PRAMANA
— journal of
physics

Vol. 74, No. 3
March 2010
pp. 365–376

Influence of pairing in double beta decay of ^{48}Ca

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MS received 4 August 2009; accepted 9 October 2009

Abstract. Two-neutrino $\beta\beta$ decay matrix elements and half-life of ^{48}Ca are calculated after including neutron–proton pairing correlations in projected Hartree–Fock–Bogoliubov (PHFB) formalism. The GT matrix elements in $2\nu\beta\beta$ decay are reduced due to broader smearing of Fermi surfaces. Half-life results for $2\nu\beta\beta$ decay of ^{48}Ca with np pairing are better than without pairing.

Keywords. Neutrinoless double β decay; Hartree–Fock–Bogoliubov formalism.

PACS Nos 23.40.Hc; 27.40.+z

1. Introduction

Proton–neutron pairing is expected to play a significant role in the calculation of nuclear matrix elements of $\beta\beta$ decay [1–6]. The recent experimental confirmation of neutrino oscillation has reinforced the interest in nuclear processes involving neutrinos [7,8]. From particle physics point of view, an essential step in studying the nature of the neutrino is to observe neutrinoless double β decay [9–15]. The $0\nu\beta\beta$ decay demands an extension of the Standard Model of electroweak interaction to grand unified theories where the neutrino prefers to be a Majorana particle and therefore the phase is identical with its antiparticle. In such grand unified models, the neutrinos have a finite mass and a slight right-handed weak interaction. Thus, $0\nu\beta\beta$ represents a unique tool to measure the neutrino masses. Experiments at Super-Kamiokande [16], SNO [17], KamLAND [18] and NEMO3 [19] gave recent measurements of the massive character of the neutrinos and hence, have opened up a new era in the neutrino physics.

The experimental search for $\beta\beta$ decay of ^{48}Ca was among the first to be attempted in live-time experiments beginning in the early 1950. With the largest energy release among all $\beta\beta$ candidates, ^{48}Ca ($Q_{\beta\beta} = 4.271 \pm 0.0004$ MeV) [20] has a $\beta\beta_{2\nu}$ sum energy spectrum that extends to higher energies than most radioactive backgrounds. Recently, Bardin *et al* [21] gave the lower limit of $(\beta\beta)_{2\nu}$ decay half-life measurement to be $>3.6 \times 10^{19}$ yr and Balysh *et al* [22] used time projection chamber (TPC) background to give $T^{2\nu}_{1/2} = (4.3^{2.4}_{-1.1}[\text{stat}] \pm 1.4[\text{syst}]) \times 10^{19}$. For

$0\nu\beta\beta$ decay mode the lower limit of decay half-life was given by Bardin *et al* to be $>2.0 \times 10^{21}$ yr. The (p,n)-type charge-exchange reaction (^{3}He,t) on $\beta\beta$ decay for ^{48}Ca nucleus was presented in refs [23,24]. Theoretically, the shell model [2,25–30] is the natural calculation scheme for this nucleus without truncation. Recently proposed GXPF1A two-body effective interaction has been successfully tested for the fp shell [31–33] to perform $2\nu\beta\beta$ decay calculations for ^{48}Ca. Recently, Horoi [30] gave the $2\nu\beta\beta$ decay matrix elements and half-lives of ^{48}Ca using full fp shell model space and GXPF1A interactions to be $T_{1/2}(0^+ \to 0^+) = 3.3 \times 10^{19}$ yr and $T_{1/2}(0^+ \to 0^+) = 8.5 \times 10^{23}$ yr respectively.

Double beta decay [34], though being the rarest nuclear weak interaction process, has a great potential to explore the physics beyond the Standard Model. The decay mechanism is a combination of two sequential virtual decays first from a parent nucleus to the adjacent intermediate nucleus, to which ordinary β decay is either energetically forbidden or suppressed by angular momentum consideration, followed by the transition to the daughter nucleus which then lies energetically below the parent.

If one knows the nuclear wave functions and an upper limit of the $0\nu\beta\beta$ decay probability, one can only extract upper limit for the effective electron–neutrino mass and the effective mixing angle between the vector bosons mediating left- and right-handed weak interaction [35]. These effective values contain information about the mixing of different neutrinos, so that the upper limits for the bare values can only be extracted if the mixing coefficients and therefore the detailed structure of the grand unified theory are known. But even to extract these effective values, mixing coefficients and half-life, one must know the nuclear matrix elements of the double beta decay very reliably. Thus, the important point at this stage is the precise computation of the nuclear matrix elements (NMEs).

The objective of this paper is to study the effect of np correlations on the two-neutrino double β matrix elements which can be further included in the studies of $0\nu\beta\beta$ decay. Neutrinoless double β decay has fundamental significance in elementary particle physics. Measurements of the neutrinoless double beta decay in different nuclei will help in determining the underlying physics mechanism [36,37]. Our aim is to study $2\nu\beta\beta$ decay and $0\nu\beta\beta$ decay of ^{48}Ca nucleus in PHFB formalism by employing effective interactions such as P + Q · Q, KB and BonnC.

2. Method of calculation

The HFB method is capable of describing all kinds of $\beta\beta$ decay transitions within the mean-field approximation. Particularly, the inclusion of short-range repulsive correlation due to the nucleon hard core is important for calculating the nuclear matrix element of neutrinoless double beta decays ($0\nu\beta\beta$). The technique of angular momentum projection and variation after projection on Hartree–Fock–Bogoliubov (VAP-HFB) method is especially suited for treating heavier deformed nuclei using suitable interaction and a large number of single-particle orbitals. We apply these methods for the ground state and excited state transitions.

We start from isospin-invariant P + Q · Q interaction accompanied by quadrupole pairing, isoscalar interactions and monopole interactions $V_{\rm m}$

$$
\begin{aligned}
H &= H_0 + H_{P_0} + H_{\mathrm{QQ}} + H_{\mathrm{OO}} + V_{\mathrm{m}} \\
&= \sum_\alpha \varepsilon_\alpha C_\alpha^\dagger c_\alpha - \frac{1}{2} g_o \sum_\kappa P_{001\kappa}^\dagger P_{001\kappa}^\dagger - \frac{1}{2}\chi_2 \sum_M : Q_{2M}^\dagger Q_{2M} : \\
&\quad - \frac{1}{2}\chi_3 \sum_M : O_{3M}^\dagger O_{3M} : - \kappa^0 \sum_{JM,ab} A_{JM00}^\dagger(ab) A_{JM00}(ab) + V_{\mathrm{m}}, \qquad (1)
\end{aligned}
$$

$$
P_{001\kappa}^\dagger = \sum_a \sqrt{j_a + 1/2}\, A_{001\kappa}^\dagger(aa), \qquad (2)
$$

$$
A_{JMT_\kappa}^\dagger(ab) = [c_a^\dagger c_b^\dagger]_{JMT_\kappa}/\sqrt{2}, \qquad (3)
$$

where ε_a is the single-particle energy, $P_{001\kappa}$ is the $T = 1$, $J = 0$ pair operator and $Q_{2M}(O_{3M})$ is the isoscalar quadrupole operator. Each term includes p-n components which play important roles in the $N = Z$ nuclei, due to an isospin invariance. Employing the HFB equations, one obtains the following expression for double beta decay nuclear matrix elements:

$$
\begin{aligned}
\langle M_{\mathrm{GT}} \rangle &= (n_{N-2,Z+2}^{J_f} n_{N,Z}^{J_i})^{1/2} \int_0^\pi n_{(N,Z),(N-2,Z+2)}(\theta) \\
&\quad \times \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \langle S_{\alpha\beta\gamma\delta} \rangle \sum_{\varepsilon\eta} [(1 + F_{N,Z}^\pi(\theta) f_{N-2,Z+2}^\pi)]_{\varepsilon\alpha}^{-1} \\
&\quad \times (f_{(N-2,Z+2)}^{\pi^*})_{\alpha\beta} [(1 + F_{N,Z}^\nu(\theta) f_{N-2,Z+2}^{\nu^*})]_{\varepsilon\alpha}^{-1} (F_{N,Z}^{\nu^*})_{\eta\delta} \sin\theta \,\mathrm{d}\theta, \\
&\qquad (4)
\end{aligned}
$$

where

$$
\begin{aligned}
n^J &= \int_0^\pi [\det(1 + F_{N,Z}^\pi F_{N-2,Z+2}^{\pi^*})]^{1/2} [\det(1 + F_{N,Z}^\nu f_{N-2,Z+2}^{\nu^*})]^{1/2} \\
&\quad \times \mathrm{d}_{MK}^J(\theta) \sin\theta \,\mathrm{d}\theta \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
n_{(N,Z),(N-2,Z+2)}(\theta) &= [\det(1 + F_{N,Z}^\nu f_{N-2,Z+2}^{\nu^*})]^{1/2} \\
&\quad \times [\det(1 + F_{N,Z}^\pi f_{N-2,Z+2}^{\pi^*})]^{1/2} \qquad (6)
\end{aligned}
$$

π (ν) represents the proton (neutron) of nuclei involved in the double beta decay processes. The matrices for $(F_{N,Z}(\theta))_{\alpha\beta}$ and $(f_{N,Z})_{\alpha\beta}$ have been developed. In the case of $2\nu\beta\beta$ decay the two-body transition operator $S_{\alpha\beta\gamma\delta}$ for $0^+ \to 0^+$ transition is given by

$$
\langle S_{\alpha\beta\gamma\delta} \rangle = \langle \alpha\beta | \sigma_1 \cdot \sigma_2 | \gamma\delta \rangle \qquad (7)
$$

and for $0^+ \to 2^+$ transition $S_{\alpha\beta\gamma\delta}$ is given by

$$
\langle S_{\alpha\beta\gamma\delta} \rangle = \langle \alpha\beta | [\sigma_1 \times \sigma_2]_0^2 | \gamma\delta \rangle, \qquad (8)
$$

where the GT transition operators for $0^+ \to 0^+$ and $0^+ \to 2^+$ may be written as

$$\begin{aligned}\langle J_1 J_2|\sigma_1\cdot\sigma_2|J_3 J_4\rangle = 3(-1)^{J_1-J_2} \\ \times\sqrt{(2J_3+1)(2J_4+1)} \\ \times\langle J_3 1/2, 1 J_1 - J_3 \mid J_1 1/2\rangle \\ \times\langle J_3 1/2, 1 J_2 - J_4 \mid J_2 1/2\rangle\end{aligned} \tag{9}$$

$$\begin{aligned}\langle J_1 J_2|[\sigma_1\otimes\sigma_2]^2_m|J_3 J_4\rangle = 3\delta_{m,J_1+J_2-J_3-J_4} \\ \times\langle J_3 1/2, J_1 - J_3 1 \mid J, 1/2\rangle \\ \times\langle J_4 1/2, 1 J_2 - J_4 \mid J_2 1/2\rangle \\ \times\langle 1 J_1 - J_3, 1 J_2 - J_4 \mid 2m\rangle.\end{aligned} \tag{10}$$

The $2\nu\beta\beta$ decay rate can be obtained by making the standard assumptions, that the outgoing leptons are in s-state and in the hadronic current the dominant allowed terms are $\sum\sigma_i\tau^i_+$ (Gammow–Teller) and $\sum\tau^i_+$ (Fermi) transitions. The isospin mixing in nuclear ground state is very small. Therefore the Fermi part of the transition will vanish and can be neglected. Under these assumptions, the standard formula for $2\nu\beta\beta$ half-life can be written as

$$[T^{\nu}_{1/2}(0^+\to f^+)]^{-1} = G^{2\nu}|M^{2\nu}_{\rm GT}(f^+)|^2, \tag{11}$$

where $f^+ = 0^+$ ground state or 2^+ state of the final nucleus, $G^{2\nu}$ is the integrated phase space factor containing all relevant constants and $M^{2\nu}_{\rm GT}$ is the Gammow–Teller nuclear matrix elements.

$$M^{2\nu}_{\rm GT} = \frac{\sum_{\rm m}\langle 0^+_{\rm f}|\sum_l \sigma(l)\cdot\tau^+(l)|1^+_{\rm m}\rangle\langle 1^+_{\rm m}|\sum_k\sigma(k)\cdot\tau^+(k)|0^+_{\rm i}\rangle}{E_{\rm m}+\frac{1}{2}Q_{\beta\beta}+m_e-(M_{\rm i}+M_{\rm f})/2}. \tag{12}$$

Here $|0^+_{\rm i}\rangle(|0^+_{\rm f}\rangle)$ is the ground state of the initial (final) nucleus with mass energies $M_{\rm i}$ and $M_{\rm f}$ respectively, $|1^+_{\rm m}\rangle$ are the 1^+ states in the intermediate odd–odd nucleus with energies $E_{\rm m}$. $\sigma(l)$ is the usual Pauli spin operator for the lth nucleon and $\tau^+(l)$ is the isospin raising operators changing a neutron into proton. The function $F^{2\nu}(E,Z)$ is the lepton phase space integral containing all the relevant constants and energy denominators. $F^{2\nu}(E,Z)$ is not very sensitive to the choice of the actual value of the energy of the intermediate mass.

Now let us consider the $0\nu\beta\beta$ decay mode caused by the weak interactions involving couplings to the right-handed lepton or hadron currents. The differential $0\nu\beta\beta$ decay rate is given by

$$\mathrm{d}R_{0\nu} = 2\pi\sum ij\sum_{\rm spin}|M_{0\nu}|^2\delta(\epsilon e_1+\epsilon e_2+E_F-E_I)\times\frac{\mathrm{d}p_1}{(2\pi)^3}\cdot\frac{\mathrm{d}p_2}{(2\pi)^3}, \tag{13}$$

where $M_{0\nu}$ is the decay amplitude given by

$$M_{0\nu} = \sqrt{\frac{1}{2}}\left(\frac{G\cos\theta_c}{\sqrt{2}}\right)^2 \sum_{i=1}^{2n}\sum_{N}\sum_{\alpha\beta=\mathrm{L,R}}\sum_{s}\int \mathrm{d}x\,\mathrm{d}y$$
$$\times \int \frac{\mathrm{d}k}{(2\pi)^3}\langle F|J^{\nu^\dagger}_{\beta i(y)}|N\rangle\langle N|J^{\mu^\dagger}_{\alpha i}(x)|I\rangle \times [1-P(l_1,l_2)]$$
$$\times \frac{\bar{e}_{p_2 s_2}(y)\gamma_\nu 2P_\beta N_{iks}(y)\bar{e}_{p_1 s_1}(x)\gamma_\mu 2P_\alpha N^c_{iks}(x)}{\epsilon e_1 + \epsilon\nu_1 + E_N - E_I}, \tag{14}$$

where $P_\alpha(\alpha = \mathrm{L,R})$ is the projection operator and the summation $\sum_{\mathrm{spin}}$ is taken over the spin projections of the electrons (s_1, s_2) and the final nuclear state. The terms $e_{ps}(x)$ and $N_{iks}(x)$ are the wave functions which are used to expand the field operators e and N_i. We obtain the following combinations of nuclear matrix elements:

$$X_1 = \left(\frac{\langle m_\nu\rangle}{m_e}\right)(\chi_F - 1)M^{(0\nu)}_{\mathrm{GT}}, \tag{15}$$

$$X_3 = (\langle\lambda\rangle\tilde{\chi}_- + \langle\eta\rangle\tilde{\chi}_+)M^{(0\nu)}_{\mathrm{GT}}, \tag{16}$$

$$X_4 = (\langle\lambda\rangle\tilde{\chi}'_- + \langle\eta\rangle\chi'_+)M^{0\nu}_{\mathrm{GT}}, \tag{17}$$

$$X_5 = \langle\eta\rangle\chi'_p M^{(0\nu)}_{\mathrm{GT}}, \tag{18}$$

$$X_6 = \langle\eta\rangle\chi'_R M^{(0\nu)}_{\mathrm{GT}}, \tag{19}$$

where

$$\langle m_\nu\rangle = \sum_j |m_j U^2_{ej}|, \tag{20}$$

$$\langle\lambda\rangle = \lambda\left|\sum_j U_{ej}V_{ej}\right|, \tag{21}$$

$$\langle\eta\rangle = \eta\left|\sum_j U_{ej}V_{ej}\right|, \tag{22}$$

where U_{ej} and V_{ej} are the unitary matrices describing the mixing of the neutrino mass eigenstate to electron neutrinos and m_j is the neutrino mass eigenvalue. The nuclear matrix elements $M^{(0\nu)}_{\mathrm{GT}}$ and χ's in the above equations are given by

$$M^{(0\nu)}_{\mathrm{GT}} = \langle H(r_{12})(\sigma_1\cdot\sigma_2)\rangle \tag{23}$$

$$\left.\begin{matrix}\tilde{\chi}_{\mathrm{GT}}\\ \chi'_{\mathrm{GT}}\end{matrix}\right\} = \left\langle \left\{ \begin{matrix}\tilde{H}(r_{12})\\ -r_{12}H'(r_{12})\end{matrix}\right\} \sigma_1 \cdot \sigma_2 \right\rangle (M_{\mathrm{GT}}^{(0\nu)})^{-1} \tag{24}$$

$$\left.\begin{matrix}\chi_F\\ \tilde{\chi}_F\\ \chi'_F\end{matrix}\right\} = \left\langle \left\{ \begin{matrix}H(r_{12})\\ \tilde{H}(r_{12})\\ -r_{12}H'r(12)\end{matrix}\right\} \right\rangle (M^{(0\nu)}{}_{\mathrm{GT}})^{-1} \tag{25}$$

$$\chi'_T = \langle -r_{12}H'(r_{12})S_{12}\rangle / M_{\mathrm{GT}}^{(0\nu)} \tag{26}$$

$$\chi'_P = \left(\frac{g_v}{g_A}\right)\left\langle -\frac{1}{2} r_{+12}H'(r_{12})i(\sigma_1-\sigma_2)\cdot(\hat{r}_{12}\times\hat{r}_{+12})\right\rangle / M_{\mathrm{GT}}^{0\nu} \tag{27}$$

$$M_{\mathrm{R}}'^{(0\nu)} = \left\langle -\frac{1}{2} m_e^{-1} H' r_{(12)}\hat{r}_{12}\cdot(\sigma_1\times D_2 - \sigma_2\times D_1)\right\rangle \tag{28}$$

$$\chi'_R = \left(\frac{g_v}{g_A}\right) M_R'^{0\nu} / M_{\mathrm{GT}}^{(0\nu)} \tag{29}$$

and their combinations

$$\tilde{\chi}_\pm = \tilde{\chi} \pm \tilde{\chi}_{\mathrm{GT}}, \quad \chi'_\pm = -\chi'_F \pm \left(\frac{1}{3}\chi'_{\mathrm{GT}} - 2\chi'_T\right). \tag{30}$$

The inverse half-life formula for the $0^+ \to 0^+$ transition of $0\nu\beta\beta$ decay can be written as

$$[T_{1/2}^{0\nu}(0^+\to 0^+)]^{-1} = \left\{ C_{mm}\left|\frac{\langle m_\nu\rangle}{m_e}\right|^2 + 2C_{m\lambda}\langle\lambda\rangle\frac{\langle m_\nu\rangle}{m_e} + 2C_{m\eta}\langle\eta\rangle\frac{\langle m_\eta\rangle}{m_\nu} + C_{\lambda\lambda}\langle\lambda\rangle^2 + C\eta\eta\langle\eta\rangle^2 + 2C_{\lambda\eta}\langle\lambda\rangle\langle\eta\rangle \right\}, \tag{31}$$

where $\langle m_\nu\rangle$, $\langle\lambda\rangle$, $\langle\eta\rangle$ etc. are the effective lepton number violation parameters and C_{ij} coefficients are the linear combinations of nuclear matrix elements and kinematical integral factors. $M_{\mathrm{GT}}^{0\nu}$ is the Gammow–Teller nuclear matrix element.

$$C_{mm} = (M_{\mathrm{GT}}^{(0\nu)})^2(\chi_F - 1)^2 F_{11} \tag{32}$$

$$C_{m\lambda} = \left(M_{\mathrm{GT}}^{(0\nu)}\right)^2 (\chi_F - 1)[\tilde{\chi}_- F_{13} - \chi'_- F_{14}] \tag{33}$$

$$C_{m\eta} = (M_{\mathrm{GT}}^{(0\nu)})^2(\chi_F - 1)[\tilde{\chi}_+ F_{13} + \chi' + F_{14} + \chi'_P F_{15} + \chi'_R F_{16}] \tag{34}$$

$$C_{\lambda\lambda} = (M_{\rm GT}^{(0\nu)})^2(\tilde{\chi}_-^2 F_{33} + \chi_-'^2 F_{44} + 2\tilde{\chi}_-\chi_-' F_{34}) \tag{35}$$

$$C_{\eta\eta} = (M_{\rm GT}^{(0\nu)})^2(\tilde{\chi}_+^2 F_{33} + \chi_+'^2 F_{44} + 2\tilde{\chi}_+\chi_+' F_{34} + \chi_P'^2 F_{55} + \chi_R'^2 F_{66} + 2\chi_P'\chi_R' F_{56}) \tag{36}$$

$$C_{\lambda\eta} = (M_{\rm GT}^{(0\nu)})^2(\tilde{\chi}_-\tilde{\chi}_+ F_{33} + \chi_-'\chi_+' F_{44} + (\tilde{\chi}_-\chi_+' + \tilde{\chi}_+\chi_-')F_{34}). \tag{37}$$

Relativistic corrections:

The $0\nu\beta\beta$ nuclear matrix elements $M_R^{(0\nu)}$ due to the relativistic correction is decomposed into central, tensor, momentum dependent and spin orbit part as

$$M_R'^{0\nu} = M_{\rm RC} + M_{\rm RT} + M_{\rm RP} + M_{\rm RLS}, \tag{38}$$

$$M_{\rm RC} = \langle H_{\rm RC}(r_{12})\sigma_1 \cdot \sigma_2 \rangle, \tag{39}$$

$$M_{\rm RT} = \langle H_{\rm RT}(r_{12})S_{12} \rangle, \tag{40}$$

$$M_{\rm RP} = \left\langle \frac{1}{4m_e M} H'(r_{12})\langle \hat{r}_{12} \times P_{12}\rangle \cdot (\sigma_1 - \sigma_2) \right\rangle, \tag{41}$$

$$M_{\rm RLS} = \left\langle -\frac{1}{2m_e M r_{12}} H'(r_{12}) I_{12} \cdot (\sigma_1 + \sigma_2) \right\rangle, \tag{42}$$

where $H(r)$ is the neutrino potential.

3. Results and discussion

To describe the structure of ^{48}Ca, ^{48}Ti and ^{48}Cr nuclei, we have used the PHFB method by employing the full 0f–1p shell valence space spanned by the orbitals $0f_{7/2}$, $1p_{3/2}$, $0f_{5/2}$ and $1p_{1/2}$. The doubly closed ^{40}Ca nucleus is treated as the core for proton and neutron and the set of single-particle energies (SPEs) used here is given in ref. [38]. The Kuo–Brown two-body effective interaction matrix elements have been employed with the following modifications in the monopole strength of the interaction matrix elements: $V_{ff}^1(\text{MKB}) = V_{ff}^1(\text{KB}) - 110$ keV for $TJ = 10, 14, 16$; $V_{ff}^1(\text{MKB}) = V_{ff}^1(\text{KB}) - 310$ keV for $TJ = 12$; $V_{ff}^0(\text{MKB}) = V_{ff}^0(\text{KB}) - 650$ keV, for $TJ = 1, 3$; $V_{ff}^0(\text{MKB}) = V_{ff}^0(\text{KB}) - 350$ keV, for $TJ = 5, 6$; $V_{fr}^1(\text{MKB}) = V_{fr}^1(\text{KB}) - 300$ keV, $\forall J$ even and $V_{fr}^0(\text{MKB}) = V_{fr}^0(\text{KB}) + 400$ keV; $\forall J$ odd values, where f stands for $f_{7/2}$ orbital, r for all of the other sub-shells ($p_{1/2}$, $p_{3/2}$, $f_{5/2}$) and V_{ij}^T centroids defined for any two sub-shells by Poves and Zuker [39]. Using these

Table 1. Yrast spectrum energies and γ-ray energies $E_\gamma = E(J) - E(J-2)$.

	J^π	0^+	2^+	4^+	6^+	8^+	10^+	12^+	14^+	16^+
	Exp.	0.00	0.983	2.295	3.333	–	–	–	–	–
^{48}Ti	MKB	0.445	0.983	1.815	2.793	4.031	5.565	7.278	9.069	12.069
	E_γ	0.000	0.538	0.827	0.983	1.238	1.534	1.713	1.790	3.000
	Exp.	0.00	0.752	1.859	3.452	5.322	7.062	8.402	10.602	–
^{48}Cr	MKB	0.00	0.366	1.151	2.822	4.865	5.792	8.121	10.886	14.193
	E_γ	0.00	0.366	0.785	1.671	2.043	1.427	1.329	2.765	3.307

Table 2. The calculated and observed $B((E2);(0_1^+ \rightarrow 2_1^+)) \times 10^{-50} e^2 fm^4$ values.

			$B(E2);(0_1^+ \rightarrow 2_1^+)$		β_2	
	Q_{HFB}	E_{HFB}	Theor.	Exp.	Theor.	Exp.
^{48}Ca	1.1780	−11.296	64.21(0.4)	84	0.0993	0.101
^{48}Ti	29.0170	−29.196	787.80(0.4)	720	0.2800	0.267
^{48}Cr	32.1450	−37.977	1290.00(0.4)	1330	0.3285	0.335

modifications in KB interaction, the PHFB method is able to reproduce the energy spectrum of ^{50}Cr nucleus upto $J^\pi = 18$ levels and the phenomenon of backbending at $J^\pi = 10$, as observed.

In table 1, we present the experimental Yrast spectrum energies and the results obtained with modified KB interaction, of ^{48}Ti and ^{48}Cr isotopes, and γ-ray energies $E_\gamma = E(J) - E(J-2)$ as the function of angular momentum J. The PHFB method with modified KB interaction has nicely reproduced the spectra as the experimental spectra upto $J^\pi = 16^+$ levels, but there is a small difference in energy of approximately 0.5 MeV for levels upto $J^\pi = 8^+$ for both isotopes' experimental observations. It can be seen from third row of ^{48}Cr that the calculated results of γ-ray energies show backbending at $J^\pi = 10^+$ as observed in the shell model calculations and experiments.

Table 2 shows the PHFB results for $B((E2);(0_1^+ \rightarrow 2_1^+)) \times 10^{-50} e^2 fm^4$ and deformation parameter β_2 at the effective charge 0.4 for neutron and the results are in good agreement with observation. The intrinsic quadrupole moments and HFB energies are also displayed in table 2. In the case of ^{48}Ti nucleus, the electric quadrupole moment $Q(b) = 0.1724$ at lower effective charge 0.15 is in agreement with the observed value of $Q(b) = 0.177 \pm 0.007$, whereas to the best of our knowledge the experimental data for $Q(b)$ is not available for ^{48}Ca and ^{48}Cr isotopes.

In figures 1 and 2, we present the variation of matrix elements of $2\nu\beta\beta$ decay with variation in interaction strength parameter χ_{pn} and monopole interaction in terms of BonnC interaction respectively. In principle, np pairing occurs with total isospin $T = 0$ and $T = 1$. The treatment of these pairing correlations in spherical nuclei within a Bogoliubov transformation is only possible for $T = 1$ pairs. The $T = 0$ pairing can only be taken into account in a spherical BCS (or HFB) type

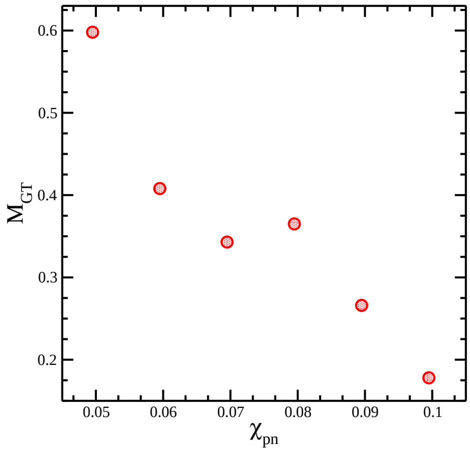

Figure 1. Variation of GT transition matrix elements of $2\nu\beta\beta$ decay of ^{48}Ca with variation in interaction strength parameter $\chi_{\rm pn}$.

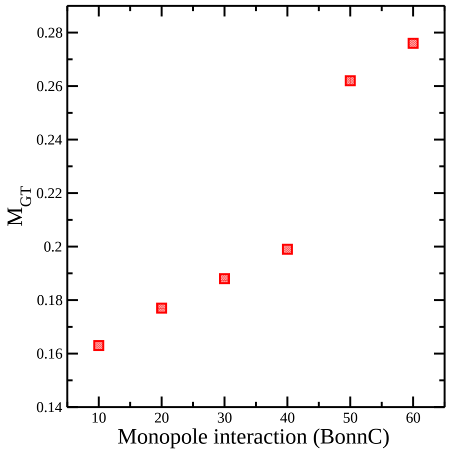

Figure 2. Variation of GT transition matrix elements of $2\nu\beta\beta$ decay of ^{48}Ca with variation of monopole interaction in terms of BonnC interaction.

formalism by the renormalization of the $T = 1$, $J = 0$ pairing force. By including np pairing correlations, the GT transition matrix elements in the $2\nu\beta\beta$ decay are strongly reduced compared to the results without np pairing. The reduction of the GT matrix element comes mainly from the fact that for the modified BCS vacua the Fermi surface is smeared out much broader due to additional np pairing correlations compared to the usual BCS calculation (nn or pp pairing) and Hartree–Fock (non-pairing) approach with a sharp Fermi surface. This broader smearing reduces the GT matrix elements and also its dependence of particle–particle strength parameter. Without np pairing correlations, the rapid decrease of the Gammow–Teller matrix elements is due to the rapid increase of the ground state (spin–isospin) correlations in the final nucleus with increasing particle–particle strength. The np pairing correlations produce a pairing gap which is (20–30)% of the nn or pp pairing gap. This lowers the ground state. Quasiparticles defined by a Bogoliubov transformation with np mixing have additional interactions due to a strong mixing of neutrons and protons. This additional $|T_Z| = 1$ interaction suppresses the rapid increase of the ground-state correlations in the final nucleus. Such a suppression prevents the rapid decrease of the GT matrix elements.

In table 3, we present our results of $2\nu\beta\beta$ decay half-lives for ^{48}Ca for different interactions and in the PHFB model and compared it with the shell model and experimental values. The results show that the present calculation with Q–Q interaction and in the PHFB model are in good agreement with the shell model calculations. The results with BonnC interaction agree well with the experimental results given by Balysh *et al.* The calculated and observed half-lives have been presented and compared for ground state-to-ground state transition. Thus, the theoretical results for the half-lives with np pairing correlations show a better agreement than the results without these correlations.

Table 3. Calculated $2\nu\beta\beta$ decay half-lives for ^{48}Ca nuclei for Q–Q interaction, Bonn interaction and in PHFB and compared with experimental results.

	$^{48}\mathrm{Ca} \to ^{48}\mathrm{Ti}\ T_{1/2}^{2\nu}(0^+ \to 0^+) \times 10^{19}$ yr		
Interactions	Present cal.	Shell model	Expt.
Q–Q	2.70×10^{19}	2.9×10^{19}	$(4.2^{+3.3}_{-1.3} \times 10^{19})$
		1.9×10^{19}	$> 3.6 \times 10^{19}$
BonnC	4.70×10^{19}	–	–
PHFB	2.46×10^{19}	–	$4.3^{+2.4}_{-1.1} \times 10^{19}$

Table 4. Nuclear matrix elements (NME) for $0\nu\beta\beta$ $^{48}\mathrm{Ca} \to ^{48}\mathrm{Ti}$.

NME	Present cal.	Shell model	QRPA
$M_{\mathrm{GT}}^{0\nu}$	0.170	0.681	−0.405
χ_F	−0.269	−0.171	0.158
χ'_{GT}	1.130	1.126	1.226
χ'_F	−0.307	−0.182	0.184
χ^w_{GT}	1.000	0.874	0.775
χ^w_F	−0.269	−0.160	0.131
χ'_T	–	−0.172	0.130
$M_{\mathrm{GT}}^{0\nu}(1-\chi_F)$	0.2162	0.7974	0.341

Table 5. The HFB predictions for the upper bound to the effective neutrino mass and half-lives around $\langle m_\nu \rangle = 1$ eV.

	Present cal.	Shell model	QRPA
$T_{1/2}^{0\nu}$	4.58×10^{25}	1.00×10^{25}	2.8×10^{25}
$\langle m_\nu \rangle$	≤ 2.66	≈ 1.00	≤ 54

In the case of ^{48}Ca $\beta\beta$ decay, observation for only the lower limit of half-life 9.5×10^{21} yr is available. Therefore, we assume a half-life of 10^{25} years as the upper bound of the three lepton number violation parameters. In table 4, we present the nuclear matrix elements $M_{\mathrm{GT}}^{0\nu}$, χ_F, χ'_{GT}, χ'_F, $\chi^\omega_{\mathrm{GT}}$, χ^ω_F, χ'_T and $M_{\mathrm{GT}}^{0\nu}(1-\chi_F)$ by employing the HFB model in the 0f–1p space. These nuclear matrix elements are compared with other theoretical calculation of the shell model and the QRPA approximation. The nuclear matrix elements presented in table 4 have been calculated using eqs (23)–(42). Our results for NMEs are reasonably comparable with NME calculated by employing the shell model, whereas NME calculated with the QRPA have opposite signs. The limits for half-life have been calculated by assuming the mass of neutrino to be 1 eV.

In table 5, we present the $0\nu\beta\beta$ half-life limits and upper limit of neutrino mass.

4. Conclusions

We summarize by saying that the HFB model equations are used to set the ^{48}Ca nucleus with a modified version of Kuo–Brown interaction, solved for amplitudes and expansion coefficients. The wave function obtained in the PHFB framework with modified KB interaction is successfully used to calculate the static properties of ^{48}Ca, ^{48}Ti as well as electron scattering data for quadrupole excitations, providing an answer to some observed anomalous features. Two-neutrino $\beta\beta$ decay matrix elements and half-life of ^{48}Ca are calculated after including neutron–proton pairing correlation in describing the ground states of initial and final nuclei in PHFB formalism. Results with these pairing correlations yield stronger bound nuclei as a solution of the HFB equation. One obtains a lower energy minimum as a solution of the HFB equation if one renormalizes $T = 1$ np pairing to the empirical mass differences in order to include $T = 0$ pairing. The GT matrix elements in $2\nu\beta\beta$ decay are reduced due to this broader smearing of the Fermi surface. $\chi_{\rm pn} = 0.0795$ gives $M_{\rm GT} = 0.365$ which compares well with the shell model calculations. BonnC gives $T_{1/2}$ which compares well with the experimental data. One obtains better half-lives for the $2\nu\beta\beta$ decay than the results without np pairing. Thus neutron–proton pairing influences the neutrinoless double beta decay rates significantly, allowing for larger values of the expectation value of light neutrino masses.

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