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Chameleon field and the late time acceleration of the Universe

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Abstract. In the present work, it is shown that a chameleon scalar field having a non-minimal coupling with dark matter can give rise to a smooth transition from a decelerated to an accelerated phase of expansion for the Universe. It is surprising to note that the coupling with the chameleon scalar field hardly affects the evolution of the dark matter sector, which still redshifts as a^{-3}.

Keywords. Cosmology; acceleration of the Universe; chameleon field.

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1. Introduction

In the absence of a clear verdict in favour of any 'dark energy' candidate, which drives the alleged late time acceleration of the Universe [1], every possibilities are being thoroughly investigated so as to find out what can really negotiate this latest challenge that theoretical physics is exposed to. The accelerated expansion is counter-intuitive, as the dominating interaction in the dynamics of the Universe is gravity which is attractive. So the 'dark energy' sector has to produce an effective anti-gravity effect. However, the actual problem is even more stringent, both observations and theoretical requirements demand that the acceleration must have set in quite late in the evolution of the Universe [2]. The models are also required to be consistent with other observational data like those on the cosmic microwave background radiation. There are already some excellent reviews which summarize the problem and the efforts towards finding a consistent solution [3]. Amongst the most talked about models, the quintessence models enjoy a fair deal of popularity. These models contain a scalar field endowed with a potential, so that the contribution to the pressure sector, $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$, can evolve to attain an adequately large negative value and drive the observed accelerated expansion. Given a particular temporal behaviour of the scale factor of the Universe, it is always possible to find the requisite potential [4]. Naturally, there is a long list of potentials that can do the trick [5]. Reconstruction of the potential $V(\phi)$ from observational data also has been worked out by many authors [6]. None of them however can boast of a proper theoretical support from field theory.

In these scalar field models, the cold dark matter and dark energy are normally allowed to evolve independently. However, there are attempts to include an interaction amongst them so that one grows at the expense of the other (see for example [7]). Nonminimally coupled scalar fields, where there is a coupling between the scalar field and geometry, also posed themselves as possible candidates for explaining the present acceleration. For example, Brans–Dicke theory cropped up as a possible arena [8].

A modification of the Brans–Dicke theory, where an interaction between the scalar field and the dark matter could be allowed, showed that the matter-dominated era can have a transition from a decelerated to an accelerated expansion without the requirement of any dark energy [9]. On the other hand, a dark energy interacting with Brans–Dicke scalar field can give rise to a late time acceleration for a very wide range of potentials [10].

Very recently, a completely different approach has been advocated by Khoury and Weltman [11], where a scalar field couples to the matter field directly. The relevant action is

$$\begin{aligned} S = &\int \left[\frac{1}{16\pi G} R - \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi) \right] \sqrt{-g}\ \mathrm{d}^4 x \\ &- \int L_m(\psi_m^{(i)}, g_{\mu\nu}^{(i)})\ \mathrm{d}^4 x, \end{aligned} \tag{1}$$

where each matter field $\psi_m^{(i)}$ is coupled to a conformally transformed metric $g_{\mu\nu}^{(i)}$ which is connected to the actual spacetime metric as

$$g_{\mu\nu}^{(i)} = \mathrm{e}^{8\pi G\beta_i \phi} g_{\mu\nu}.$$

Here β_i's are constants, characteristic to the matter field $\psi_m^{(i)}$. The scalar field is allowed to evolve cosmologically and the coupling with matter is used to effect a variation of the effective mass of the scalar field with local inhomogeneity. The ϕ-field, called the chameleon field [11–13], remains practically massless so as to account for the observational limits on the variation of the universal constants like the fine structure constant α but becomes massive in contact with massive objects (such as the Earth) so as to attain a high mass so that it can respect the stringent limits on the violation of the equivalence principle. This chameleon field, in a slightly alternative version [13], is modelled as a quintessence field also, which is nonminimally coupled to dark matter via a term like $f(\phi)L_m$ in the action. Many interesting cosmological possibilities with this type of chameleon field have been recently pointed out by Das *et al* [14]. For some gravitational implications of such a field, other interesting results and reviews, see [15].

In the present work we adopt the second version and show that a simple chameleon field can indeed make room for a decelerated expansion in the early matter-dominated era and allow for an accelerated expansion at the present epoch. This transition can take place quite smoothly at a recent past.

An intriguing feature of this field, as will be shown later, is that in spite of the interaction with the chameleon field the dark matter density falls off as $1/a^3$, where a is the scale factor of the Universe, exactly similar to the behaviour expected where the dark matter sector does not interact with dark energy and satisfies its conservation equation by itself. So the apprehension that there could be discrepancy between a chameleon model and the expected $(1+z)^3$ dependence in the clustering of the cold dark matter [14] is ruled out.

2. Field equations and results

The relevant action is

$$A = \int \left[\frac{R}{16\pi G} - \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi) + f(\phi)L_m \right] \sqrt{-g}\, \mathrm{d}^4x, \tag{2}$$

where R is the Ricci scalar, G is the Newtonian constant of gravity and ϕ is the chameleon scalar field with a potential $V(\phi)$. Unlike the usual Einstein–Hilbert action, the matter Lagrangian L_m is modified as $f(\phi)L_m$, where $f(\phi)$ is an analytic function of ϕ. This term brings about the nonminimal interaction between the cold dark matter and chameleon field. It deserves mention that a string-inspired dilaton field also has a similar coupling with the matter sector [16]. For a spatially flat FRW metric, the line element is given by

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)[\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\, \mathrm{d}\phi^2]. \tag{3}$$

Variation of action (2) with respect to the metric tensor components yields the field equations as

$$3\frac{\dot{a}^2}{a^2} = \rho_m f + \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{4}$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = -\frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{5}$$

where $8\pi G = 1$. As we are interested in a matter-dominated Universe, the fluid is taken in the form of pressureless dust ($p_m = 0$); ρ_m stands for the contribution from the cold dark matter to the energy density and the overhead dot indicates differentiation with respect to the cosmic time t.

Also, variation of the action (2) with respect to ϕ provides the wave equation for the chameleon field as

$$\ddot{\phi} + 3H\dot{\phi} = -V' - \rho_m f' \tag{6}$$

where the prime indicates differentiation with respect to ϕ.

The matter conservation equation is not an independent equation in view of Bianchi identities and hence should follow from eqs (4)–(6). By subtracting eq. (4) from (5), one obtains

$$2\left(\frac{\ddot{a}}{a}-\frac{\dot{a}^2}{a^2}\right)=-\rho_m f-\dot{\phi}^2, \tag{7}$$

and eq. (4), if differentiated with respect to the cosmic time t, yields

$$6\frac{\dot{a}}{a}\left(\frac{\ddot{a}}{a}-\frac{\dot{a}^2}{a^2}\right)=(\rho_m f)^{\cdot}+\dot{\phi}\ddot{\phi}+V'\dot{\phi}. \tag{8}$$

By using (7) and substituting $\ddot{\phi}$ from eq. (6) in eq. (8), one can arrive at the relation

$$(\rho_m f)^{\cdot}+3H\rho_m f=\rho_m\dot{\phi}f', \tag{9}$$

which readily integrates to yield

$$\rho_m=\frac{\rho_0}{a^3}\ , \tag{10}$$

ρ_0 being a constant.

So it is quite clear, as indicated before, that the cold dark matter (CDM) indeed redshifts as $(1+z)^3$. The coupling factor $f(\phi)$ in eq. (4) can only tune the epoch at which the baton of dominance shifts from the CDM to the dark energy sector. It deserves mention at this stage that if the scalar field is nonminimally coupled with both the matter sector and the Ricci scalar, the dark matter density would show a different evolution [17]. Now, out of eqs (4), (5), (6) and (10), only three are independent equations as the fourth one can be derived from the other three by using Bianchi identity. So one is left with four unknowns a, ϕ, $V(\phi)$, $f(\phi)$ and three independent equations. So we make an ansatz

$$H=\frac{\dot{a}}{a}=\mathrm{e}^{(1-\gamma a^2)/\alpha a}, \tag{11}$$

where γ and α are positive constants which ensures that H attains an infinite value only when a goes to zero. So this model avoids a big rip. Obviously, this choice is made to track the observed accelerated expansion of the Universe. The system of equations is now closed, but the high degree of nonlinearity makes it difficult to get a set of solutions. In order to present a complete model, we make the simplifying assumption

$$V=\beta\dot{\phi}^2, \tag{12}$$

β being a positive constant. This assumption that the kinetic and the potential parts of the chameleon field are proportional to each other is indeed restrictive and not commonly used for a quintessence field. It deserves mention that as the system of equations is already closed, the ansatz (11) already serves the purpose of providing a model which has an early deceleration and late-time acceleration. We employ relation (12) just to solve the system completely as an example. Although eq. (12) absorbs the potential in the kinetic term, it does not make the chameleon a free field as the kintic term is no longer a simple $\frac{1}{2}\dot{\phi}^2$, but rather $(\beta+\frac{1}{2}\dot{\phi}^2)$. For a positive β, the chameleon field never mimics a phantom field, and in the limit of an infinite β, it resembles a cosmological constant.

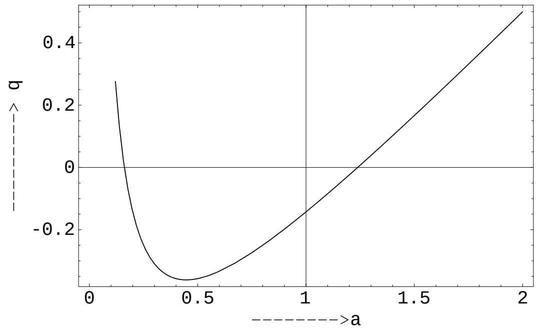

Figure 1. Plot of q vs. a for $\alpha = 7$ and $\gamma = 5$.

Also the field equations are satisfied. The constant β must be greater than $\frac{1}{2}$, so that the effective pressure is negative (eq. (5)). With the assumption (12), the field equations are satisfied with

$$\rho_m f = \frac{H^2}{(2\beta - 1)} \left[(4\beta + 2) \left(\frac{1}{\alpha a} + \frac{\gamma a}{\alpha} \right) - 6 \right] , \tag{13}$$

$$\dot{\phi}^2 = \frac{H^2}{(2\beta - 1)} \left[6 - 4 \left(\frac{1}{\alpha a} + \frac{\gamma a}{\alpha} \right) \right] \tag{14}$$

and

$$f = \frac{2a^3 H^2}{\rho_0 (2\beta - 1)} \left[(2\beta + 1) \left(\frac{1}{\alpha a} + \frac{\gamma a}{\alpha} \right) - 3 \right] . \tag{15}$$

With H given by eq. (11), all the relevant quantities are now known in terms of the scale factor a, and the evolution of each quantity can be found out.

Knowing that the ansatz (11) and (12) can yield a consistent set of solutions, one can now talk about the deceleration parameter q, which can be written (from eq. (11)) as

$$q = \frac{1 + \gamma a^2}{\alpha a} - 1 . \tag{16}$$

Equation (16) indicates that one can obtain a negative q at present ($a = 1$) if $\alpha > 1 + \gamma$. We work out the model for $\alpha = 7$ and $\gamma = 5$ which satisfies this condition. A plot of q vs. a shows that q enters into a negative value regime from a positive value and one has the desired feature of negative q at $a = 1$, i.e., the present epoch. This indicates that the Universe has undergone a transition from a decelerated phase of expansion to an accelerated one in the recent past in the matter-dominated era itself as expected both theoretically [2] and observationally [18]. From figure 1 it is seen that this flip in q takes place at around $a \approx 0.17$ which corresponds to $z \approx 4.9$ which is quite reasonable as argued by Amendola [19].

The choice of the ansatz (11) ensures the additional feature of this model that q has another signature flip from a negative to positive direction at around $a \approx 1.25$

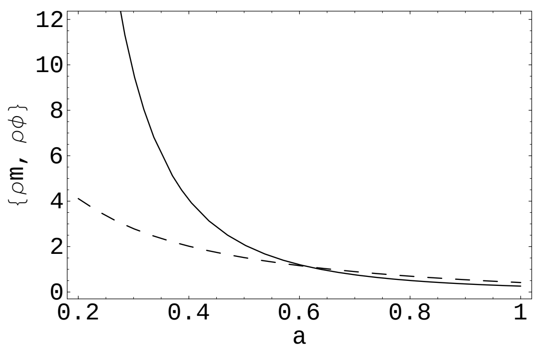

Figure 2. Plot of $\{\rho_m, \rho_\phi\}$ vs. a for $\alpha = 7$, $\gamma = 5$ and $\beta = 10$. The solid line shows the evolution of the matter density sector whereas the dashed line indicates the same for the scalar field sector.

which indicates that the Universe re-enters a decelerated phase of expansion in 'future' and thus 'phantom menace' can be avoided – the Universe does not have a singularity of infinite volume and infinite rate of expansion in a 'finite' future. Also the nature of the plot is not crucially sensitive to values of α and γ chosen; the time when flip occurs shifts a little with different choices of α and γ.

As all the relevant quantities are known in terms of the scale factor a, one can easily compute ρ_m and ρ_ϕ for this model. A plot of $\{\rho_m, \rho_\phi\}$ as a function of a (figure 2) shows that one can obtain a sufficiently long matter-dominated era and ρ_ϕ dominates over ρ_m quite late during the evolution. Figure 2 indicates that the cross-over takes place at around $a \sim 0.65$ which corresponds to $z \sim 0.54$. It is quite apparent that during the later stage of evolution, ρ_m and ρ_ϕ are almost parallel. So the model alleviates the coincidence problem to a certain extent.

The statefinder parameters $\{r, s\}$, introduced by Sahni *et al* [20], have also been computed for this model. These parameters can effectively differentiate between different forms of dark energy and provide a simple diagnostic tool to verify whether a particular model fits into the basic observational data. These parameters are defined as

$$r = \frac{\dddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3(q-1/2)}. \tag{17}$$

As present observations facilitate the study of the evolution of the deceleration parameter q, these statefinder parameters have become useful as they involve third-order derivative of the scale factor 'a'.

For the present model, the statefinder pair comes out as

$$r = -2\left[\frac{2\gamma a}{\alpha} + \frac{1}{\alpha a}\right] + 2\left[\frac{1}{\alpha a} + \frac{\gamma a}{\alpha}\right]^2 + 1 \tag{18}$$

and

$$s = \frac{4(\gamma^2 a^4 - 2\gamma\alpha a^3 + 2\gamma a^2 - \alpha a + 1)}{3(2\gamma\alpha a^3 - 3\alpha^2 a^2 + 2\alpha a)}\ . \tag{19}$$

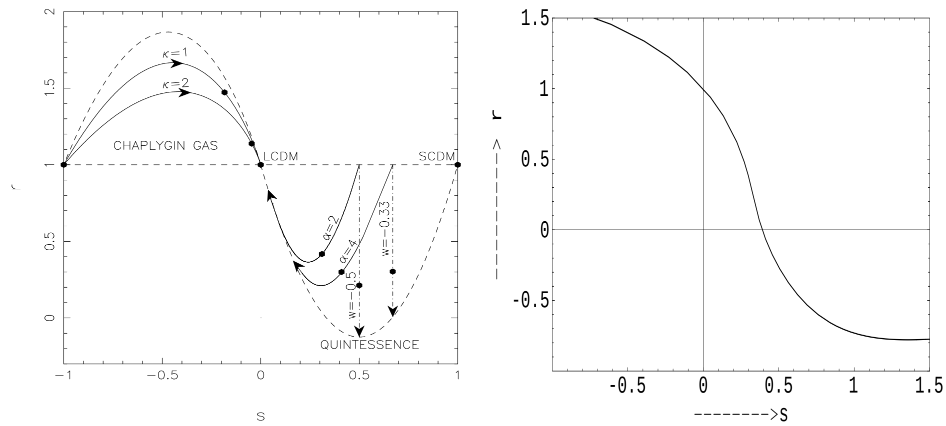

Figure 3. The plot on the left panel shows the time evolution of the statefinder pair $\{r, s\}$ for quintessence models and the chaplygin gas [20]. The right panel of the plot shows the same for our model with $\alpha = 7$ and $\gamma = 5$.

We now plot r vs. s (figure 3) for the same values of α and γ. The plot shows that this model does not mimic the most talked about models like the ΛCDM or quintessence. So the model has distinguishing features, and hence open to the judgement regarding the suitability compared to the other competing models.

3. Discussion

The smooth transition from a decelerated to an accelerated phase of expansion driven by the chameleon field makes the latter worthy of attention. The model presented here is restricted, but it clearly shows that the nonminimal coupling of the field with the CDM sector governs the onset of the acceleration, but it hardly affects the a^{-3} behaviour of the CDM itself. However, the contribution to the Hubble evolution from the dark matter reads like $f(\phi)/a^3$. Equation (4) clearly indicates that this field thus warrants more attention. If one could find a coupling $f(\phi)$ so that the chameleon field has an oscillatory behaviour (with a small amplitude) at the beginning of the matter-dominated epoch, but grows later to dictate the dynamics of the Universe, it would solve many a fine tuning problems.

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