beta s\psi)\exp(-\beta s\phi) \\ &\quad - \left[1 + \frac{4\alpha}{1+3\alpha}(\beta_1 s\phi + \beta_1^2 s^2\phi^2)\right]\exp[\beta_1 s(\phi + \psi)] = 0 \end{aligned} \tag{9}$$

$$\psi = \frac{e\Phi}{T_{\rm eff}} \quad \text{and} \quad \mu_i = \frac{m_{\rm i}}{m_e} \cong 1840.$$

The dust charge $Q_{\rm d} = c\Phi$ is calculated by using (7) in which c is the capacitance of dust grains. $Z_{\rm d}$ is defined as

$$Z_{\rm d} = \frac{\psi}{\psi_0}, \tag{10}$$

where $\psi_0 = \psi(\phi = 0)$ is the dust surface floating potential with respect to the unperturbed plasma potential at an infinite region. By substituting $\phi = 0$ into (9) we have

$$b_1\delta_1(1 - s\psi_0) + b_2\delta_2(1 - \beta s\psi_0) = \exp(\beta_1 s\psi_0), \tag{11}$$

where

$$b_1 = \sqrt{\beta_1/\mu_{\rm i}} \quad \text{and} \quad b_2 = \sqrt{\beta_2/\mu_{\rm i}}.$$

$Z_{\rm d}$ can be expanded with respect to ϕ as follows:

$$Z_{\rm d} = 1 + \gamma_1\phi + \gamma_2\phi^2 + \gamma_3\phi^3 + \cdots, \tag{12}$$

where $\gamma_1 \equiv \psi_0'/\psi_0$, $\gamma_2 \equiv \psi_0''/2\psi_0$ and $\gamma_3 \equiv \psi_0'''/6\psi_0$ come from expanding ψ near ψ_0 so that we obtain

$$\begin{aligned}
\psi_0' &= \frac{(b_1\delta_1 + b_2\delta_2\beta^2 s)\psi_0 - [b_1\delta_1 + b_2\delta_2\beta - (1+G)\beta_1 \exp(\beta_1 s\psi_0)]}{\delta_1 b_1 + \delta_2 b_2 \beta + \beta_1 \exp(\beta_1 s\psi_0)}, \\
\psi_0'' &= \frac{[2s(\delta_1 b_1 + \delta_2 b_2\beta^2) - \beta_1(1+sG)\exp(\beta_1 s\psi_0)]\psi_0' - s[b_1\delta_1 + \delta_2 b_2\beta^3 s]\psi_0}{\delta_1 b_1 + \delta_2 b_2 \beta + \beta \exp(\beta_1 s\psi_0)} \\
&\quad + \frac{s(\delta_1 b_1 s + \delta_2 b_2\beta^2) - (1+3\beta_1 sG)\beta_1 \exp(\beta_1 s\psi_0)}{\delta_1 b_1 + \delta_2 b_2 \beta + \beta_1 \exp(\beta_1 s\psi_0)}, \\
\psi_0''' &= \frac{3s\delta_1 b_1 + \delta_2 b_2\beta(1+2\beta s) - \beta_1[\beta_1 s(1+\psi_0) + 1 + 2\beta_1 sG]\exp(\beta_1 s\psi_0)}{\delta_1 b_1 + \delta_2 b_2 \beta + \beta_1 \exp(\beta_1 s\psi_0)} \\
&\quad - \frac{3\delta_1 b_1 s^2 + (\beta s + 2\delta_2 b_2)\beta^2 s - [\beta_1 Gs(3+\psi_0) - (1+\psi_0)]\beta_1^2 s \exp(\beta_1 s\psi_0)}{\delta_1 b_1 + \delta_2 b_2 \beta + \beta_1 \exp(\beta_1 s\psi_0)} \\
&\quad - \frac{\begin{array}{r}\delta_1 b_1 s^2(1 - s\psi_0) + \delta_2 b_2\beta^2 s(1-\beta s\psi_0) - [(1+3G\beta_1 s)(1+\psi_0) \\ +4\beta_1 Gs]\beta_1^2 s \exp(\beta_1 s\psi_0)\end{array}}{\delta_1 b_1 + \delta_2 b_2 \beta + \beta_1 \exp(\beta_1 s\psi_0)},
\end{aligned} \tag{13}$$

where

$$G = \frac{4\alpha}{1+3\alpha}. \tag{14}$$

3. The derivation of KP equation

According to the general method of reductive perturbation theory, we choose the independent variables as

$$\xi = \varepsilon(x - \lambda t), \quad \tau = \varepsilon^3 t, \quad \eta = \varepsilon^2 y \tag{15}$$

ε is a small dimensionless expansion parameter which characterizes the strength of nonlinearity in the system and λ is the phase velocity of the wave along the x direction. We can expand physical quantities which have been appeared in (2), in terms of the expansion parameter ε as

$$\begin{aligned}
&n_{\mathrm{d}} = 1 + \varepsilon^2 n_{1\mathrm{d}} + \varepsilon^4 n_{2\mathrm{d}} + \cdots, \\
&u_{\mathrm{d}} = \varepsilon^2 u_{1\mathrm{d}} + \varepsilon^4 u_{2\mathrm{d}} + \cdots, \\
&\nu_{\mathrm{d}} = \varepsilon^3 \nu_{1\mathrm{d}} + \varepsilon^5 \nu_{2\mathrm{d}} + \cdots, \\
&\phi = \varepsilon^2 \phi_1 + \varepsilon^4 \phi_2 + \cdots, \\
&P_{\mathrm{d}} = 1 + \varepsilon^2 P_{1\mathrm{d}} + \varepsilon^4 P_{2\mathrm{d}} + \cdots \\
&Z_{\mathrm{d}} = 1 + \varepsilon^2 Z_{1\mathrm{d}} + \varepsilon^4 Z_{2\mathrm{d}} + \cdots.
\end{aligned} \tag{16}$$

Substituting (15) and (16) into eq. (2) and collecting terms with same powers of ε, from the coefficients of lowest order we have

$$n_{1\mathrm{d}}=\frac{\phi_1}{3\sigma-\lambda^2},\quad n_{1\mathrm{d}}=\frac{u_{1\mathrm{d}}}{\lambda},$$
$$\lambda^2=3\sigma+\left[1+\gamma_1-\frac{4\alpha\beta_1}{(1+3\alpha)(\delta_1+\delta_2\beta+\beta_1)}\right]^{-1},$$
$$P_{1\mathrm{d}}=\frac{3}{\lambda}u_{1\mathrm{d}},\quad \frac{\partial\nu_{1\mathrm{d}}}{\partial\xi}=-\frac{1}{\nu_0}\frac{\partial\phi_1}{\partial\eta} \tag{17}$$

and for higher orders of ε

$$-\lambda\frac{\partial n_{2\mathrm{d}}}{\partial\xi}+\frac{\partial n_{1\mathrm{d}}}{\partial\tau}+\frac{\partial(u_{2\mathrm{d}}+n_{1\mathrm{d}}u_{1\mathrm{d}})}{\partial\xi}+\frac{\partial\nu_{1\mathrm{d}}}{\partial\eta}=0$$
$$-\lambda\frac{\partial u_{2\mathrm{d}}}{\partial\xi}+\frac{\partial u_{1\mathrm{d}}}{\partial\tau}+u_{1\mathrm{d}}\frac{\partial u_{1\mathrm{d}}}{\partial\xi}+\sigma\frac{\partial P_{2\mathrm{d}}}{\partial\xi}-\sigma n_{1\mathrm{d}}\frac{\partial P_{1\mathrm{d}}}{\partial\xi}=\gamma_1\phi_1\frac{\partial\phi_1}{\partial\xi}+\frac{\partial\phi_2}{\partial\xi}$$
$$-\lambda\frac{\partial v_{1\mathrm{d}}}{\partial\xi}+\sigma\frac{\partial P_{1\mathrm{d}}}{\partial\eta}=\frac{\partial\phi_1}{\partial\eta}$$
$$-\lambda\frac{\partial P_{2\mathrm{d}}}{\partial\xi}+u_{1\mathrm{d}}\frac{\partial P_{1\mathrm{d}}}{\partial\xi}+3P_1\frac{\partial u_{1\mathrm{d}}}{\partial\xi}+3\frac{\partial v_{1\mathrm{d}}}{\partial\eta}+3\frac{\partial u_{2\mathrm{d}}}{\partial\xi}+\frac{\partial P_{1\mathrm{d}}}{\partial\tau}=0$$
$$\frac{\partial^2\phi_1}{\partial\xi^2}=\gamma_1 n_{1\mathrm{d}}\phi_1+n_{2\mathrm{d}}+\left[1+\gamma_1-\frac{4\alpha\beta_1}{(1+3\alpha)(\delta_1+\delta_2\beta+\beta_1)}\right]\phi_2$$
$$+\left[\gamma_2-\frac{1}{2}\frac{(\delta_1+\delta_2-1)^2(\delta_1+\delta_2\beta^2-\beta_1^2)}{(\delta_1+\delta_2\beta+\beta_1)^2}\right]\phi_1^2. \tag{18}$$

The KP equation is derived from the above equations.

$$\frac{\partial}{\partial\xi}\left[\frac{\partial\phi_1}{\partial\tau}+A\phi_1\frac{\partial\phi_1}{\partial\xi}+B\frac{\partial^3\phi_1}{\partial\xi^3}\right]+C\frac{\partial^2\phi_1}{\partial\eta^2}=0, \tag{19}$$

where

$$A=\frac{1}{2\lambda}\left\{-2+(\lambda^2-3\sigma)^2\left[(\delta_1+\delta_2\beta^2-\beta_1^2)\frac{(\delta_1+\delta_2-1)}{(\delta_1+\delta_2\beta+\beta_1)^2}-2\gamma_2\right]\right.$$
$$\left.+3\gamma_1(\lambda^2-3\sigma)-\frac{\lambda^2+9\sigma}{\lambda^2-3\sigma}\right\}$$
$$B=\frac{1}{2\lambda}(\lambda^2-3\sigma)^2,\quad C=\frac{\lambda}{2}. \tag{20}$$

If the charge of the dust particles was constant ($\gamma_1=\gamma_2=0$),

$$A=\frac{1}{2\lambda}\left\{-3+(\lambda^2-3\sigma)^2\left[(\delta_1+\delta_2\beta^2-\beta_1^2)\frac{(\delta_1+\delta_2-1)}{(\delta_1+\delta_2\beta+\beta_1)^2}\right]\right.$$
$$\left.-\frac{12\sigma}{\lambda^2-3\sigma}\right\} \tag{21}$$

where

$$\lambda^2=3\sigma+\left[1-\frac{4\alpha\beta_1}{(1+3\alpha)(\delta_1+\delta_2\beta+\beta_1)}\right]^{-1}.$$

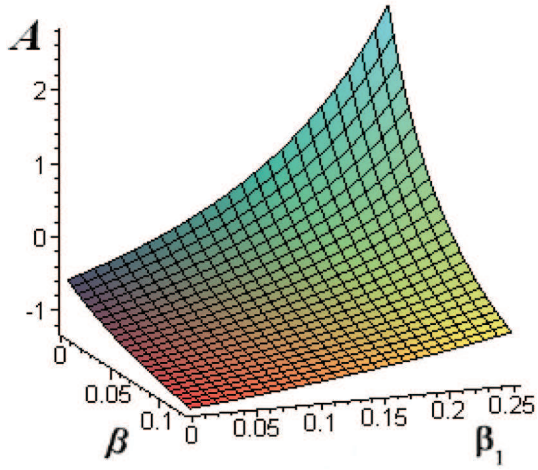

Figure 1. The parameter A as a function of β and β_1 for $\alpha = 0.0125$, $\sigma = 0.1$, $\delta_1 = 0.5$, $\delta_2 = 2$, $\gamma_1 = 0.02$ and $\gamma_2 = 0.01$.

The KP equation has many applications in the plasma and theoretical physics. Duan *et al* [27] have studied resonance of the KP equation, theoretically. Exact solutions of the KP equation have been derived by Zhang [28]. The stationary solution of (19) can be written as [19]

$$\phi_1 = \phi_0 \operatorname{sech}^2\left(\frac{\xi + \eta - U\tau}{W}\right), \quad \phi_0 = \frac{3(U - C)}{A}, \quad W = 2\sqrt{\frac{B}{U - C}}. \tag{22}$$

ϕ_0 and W are amplitude and width of soliton, respectively.

Solitonic solutions of (19) can be compared with other authors' results. The above results are reduced to [19] for Maxwell distributed electrons ($\alpha = 0$) and cold plasma ($\sigma = 0$). The above-mentioned equations for the warm plasma with one ion and without fast electron ($\alpha = 0$), when $\gamma_1 = \gamma_2 = 0$, agree with those of Duan [17]. Also, the above results are comparable with the results of Xie *et al* in [12].

Figure 1 shows the strength of nonlinear term of the KP equation with different values of the plasma parameters. In figure 1, A is plotted as a function of β and β_1 when $\alpha = 0.0125$, $\sigma = 0.1$, $\delta_1 = 0.5$, $\delta_2 = 2$, $\gamma_1 = 0.02$ and $\gamma_2 = 0.01$. This figure clearly shows that A can be positive or negative for different values of parameters.

4. Energy of soliton and linear dispersion relation

The soliton energy can be obtained from the following equation [29]:

$$E = \int_{-\infty}^{+\infty} u_{1\mathrm{d}}^2 \, \mathrm{d}\xi. \tag{23}$$

After the integration, we obtain [29]

$$E = \frac{4}{3}u_m^2 W = \frac{24\lambda^2}{(3\sigma-\lambda^2)^2}\frac{(u-c)^2}{A^2}\sqrt{\frac{B}{u-c}}. \tag{24}$$

Figure 2 indicates that the energy of soliton increases by increasing γ_1. This figure also shows that the energy of the soliton decreases when the nonthermal parameter decreases.

Linear dispersion relation can be obtained as follows. According to the standard normal-mode analysis, by linearization of dependent variables n_d, ϕ and Z_d in terms of their equilibrium and perturbed parts [30,31], we have

$$n_\mathrm{d} = 1 + n_{1\mathrm{d}}, \quad \phi = \phi_1, \quad u_\mathrm{d} = u_{1\mathrm{d}}, \quad Z_\mathrm{d} = 1 + Z_{1\mathrm{d}} = 1 + \gamma_1\phi_1. \tag{25}$$

Then, we may assume that all the perturbed quantities are proportional to $\mathrm{e}^{i(kx-\omega t)}$ with k as the wave propagation constant in the direction of x-axis. So we have $\frac{\partial}{\partial t} = -i\omega$, $\frac{\partial}{\partial x} = ik$. Substituting (25) into (2) and using their linear terms one obtains linear dispersion relation as

$$\omega^2 = \frac{k^2}{k^2+H} + 3k^2\sigma, \tag{26}$$

where

$$H = 1 + \gamma_1 - \frac{G\beta_1}{\delta_1 + \delta_2\beta + \beta_1}. \tag{27}$$

Figure 3 shows the angular frequency (ω) as a function of k for $\gamma_1 = 0$, 0.2, 0.4 and $\gamma_2 = 0$. Figure 3 indicates that increasing k (γ_1) leads to increasing (decreasing)

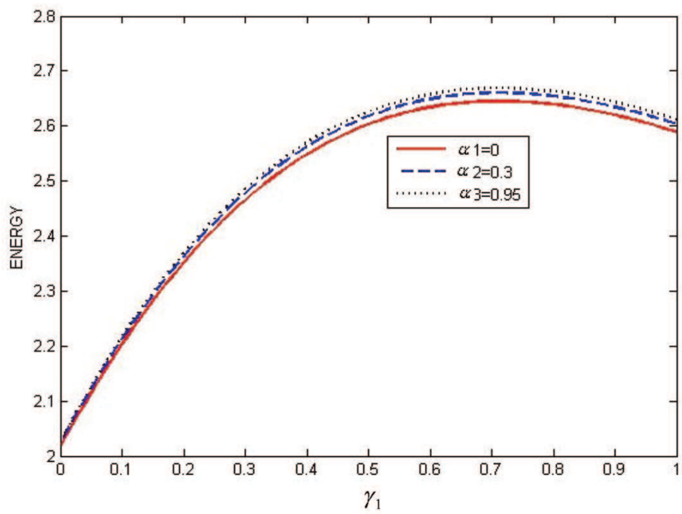

Figure 2. Energy of the soliton as a function of γ_1 for $\delta_1 = 0.8$, $\delta_2 = 0.6$, $\sigma = 0.1$, $\beta = 0.4$, $\beta_1 = 0.01$, $\gamma_2 = 0$, $u = 1$ and different values of α.

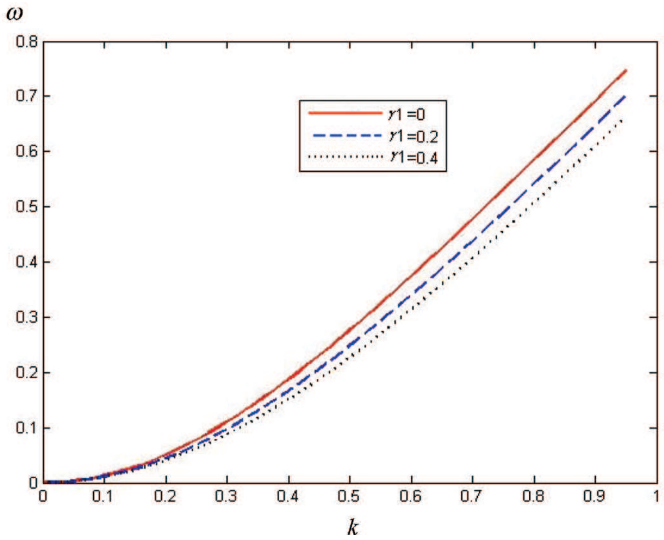

Figure 3. The angular frequency with respect to k for $\alpha = 0.2$, $\delta_1 = 0.8$, $\delta_2 = 0.6$, $\sigma = 0.1$, $\beta = 0.4$, $\beta_1 = 0.01$, $\gamma_1 = 0$, 0.2, 0.4, $\gamma_2 = 0$, $u = 1$.

values for ω. For real values of ω, all perturbation variables oscillate harmonically and if any or all ω's have positive imaginary parts, then the system is unstable since those normal modes will grow in time [31,32].

5. Conclusion and remarks

The KP equation was obtained in unmagnetized dusty plasma with variable dust charge, two-temperature ions and nonthermal distributed electrons. In this equation, nonthermal parameter, relative densities, relative temperatures and charge of dust particles (γ_1 and γ_2) appear. The coefficient of nonlinear term, A, can be positive or negative. This means that both rarefactive and compressive solitons can be appeared in the plasma. Analytically, the coefficients of the dispersive terms, B and C, depend on the parameter γ_1. Indeed, dispersion decreases when γ_1 is increased. On the other hand, A is a function of γ_1 and γ_2. Therefore, it is possible that the competition between the nonlinear and dispersion terms leads to the formation of a soliton. The energy of soliton and linear dispersion relation have been derived and discussed too. The energy of solitons increases when the dust charge increases. We showed that increasing the charge of the dust grains leads to decreasing values of ω and also increasing values of the soliton energy. Since the parameter A can be positive or negative, it can also be zero. But a solitonic solution cannot be established when A is zero and therefore A has critical values. In this situation we can obtain modified KP equation, and solitonic solution of this equation has finite amplitude.

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