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Intensity-dependent change in polarization state of light in normal incidence on an isotropic nonlinear Kerr medium

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Abstract. It is shown that all optical polarization states of light except plane and circular polarization states undergo an intensity-dependent change in normal incidence of light in an isotropic nonlinear Kerr medium. This effect should be detectable and we propose an experiment for detecting nonlinear susceptibility involved in that part of nonlinear polarization, which depends on the polarization state of light also.

Keywords. Intensity-dependent change in polarization state of light; isotropic nonlinear medium; refraction at isotropic nonlinear medium.

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Nonlinear interaction of coherent radiation with a Kerr medium has been studied by many authors. Maker *et al* [1,2] reported that when strong elliptically polarized light propagates through a lossless isotropic nonlinear medium, self-rotation of the polarization ellipse occurs. Prakash and Chandra [3] showed that if this Kerr medium is lossy, the ellipticity also changes; only plane and circular polarizations are stable and others slowly tend to one of these. Prakash *et al* [4] studied this problem for a lossy isotropic nonlinear medium under slowly varying envelope approximation and obtained analytic results without taking any further approximation. Goyal and Prakash [5] showed conical refraction of a weak light beam in the presence of a co-propagating intense light beam. Prakash and Singh [6] showed that in the presence of intense light beam, change in polarization state and intensity of a weak light beam in a lossy isotropic nonlinear Kerr medium may occur. Nasalski [7] studied the reflection of intense light at the nonlinear–linear interface near the critical angle of total internal reflection and reported that reflection coefficient differ from the standard Fresnel reflection coefficient owing to the nonlinear modifications in the beam of light.

In this paper, we study intensity-dependent change in polarization state in the reflection and transmission of intense beam in normal incidence on an isotropic nonlinear Kerr medium and show that all optical polarization states of light except plane and circular polarization undergo an intensity-dependent change. This effect should be detectable and we propose an experiment for detecting nonlinear susceptibility involved in that part of nonlinear polarization, which depends on the polarization state of light also. We discuss a numerical example.

It is well known [1–6,8] that, in an isotropic nonlinear medium, a plane electromagnetic wave of frequency ω produces electric polarization of frequency ω given by

$$\mathcal{P} = \chi\mathcal{E} + A(\mathcal{E}^* \cdot \mathcal{E})\mathcal{E} + \frac{1}{2}B(\mathcal{E} \cdot \mathcal{E})\mathcal{E}^*, \tag{1}$$

where $\mathcal{E}$ and $\mathcal{P}$ are analytic signals [9–12] of electric field and polarization, χ is the susceptibility in the linear regime, A and B are nonlinear susceptibilities. For studying plane waves propagating along z-direction, the two linear polarizations, say, along the x- and y-directions, do not form a convenient basis as $\mathcal{P}_x(\mathcal{P}_y)$ are not of the form $\mathcal{E}_x(\mathcal{E}_y)$ multiplied by some expression. A convenient basis is in terms of right- and left-handed circular polarizations [13] defined by unit vectors $\mathbf{e}_{\mathrm{R,L}} = (\mathbf{e}_x \pm \mathrm{i}\mathbf{e}_y)/\sqrt{2}$. If we write $\mathcal{P} = \mathcal{P}_{\mathrm{R}}\mathbf{e}_{\mathrm{R}} + \mathcal{P}_{\mathrm{L}}\mathbf{e}_{\mathrm{L}}$ and $\mathcal{E} = \mathcal{E}_{\mathrm{R}}\mathbf{e}_{\mathrm{R}} + \mathcal{E}_{\mathrm{L}}\mathbf{e}_{\mathrm{L}}$, eq. (1) gives

$$\mathcal{P}_{\mathrm{R,L}} = \chi\mathcal{E}_{\mathrm{R,L}} + A|\mathcal{E}_{\mathrm{R,L}}|^2\mathcal{E}_{\mathrm{R,L}} + (A+B)|\mathcal{E}_{\mathrm{L,R}}|^2\mathcal{E}_{\mathrm{R,L}}. \tag{2}$$

This leads to the refractive indices,

$$n_{\mathrm{R,L}}^2 = n_0^2 + 4\pi A|\mathcal{E}_{\mathrm{R,L}}|^2 + 4\pi(A+B)|\mathcal{E}_{\mathrm{L,R}}|^2, \tag{3}$$

where $n_0 = (1+4\pi\chi)^{1/2}$ is the refractive index for small intensities. In terms of the intensities $I_{\mathrm{R,L}}$ of right- and left-handed circularly polarized components given by [14] $I_{\mathrm{R,L}} = (1/8\pi)|\mathcal{E}_{\mathrm{R,L}}|^2$ and the total intensity incident on the nonlinear medium $I = I_{\mathrm{R}} + I_{\mathrm{L}}$, to first order in the nonlinear susceptibilities, eq. (3) gives

$$\begin{aligned} n_{\mathrm{R,L}} &= \bar{n} \pm \frac{1}{2}\Delta n, \quad \bar{n} = n_0 + \left(\frac{8\pi^2}{n_0}\right)(2A+B)I, \\ \Delta n &= \left(\frac{16\pi^2}{n_0}\right)B(I_{\mathrm{L}} - I_{\mathrm{R}}). \end{aligned} \tag{4}$$

Consider a perfectly polarized plane electromagnetic wave travelling along z-axis described by the analytic signal $\mathcal{E}$ of electric field expressed in terms of complex amplitudes $\underline{\mathbf{A}}$ in the form

$$\begin{aligned} \mathcal{E} = \underline{\mathbf{A}}\mathrm{e}^{-\mathrm{i}\psi} &= (\underline{A}_x\mathbf{e}_x + \underline{A}_y\mathbf{e}_y)\mathrm{e}^{-\mathrm{i}\psi} \\ &= (\underline{A}_{\mathrm{R}}\mathbf{e}_{\mathrm{R}} + \underline{A}_{\mathrm{L}}\mathbf{e}_{\mathrm{L}})\mathrm{e}^{-\mathrm{i}\psi}, \quad \psi = \omega t - kz, \end{aligned} \tag{5}$$

$$\underline{A}_{\mathrm{R,L}} = \frac{(\underline{A}_x \pm \mathrm{i}\underline{A}_y)}{\sqrt{2}}, \quad \underline{A}_x = \frac{(\underline{A}_R + \underline{A}_{\mathrm{L}})}{\sqrt{2}}, \quad \underline{A}_y = \mathrm{i}\frac{(\underline{A}_{\mathrm{R}} - \underline{A}_{\mathrm{L}})}{\sqrt{2}}. \tag{6}$$

For perfect polarization, although $\underline{\mathbf{A}}$ is random, ratio of the real amplitudes $|\underline{A}_y|/|\underline{A}_x|$ and the phase difference $\arg(\underline{A}_y) - \arg(\underline{A}_x)$ are constants. This is equivalent to having a constant ratio of the complex amplitudes, i.e., $p = \underline{A}_y/\underline{A}_x$ is constant. The polarization state can be characterized by this constant, which we call the polarization index. If we write $p = \tan(\theta/2)\mathrm{e}^{\mathrm{i}\delta}$, then $\tan(\theta/2)$ is the ratio of the random real amplitude of the y-component to the random real amplitude of the x-component and δ is the phase difference (lag of random phase of y-component behind that of the x-component). Complex polarization index p, or the angles, $\theta(0 \leq \theta \leq \pi)$ and $\delta(-\pi < \delta \leq \pi)$ can represent all possible polarization states unambiguously. In terms of the ratio $p = \underline{A}_y/\underline{A}_x$, the ratio of intensities of polarized components are

$$\frac{I_y}{I_x} = |p|^2, \quad \frac{I_\mathrm{R}}{I_\mathrm{L}} = \left|\frac{(1-\mathrm{i}p)}{(1+\mathrm{i}p)}\right|^2 = \frac{[1+|p|^2+\mathrm{i}(p^*-p)]}{[1+|p|^2-\mathrm{i}(p^*-p)]} \tag{7}$$

and, thus, $I_{x,y}$ and $I_{\mathrm{R,L}}$ can be evaluated in terms of the total intensity $I = I_x + I_y = I_\mathrm{R} + I_\mathrm{L}$ and the polarization index p. The latter part of eq. (7) gives value of $(I_\mathrm{L} - I_\mathrm{R})/I$ in terms of p, and the latter part of eq. (4) then gives

$$\Delta n = 16\pi^2 \left[\frac{\mathrm{i}(p-p*)}{n_0(1+|p|^2)}\right] BI, \tag{8}$$

where I is the total intensity in the nonlinear medium. The dependence of refractive indices $n_{\mathrm{R,L}}$ on intensity I and the polarization state (described by p) gives interesting effects.

If we express incident, reflected and transmitted fields in the form of eq. (5) with subscript inc, ref and tra, calculations similar to the usual Fresnel's formulae calculations [15] give

$$\begin{aligned} &\underline{A}_{\mathrm{ref\ R,L}} = r_{\mathrm{R,L}}\underline{A}_{\mathrm{inc\ R,L}}, \quad \underline{A}_{\mathrm{tra\ R,L}} = t_{\mathrm{R,L}}\underline{A}_{\mathrm{inc\ R,L}}, \\ &r_{\mathrm{R,L}} = \frac{(1-n_{\mathrm{R,L}})}{(1+n_{\mathrm{R,L}})}, \quad t_{\mathrm{R,L}} = \frac{2}{(1+n_{\mathrm{R,L}})}. \end{aligned} \tag{9}$$

Using eqs (6), we get results for the x- and y-components. The results accurate to first order in Δn are

$$\underline{A}_{\mathrm{ref},x} = \bar{r}(\underline{A}_{\mathrm{inc},x} - \mathrm{i}\Delta_1\underline{A}_{\mathrm{inc},y}), \quad \underline{A}_{\mathrm{ref},y} = \bar{r}(\underline{A}_{\mathrm{inc},y} + \mathrm{i}\Delta_1\underline{A}_{\mathrm{inc},x}), \tag{10}$$

$$\underline{A}_{\mathrm{tra},x} = \bar{t}\,(\underline{A}_{\mathrm{inc},x} + \tfrac{1}{2}\mathrm{i}\Delta_2\underline{A}_{\mathrm{inc},y}), \quad \underline{A}_{\mathrm{tra},y} = \bar{t}\,(\underline{A}_{\mathrm{inc},y} - \tfrac{1}{2}\mathrm{i}\Delta_2\underline{A}_{\mathrm{inc},x}), \tag{11}$$

where $\bar{r} \equiv (1-\bar{n})/(1+\bar{n})$, $\bar{t} \equiv 2/(1+\bar{n})$, $\Delta_1 \equiv \Delta n/(\bar{n}^2-1)$, $\Delta_2 \equiv \Delta n/(\bar{n}+1)$ and $\bar{n}$ and Δn are defined by eq. (4).

Polarization states of incident, reflected and transmitted light beams can be expressed by

$$p_{\mathrm{inc}} = \frac{\underline{A}_{\mathrm{inc},y}}{\underline{A}_{\mathrm{inc},x}}, \quad p_{\mathrm{ref}} = \frac{\underline{A}_{\mathrm{ref},y}}{\underline{A}_{\mathrm{ref},x}}, \quad p_{\mathrm{tra}} = \frac{\underline{A}_{\mathrm{tra},y}}{\underline{A}_{\mathrm{tra},y}}. \tag{12}$$

Equations (10), (11) and (8) then lead to

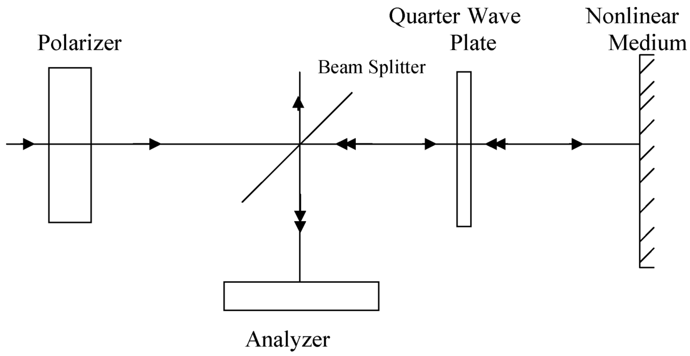

Figure 1. Block diagram for the proposed experiment.

$$p_{\mathrm{ref}} = p_{\mathrm{inc}} - 16\pi^2 \left[\frac{(1+p_{\mathrm{inc}}^2)(p_{\mathrm{inc}} - p_{\mathrm{inc}}^*)}{n_0(n_0^2-1)(1+|p_{\mathrm{inc}}|^2)} \right] BI, \tag{13}$$

$$p_{\mathrm{tra}} = p_{\mathrm{inc}} + 8\pi^2 \left[\frac{(1+p_{\mathrm{inc}}^2)(p_{\mathrm{inc}} - p_{\mathrm{inc}}^*)}{n_0(n_0+1)(1+|p_{\mathrm{inc}}|^2)} \right] BI. \tag{14}$$

These equations clearly show the intensity dependence of polarization state of the reflected and transmitted light beams. From eqs (13) and (14) it is clear that when incident light is plane polarized ($p_{\mathrm{inc}} = p_{\mathrm{inc}}^*$) or circularly polarized ($p_{\mathrm{inc}} = \pm \mathrm{i}$), then $p_{\mathrm{ref}} = p_{\mathrm{inc}}$ and $p_{\mathrm{tra}} = p_{\mathrm{inc}}$, i.e., there is no change in polarization state of light. When incident light is elliptically polarized, the intensity-dependent change in phase difference is maximum if initial phase difference is $\pm\pi/4$ or $\pm 3\pi/4$.

To understand these implications more clearly, let us consider an experiment illustrated in figure 1. A laser beam moving forward along z-direction passes through a polarizer which transmits only the vibrations along $\mathbf{e}_{\mathrm{pol}} = \mathbf{e}_x \cos(\theta/2) + \mathbf{e}_y \sin(\theta/2)$, and is then incident on a beam splitter, which reflects the fraction R. The transmitted beam passes through a quarter wave plate which has fast-vibration axis along $\mathbf{e}_x$ and is then incident normally on the nonlinear medium. Beam reflected from the nonlinear medium again passes through the quarter wave plate and is then reflected by the beam splitter. The reflected beam then passes through an analyzer which is in such a position that it extincts light at low intensities. This is possible, as, passage through quarter wave plate twice is equivalent to the passage through a half wave plate and at low intensity the plane polarized incident light remains plane polarized. For high intensities, however, there is an intensity-dependent change in polarization in reflection by the nonlinear medium and the beam incident on analyzer is elliptically polarized, and hence a nonzero intensity filtered through the analyzer is now obtained.

If the polarizer transmits fully the component polarized along $\mathbf{e}_{\mathrm{pol}} = \mathbf{e}_x \cos(\theta/2) + \mathbf{e}_y \sin(\theta/2)$, the incident light before transmission through the quarter wave plate is represented by

$$\mathcal{E} = E[\mathbf{e}_x \cos(\theta/2) + \mathbf{e}_y \sin(\theta/2)]\mathrm{e}^{-\mathrm{i}\psi_0}, \quad \psi_0 = \omega t - kz, \tag{15}$$

and has intensity $I = E^2/2\pi$. After transmission through the quarter wave plate, which has fast vibration axis along $\mathbf{e}_x$, it is given by

$$\mathcal{E} = E[\mathbf{e}_x \cos(\theta/2) + \mathrm{i}\mathbf{e}_y \sin(\theta/2)]\mathrm{e}^{-\mathrm{i}(\psi_0 - \phi_x)}, \tag{16}$$

where ϕ_x is the phase retardation of the x-vibration in the quarter wave plate. If this is reflected from the boundary of the nonlinear medium at $z = z_0$, a straight application of results (10) tells that the reflected wave is given by

$$\mathcal{E} = E\bar{r}[\mathbf{e}_x\{\cos(\theta/2) + \Delta_1 \sin(\theta/2)\} + \mathrm{i}\mathbf{e}_y\{\sin(\theta/2) + \Delta_1 \cos(\theta/2)\}]\mathrm{e}^{-\mathrm{i}\psi'}, \tag{17}$$

where $\psi' = \omega t + \omega z - \phi_x - 2\omega z_0$. Passage through the quarter wave plate changes it to

$$\mathcal{E} = E\bar{r}[\mathbf{e}_x\{\cos(\theta/2) + \Delta_1 \sin(\theta/2)\} - \mathbf{e}_y\{\sin(\theta/2) + \Delta_1 \cos(\theta/2)\}]\mathrm{e}^{-\mathrm{i}\psi''} \tag{18}$$

with $\psi'' = \omega t + \omega z - 2\phi_x - 2\omega z_0$.

If intensity I is not so high, so as to make $\Delta_1 \equiv \Delta n/(\bar{n}^2 - 1)$ appreciable, $\mathcal{E}$ in eq. (18) is along $\mathbf{e}_x \cos(\theta/2) - \mathbf{e}_y \sin(\theta/2)$. If we place an analyzer with plane of transmission along $\mathbf{e}_{\mathrm{ana}} = \mathrm{e}_x \sin(\theta/2) + \mathbf{e}_y \cos(\theta/2)$, it cuts light completely at low intensities. At high intensities, the field coming out of the analyzer is $(\mathcal{E}\cdot\hat{\mathbf{e}}_{\mathrm{ana}})\hat{\mathbf{e}}_{\mathrm{ana}} = -E\bar{r}\Delta_1 \cos\theta \hat{\mathbf{e}}_{\mathrm{ana}}\mathbf{e}^{-i\psi''}$ and has intensity $I(\bar{r}\Delta_1 \cos\theta)^2$.

In the arrangement shown in figure 1, if intense light of intensity I_0 falls on the beam splitter having reflectivity R, the light with intensity $I = (1 - R)I_0$ falls on nonlinear medium. The light reflected from the nonlinear medium passes through the quarter wave plate and is then reflected by the beam splitter. The reflected light then passes through a crossed analyzer, the light with intensity $(1 - R)RI_0(\bar{r}\Delta_1 \cos\theta)^2 = 64\pi^4 n_0^{-2}(\bar{n}+1)^{-4}B^2R(1-R)^3I_0^3 \sin^2 2\theta$ will leak through it.

This intensity is maximum for $2\theta = \pi/2$, i.e., for polarization index $\tan(\pi/8)$ of incident plane polarized light. The ratio of this intensity to reflected intensity from nonlinear medium is proportional to the square of the intensity-dependent part of the refractive index.

This effect should be detectable using laser pulses and polarizer/analyzer available presently. For example, if we take CS_2 as nonlinear medium for which we have $B = 8.33 \times 10^{-17}$ cm^2/W [1], and $n_0 = 1.628$ [16], a 100 J pulse of 1 ns duration having 1 mm^2 area of cross-section ($I_0 = 10^{13}$ W/cm^2) will be incident on a (25/75) beam splitter. The transmitted beam having an intensity of 7.5×10^{12} W/cm^2 is then incident normally on the nonlinear medium and the beam reflected from the nonlinear medium has an intensity of 0.43×10^{12} W/cm^2. After reflection from the beam splitter and transmission through the analyzer, a beam with an intensity of 2.8×10^7 W/cm^2 is obtained at the detector, which can be detected, as the intensity decreases in the ratio of 0.65×10^{-4} and the experiment has been done with polarizer/analyzer having a specified extinction ratio better than 10^{-6} [17]. The nonlinear interaction thus produces a photon flux which is about 6.5 times the flux which leaks into the orthogonal mode due to a nonperfect polarizer.

One simple application of this experiment can be a method to evaluate B as the ratio of the detected intensity to I_0^3 is a constant proportional to B^2. If I_0 is varied and the ratio of the detected intensity to I_0^3 is measured, B can be found.

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[12] The real physical signals are real parts of the complex analytic signals
[13] We follow the natural nomenclature and our right-handed photon has spin +1
[14] We work in natural system of units and take $c = 1$
[15] The well-known optical laws are treated in most textbooks of optics, see, e.g., M Born and E Wolf, *Principles of optics* (Cambridge University Press, Cambridge, 1997) p. 38
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