PRAMANA — journal of physics Vol. 74, No. 2
February 2010
pp. 219–230

Fundamental optimal relation of a generalized irreversible Carnot heat pump with complex heat transfer law

JUN LI, LINGEN CHEN* and FENGRUI SUN
Postgraduate School, Naval University of Engineering, Wuhan, 430033, People's Republic of China
*Corresponding author. E-mail: lgchenna@yahoo.com; lingenchen@hotmail.com

MS received 7 December 2008; revised 26 August 2009; accepted 9 October 2009

Abstract. The fundamental optimal relation between heating load and coefficient of performance (COP) of a generalized irreversible Carnot heat pump is derived based on a new generalized heat transfer law, which includes the generalized convective heat transfer law and generalized radiative heat transfer law, $q \propto (\Delta T^n)^m$. The generalized irreversible Carnot heat pump model incorporates several internal and external irreversibilities, such as heat resistance, bypass heat leakage, friction, turbulence and other undesirable irreversibility factors. The added irreversibilities besides heat resistance are characterized by a constant parameter and a constant coefficient. The effects of heat transfer laws and various loss terms are analysed. The heating load vs. COP characteristic of a generalized irreversible Carnot heat pump is a parabolic-like curve, which is consistent with the experimental result of thermoelectric heat pump. The obtained results include those obtained in many literatures and indicated that the analysis results of the generalized irreversible Carnot heat pump were more suitable for engineering practice than those of the endoreversible Carnot heat pump.

Keywords. Finite-time thermodynamics; entropy generation minimization; irreversible Carnot heat pump; optimal performance; heat transfer law.

PACS Nos 05.60.Cd; 05.70.-a; 05.70.Ln

1. Introduction

In the analysis of finite-time thermodynamics or entropy generation minimization [1–18], the basic thermodynamic model is Newtonian law system endoreversible one in which the irreversibility of only linear finite rate heat transfer is considered. Blanchard [19] was the first to extend the Curzon–Ahlborn analysis method [20] to the analysis of heat pump cycles, and derived the coefficient of performance (COP) bounds for the fixed heating load for an endoreversible Carnot heat pump obeying Newton's law. Goth and Feidt [21], Feidt [22], Philippi and Feidt [23] and Feidt [24] derived the optimal COP for the fixed heating load, i.e. the fundamental optimal relation of a Carnot heat pump obeying Newton's law. Chen *et al* [25] extended the characteristic parameters of heat engines to the heat pump and derived the optimization criteria of a steady-flow two-heat-reservoir heat pump. Wu [26], Chen *et al* [27] and Wu *et al* [28] investigated the specific heating load optimization of the endoreversible Carnot heat pump, derived the bounds of specific heating load and COP as well as the optimal relation between the optimal specific heating load and COP. Sahin and Kodal [29] investigated the finite-time thermoeconomic optimization for endoreversible Carnot heat pumps.

However, real heat pumps are usually devices with both internal and external irreversibilities. Besides the irreversibility of finite-rate heat transfer, there are other sources of irreversibilities also, such as bypass heat leakages, dissipation processes inside the working fluid, etc. Some authors have assessed the effect of finite-rate heat transfer, together with major irreversibilities on the performance of Carnot heat pumps using the heat resistance and heat leakage model [28,30–33] and heat resistance and internal irreversibility model [34,35]. Cheng and Chen [36] and Chen *et al* [37–39] established a generalized irreversible Carnot heat pump model which considers the effects of heat resistance, bypass heat leakage, and other internal irreversibilities. The generalized irreversible Carnot heat pump model is more similar to the real heat pump than the endoreversible Carnot heat pump model. And they derived its fundamental optimal relation between heating load and COP [36–38] and the ecological optimal performance [39] with Newtonian heat transfer law. Kodal *et al* [40] investigated the finite-time thermoeconomic optimization for the generalized irreversible Carnot heat pumps.

In general, heat transfer is not necessarily Newtonian. Some authors have assessed the effects of heat transfer law on the performance of endoreversible Carnot heat pump [41–45]. Chen *et al* [41] first derived the optimal relation between heating load and COP of an endoreversible Carnot heat pump with the linear phenomenological heat transfer law, $q \propto \Delta(T^{-1})$. Zhu *et al* [42] investigated the optimal performance of an endoreversible Carnot heat pump under the mixed heat resistance condition. Sun *et al* [43] first obtained the performance limits and the optimal relation between heating load and COP of the endoreversible Carnot heat pump with the generalized radiative heat transfer law, $q \propto (\Delta T^n)$. Other optimal performances of an endoreversible Carnot heat pump were obtained based on this heat transfer law [45]. Chen *et al* [44] first derived the optimal relation between heating load and COP of an endoreversible Carnot heat pump with the generalized convective heat transfer law, $q \propto (\Delta T)^n$. Other optimal performances of an endoreversible Carnot heat pump were obtained based on this heat transfer law [45]. Li *et al* [46] first derived the optimal relation between heating load and COP of an endoreversible Carnot heat pump with a new generalized heat transfer law, which includes the generalized convective heat transfer law and generalized radiative heat transfer law, $q \propto (\Delta T^n)^m$.

Ni *et al* [47] investigated the fundamental optimal relation between heating load and COP of the generalized irreversible Carnot heat pump with the generalized radiative heat transfer law, $q \propto (\Delta T^n)$. Kodal [48] considered the generalized convective heat transfer law, $q \propto (\Delta T)^n$, and obtained the fundamental optimal relation of irreversible heat pump without heat leakage loss. Zhu *et al* [49] obtained the fundamental optimal relation between heating load and COP of the generalized irreversible Carnot heat pump with the generalized convective heat transfer law, $q \propto \Delta(T)^n$. The effects of generalized radiative heat transfer law and generalized convective heat transfer law on the ecological optimal performance of the generalized irreversible Carnot heat pump were analysed by Zhu *et al* [50,51].

One of the aims of finite-time thermodynamics is to pursue generalized rules and results. This paper will extend the previous work to find out the fundamental optimal relationship between heating load and COP of the generalized irreversible Carnot heat pump based on refs [36–40,47,49–51] using a new generalized heat transfer law which includes the generalized convective heat transfer law and generalized radiative heat transfer law, $q \propto (\Delta T^n)^m$, in the heat transfer processes between the working fluid and the heat reservoirs of the heat pump [36].

2. Generalized irreversible Carnot heat pump model

The generalized irreversible Carnot heat pump and its surroundings considered in this paper are shown in figure 1. The following assumptions are made for this model [36–40,47,49–51]: (i) The working fluid (refrigerant) flows through the system in a steady-state fashion. The cycle consists of two isothermal and two adiabatic processes. All four processes are irreversible. (ii) Because of the heat transfer, the working fluid temperatures, $T_{\rm HC}$ and $T_{\rm LC}$, are different from the reservoir temperatures, $T_{\rm H}$ and $T_{\rm L}$. The four temperatures are in the following order: $T_{\rm HC} > T_{\rm H} > T_{\rm L} > T_{\rm LC}$. The heat transfer surface areas, F_1 and F_2, of the high- and low-temperature heat exchangers are finite. The total heat transfer surface area, F, of the two heat exchangers is assumed to be constant: $F = F_1 + F_2$. (iii) There exists a constant rate of heat leakage, q, from the heat sink to the heat source. Thus, $Q_{\rm H} = \pi = Q_{\rm HC} - q$ and $Q_{\rm L} = Q_{\rm LC} - q$, where $Q_{\rm HC}$ is due to the driving force of $T_{\rm HC} - T_{\rm H}$, $Q_{\rm LC}$ is due to the driving force of $T_{\rm L} - T_{\rm LC}$, $Q_{\rm H}$ is the rate of heat transfer released to the heat sink, i.e., the heating load π, and $Q_{\rm L}$ is the rate of heat transfer supplied by the heat source. (iv) A constant coefficient Φ is introduced to characterize the additional internal miscellaneous irreversibility effect: $\Phi = Q_{\rm LC}/Q'_{\rm LC} \geq 1$, where $Q_{\rm LC}$ is the rate of heat flow from the heat source to the cold working fluid for the generalized irreversible Carnot heat pump while $Q'_{\rm LC}$ is that for the Carnot heat pump which is endoreversible.

If $q = 0$ and $\Phi = 1$, the model will be an endoreversible Carnot heat pump [19,21–29,45–49]. If $q \neq 0$ and $\Phi = 1$, the model will be an irreversible Carnot heat pump with heat resistance and heat leakage losses [30–33] and if $q = 0$ and $\Phi > 1$, the model will be an irreversible Carnot heat pump with heat resistance and internal irreversibilities [34,35,48].

3. Generalized optimal characteristics

3.1 *Fundamental optimal relation*

The second law of thermodynamics requires that $Q_{\rm HC}/Q_{\rm LC} = \Phi T_{\rm HC}/T_{\rm LC}$. The first law of thermodynamics gives that the power input, P, to the heat pump is

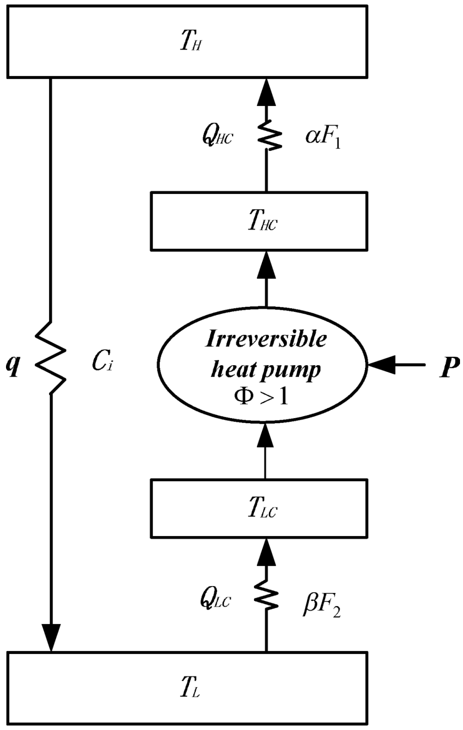

Figure 1. Generalized irreversible Carnot heat pump model [26–30, 37,39–41].

$P = Q_{\rm H} - Q_{\rm L} = Q_{\rm HC} - Q_{\rm LC}$, and the coefficient of performance (COP) of the heat pump $\varphi = Q_{\rm H}/P = \pi/P$.

Consider that the heat transfer between the heat pump and its surroundings follows a new generalized law, $q \propto (\Delta T^n)^m$. Then

$$Q_{\rm HC} = \alpha F_1 (T_{\rm HC}^n - T_{\rm H}^n)^m, \quad Q_{\rm LC} = \beta F_2 (T_{\rm L}^n - T_{\rm LC}^n)^m, \tag{1}$$

where α is the overall heat transfer coefficient, F_1 is the heat transfer surface area of the high-temperature-side heat exchanger (condenser), β is the overall heat transfer coefficient and F_2 is the heat transfer surface area of the low-temperature-side heat exchanger (evaporator).

Define ratio (f) of the heat transfer surface area and ratio (x) of working fluid temperature as follows: $f = F_1/F_2$, $x = T_{\rm LC}/T_{\rm HC}$, where $0 \le x \le T_{\rm L}/T_{\rm H}$. Combining the above conditions and eq. (1) gives

$$\frac{Q_{\rm LC}}{Q_{\rm HC}} = \frac{T_{\rm LC}}{\Phi T_{\rm HC}} = \frac{x}{\Phi} = \frac{\beta F_2 (T_{\rm L}^n - T_{\rm LC}^n)^m}{\alpha F_1 (T_{\rm HC}^n - T_{\rm H}^n)^m}. \tag{2}$$

Solving eq. (2) for $T_{\rm HC}$ yields

$$T_{\rm HC} = \left[\frac{T_{\rm L}^n + (rfx/\Phi)^{1/m} T_{\rm H}^n}{x^n + (rfx/\Phi)^{1/m}}\right]^{1/n}, \tag{3}$$

where $r = \alpha/\beta$. Combining the first and second law of thermodynamics gives

$$\pi = Q_{\rm HC} - q, \quad \varphi = \frac{(Q_{\rm HC} - q)}{[Q_{\rm HC}(1 - x/\Phi)]}. \tag{4}$$

Combining eqs (1), (3) with (4) gives

$$\varphi = \frac{\Phi\alpha f F(T_{\rm L}^n - T_{\rm H}^n x^n)^m - q\Phi(1+f)[x^n + (rfx/\Phi)^{1/m}]^m}{\alpha f F(\Phi - x)(T_{\rm L}^n - T_{\rm H}^n x^n)^m} \tag{5}$$

$$\pi = \frac{\alpha f F}{1+f}\left[\frac{T_{\rm L}^n - T_{\rm H}^n x^n}{x^n + (rfx/\Phi)^{1/m}}\right]^m - q. \tag{6}$$

Equations (5) and (6) indicate that both heating load (π) and COP (φ) are functions of f for fixed $T_{\rm H}$, $T_{\rm L}$, α, β, n, m, q, Φ and x. Taking the derivatives of π and φ with respect to f and setting them equal to zero ($\mathrm{d}\pi/\mathrm{d}f = 0$, $\mathrm{d}\varphi/\mathrm{d}f = 0$) yields

$$f_a = \left(\frac{\Phi x^{nm-1}}{r}\right)^{1/(m+1)}. \tag{7}$$

The corresponding optimal heating load and optimal COP are, respectively,

$$\pi = \frac{\alpha F(T_{\rm L}^n x^{-n} - T_{\rm H}^n)^m}{[1 + (rx^{1-mn}/\Phi)^{1/(m+1)}]^{m+1}} - q \tag{8}$$

$$\varphi = \frac{\Phi}{\Phi - x}\left[1 - \frac{q[1 + (rx^{1-nm}/\Phi)^{1/(m+1)}]^{m+1}}{\alpha F(T_{\rm L}^n x^{-n} - T_{\rm H}^n)^m}\right]. \tag{9}$$

Equations (8) and (9) are the major results of this paper. The optimal heating load and COP of the generalized irreversible Carnot heat pump for the given $T_{\rm H}$, $T_{\rm L}$, α, β, n, m, q, Φ and x can be determined by the two equations.

Eliminating x from eqs (8) and (9) gives

$$(\pi + q)\left\{1 + \left[\frac{r\pi(\varphi - 1) + r\varphi q}{\varphi(\pi + q)}\right]^{1/(m+1)}\right\}^{m+1} - \alpha F\left\{T_{\rm L}^n\left[\frac{\varphi(\pi + q)}{\Phi\pi(\varphi - 1) + \Phi\varphi q}\right]^n - T_{\rm H}^n\right\}^m = 0. \tag{10}$$

Equation (10) is the fundamental relation between optimal heating load and COP of the heat pump. It is an implicit equation. Therefore, it is not as convenient as eqs (8) and (9) in heat pump performance analysis. The maximum heating load and maximum COP bounds are different for different values of m and n.

3.2 *Effects of various losses*

(i) If there is no bypass heat leakage in cycle ($q = 0$), eqs (8) and (9) become

$$\pi = \frac{\alpha F(T_{\mathrm{L}}^n x^{-n} - T_{\mathrm{H}}^n)^m}{[1 + (r x^{1-mn}/\Phi)^{1/(m+1)}]^{m+1}} \tag{11}$$

$$\varphi = \frac{\Phi}{(\Phi - x)}. \tag{12}$$

The heating load vs. COP curve is a monotonic decreasing function when $n \neq 0$ and $m > 0$, and a monotonic increasing function when $n < 1$ and $m < -1$. The relationship between heating load and COP is irregular in other range of values of m and n.

(ii) If there are only heat resistance and bypass heat leakage losses in the cycle ($\Phi = 1$), eqs (8) and (9) become

$$\pi = \frac{\alpha F(T_{\mathrm{L}}^n x^{-n} - T_{\mathrm{H}}^n)^m}{[1 + (r x^{1-mn})^{1/(m+1)}]^{m+1}} - q \tag{13}$$

$$\varphi = \frac{1}{1-x}\left[1 - \frac{q[1 + (r x^{1-nm})^{1/(m+1)}]^{m+1}}{\alpha F(T_{\mathrm{L}}^n x^{-n} - T_{\mathrm{H}}^n)^m}\right]. \tag{14}$$

The heating load vs. COP curve is a parabolic-like curve when $n \neq 0$ and $m > 0$ and a monotonic increasing function when $n < 1$ and $m < -1$. The relationship between heating load and COP is irregular in other range of values of m and n.

(iii) If the cycle is endoreversible ($q = 0, \Phi = 1$), eqs (8) and (9) become

$$\pi = \frac{\alpha F(T_{\mathrm{L}}^n x^{-n} - T_{\mathrm{H}}^n)^m}{[1 + (r x^{1-mn})^{1/(m+1)}]^{m+1}} \tag{15}$$

$$\varphi = \frac{1}{(1 - x)}. \tag{16}$$

Equations (15) and (16) are the same results as in ref. [46] for endoreversible Carnot heat pumps. The heating load vs. COP curve is a monotonic decreasing function when $n \neq 0$ and $m > 0$ and a monotonic increasing function when $n < 1$ and $m < -1$. The relationship between heating load and COP is irregular in other range of values of m and n.

One can see that the internal irreversibility changes the performance characteristics of the heat pump quantitatively, and the bypass heat leakage changes the heating load vs. COP relationship qualitatively and quantitatively. The heating load vs. COP curve is changed from the monotonic decreasing curve to parabolic curve if $n \neq 0$ when the heat pump is with the heat leakage loss. The model presented in this paper reflects the effects of various factors on the performance of a generalized irreversible Carnot heat pump.

3.3 *Effects of heat transfer laws*

(1) When $m = 1$, eqs (8) and (9) become

$$\pi = \frac{\alpha F(T_{\rm L}^n x^{-n} - T_{\rm H}^n)}{[1 + (rx^{1-n}/\Phi)^{1/2}]^2} - q \tag{17}$$

$$\varphi = \frac{\Phi}{\Phi - x}\left[1 - \frac{q[1 + (rx^{1-n}/\Phi)^{1/2}]^2}{\alpha F(T_{\rm L}^n x^{-n} - T_{\rm H}^n)}\right]. \tag{18}$$

Equations (17) and (18) are the same results as those obtained in ref. [47]. The effects of various losses on the performance are as follows:

(i) If there is no bypass heat leak in the cycle ($q = 0$), eqs (17) and (18) become

$$\pi = \frac{\alpha F(T_{\rm L}^n x^{-n} - T_{\rm H}^n)}{[1 + (rx^{1-n}/\Phi)^{1/2}]^2} \tag{19}$$

$$\varphi = \frac{\Phi}{(\Phi - x)}. \tag{20}$$

Equations (19) and (20) are the same results as those obtained in ref. [48] which indicate that heating load is a monotonic decreasing function of COP.

(ii) If there are only heat resistance and bypass heat leakage losses in the cycle ($\Phi = 1$), eqs (17) and (18) become

$$\pi = \frac{\alpha F(T_{\rm L}^n x^{-n} - T_{\rm H}^n)}{[1 + (rx^{1-n})^{1/2}]^2} - q \tag{21}$$

$$\varphi = \frac{1}{1 - x}\left[1 - \frac{q[1 + (rx^{1-n})^{1/2}]^2}{\alpha F(T_{\rm L}^n x^{-n} - T_{\rm H}^n)}\right]. \tag{22}$$

Equations (21) and (22) indicate that the heating load vs. COP curve is parabolic.

(iii) If the cycle is endoreversible ($q = 0, \Phi = 1$), eqs (17) and (18) become

$$\pi = \frac{\alpha F(T_{\rm L}^n x^{-n} - T_{\rm H}^n)}{[1 + (rx^{1-n})^{1/2}]^2} \tag{23}$$

$$\varphi = \frac{1}{(1 - x)}. \tag{24}$$

Equations (23) and (24) are the same results as those obtained in refs [43,45] which indicate that the heating load vs. COP curve is a monotonic decreasing function.

(iv) If the total heat transfer area of the heat pump is infinite ($F \to \infty$, no heat resistance losses), eq. (10) becomes

$$\pi = \frac{q\varphi}{[\Phi T_{\rm H}/(\Phi T_{\rm H} - T_{\rm L}) - \varphi]}. \tag{25}$$

The heating load is a monotonic increasing function of COP. However, in this case, the specific heating load (π/F) is zero. In this case, the performance is independent of the heat transfer law.

(v) If $n = 1$ further, eqs ((17), (18)), ((19), (20)), ((21), (22)) and ((23), (24)) become the corresponding results of refs [36–38], [34,35], [30–33] and [19,21–29], respectively.

(2) When $n = 1$, eqs (8) and (9) become

$$\pi = \frac{\alpha F(T_{\rm L}x^{-1} - T_{\rm H})^m}{[1 + (rx^{1-m}/\Phi)^{1/(m+1)}]^{m+1}} - q \tag{26}$$

$$\varphi = \frac{\Phi}{\Phi - x}\left[1 - \frac{q[1 + (rx^{1-m}/\Phi)^{1/(m+1)}]^{m+1}}{\alpha F(T_{\rm L}x^{-1} - T_{\rm H})^m}\right]. \tag{27}$$

Equations (26) and (27) are the same results as those obtained in ref. [49]. The relationship of heating load and COP is a parabolic function. The effects of various losses on the performance are as follows:

(i) If there is no bypass heat leakage in the cycle ($q = 0$), eqs (26) and (27) become

$$\pi = \frac{\alpha F(T_{\rm L}x^{-1} - T_{\rm H})^m}{[1 + (rx^{1-m}/\Phi)^{1/(m+1)}]^{m+1}} \tag{28}$$

$$\varphi = \frac{\Phi}{(\Phi - x)}. \tag{29}$$

Equations (28) and (29) indicate that heating load is a monotonic decreasing function of COP.

(ii) If there are only heat resistance and bypass heat leakage losses in the cycle ($\Phi = 1$), eqs (26) and (27) become

$$\pi = \frac{\alpha F(T_{\rm L}x^{-1} - T_{\rm H})^m}{[1 + (rx^{1-m})^{1/(m+1)}]^{m+1}} - q \tag{30}$$

$$\varphi = \frac{1}{1 - x}\left[1 - \frac{q[1 + (rx^{1-m})^{1/(m+1)}]^{m+1}}{\alpha F(T_{\rm L}x^{-1} - T_{\rm H})^m}\right]. \tag{31}$$

Equations (30) and (31) indicate that the heating load vs. COP curve is parabolic.

(iii) If the cycle is endoreversible ($q = 0, \Phi = 1$), eqs (26) and (27) become

$$\pi = \frac{\alpha F(T_{\rm L}x^{-1} - T_{\rm H})^m}{[1 + (rx^{1-m})^{1/(m+1)}]^{m+1}} \tag{32}$$

$$\varphi = \frac{1}{(1 - x)}. \tag{33}$$

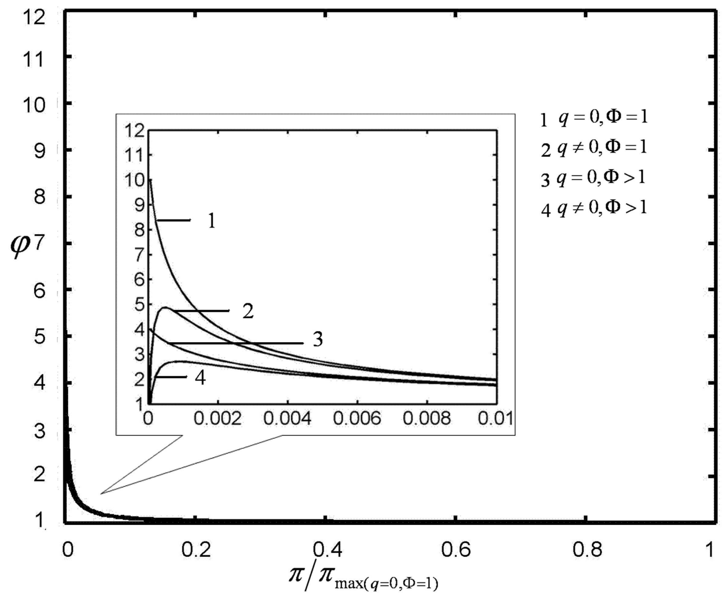

Figure 2. Effects of various losses on the relationship between heating load and COP for $m = 1.25$ and $n = 4$.

Equations (32) and (33) are the same results as those obtained in refs [44,45] which indicate that the heating load vs. COP curve is a monotonic decreasing function.

(iv) If the total heat transfer area of the heat pump is infinite ($F \to \infty$, no heat resistance losses), eq. (10) becomes eq. (25).

(v) If $m = 1$ further, eqs ((17), (18)), ((19), (20)), ((21), (22)) and ((23), (24)) become the corresponding results of refs [36–38], [34,35], [30–33] and [19,21–29], respectively.

4. Numerical example

To show the heating load vs. COP characteristic of the generalized irreversible Carnot heat pump with the generalized heat transfer law, one numerical example is provided (see figure 2). In the numerical calculation for the performance characteristics of the generalized irreversible Carnot heat pump, $T_{\rm L} = 273$ K, $T_{\rm H} = 300$ K, $\alpha F = 4$ W/K, $\Phi = 1.2$, $\alpha = \beta (r = 1)$, $n = 4$, $m = 1.25$, $q = C_i(T_{\rm H}^4 - T_{\rm L}^4)^{1.25}$ and $C_i = 0.16$ W/K are set, where C_i is the heat conductance of the heat pump. Figure 2 shows the effects of various losses on the relationship between heating load and COP of the heat pump in this case.

Curve 1 in figure 2 is the characteristic curve of the endoreversible Carnot heat pump with the sole loss of heat resistance and it shows that heating load is a monotonic decreasing function of COP. Curve 4 in figure 2 is the characteristic curve of the generalized irreversible Carnot heat pump with the losses of heat resistance, bypass heat leakage and other internal irreversibility, and it shows that the relationship between heating load and COP is a parabolic function, i.e. there exists a maximum COP and the corresponding heating load. The theoretical analysis results for real irreversible simple cycle air heat pump [13,52], real irreversible regenerated cycle air heat pump [13,53], real irreversible single-stage thermoelectric heat pump [13,27,54,55] and real irreversible two-stage thermoelectric heat pump [56,57] indicated that the relationships between heating load and COP were parabolic-like functions, and there exists a maximum COP and the corresponding heating load. The experimental results of a single-stage thermoelectric heat pump [58] show that the heating load vs. COP characteristics was also a parabolic curve. Therefore, the theoretical analysis results of this paper are consistent with those of practical heat pumps. Apparently, the analysis results of the generalized irreversible Carnot heat pump are more suitable for practical engineering devices than those of the endoreversible Carnot heat pump.

5. Conclusion

The fundamental optimal relation between optimal heating load and COP of a generalized irreversible Carnot heat pump is derived based on a new generalized heat transfer law, including generalized convective heat transfer law and generalized radiative heat transfer law, $q \propto (\Delta T^n)^m$. The generalized irreversible Carnot heat pump model incorporates several internal and external irreversibilities, such as heat resistance, bypass heat leakage, friction, turbulence and other undesirable irreversibility factors. The added irreversibilities besides heat resistance are characterized by a constant parameter and a constant coefficient. The obtained results include those obtained in many literatures, such as the optimal performance of endoreversible Carnot heat pump with different heat transfer laws ($m \neq 0$, $n \neq 0$, $q = 0$, $\Phi = 1$), the optimal performance of the Carnot heat pump with heat resistance and internal irreversibility ($m \neq 0, n \neq 0, q = 0, \Phi > 1$), the optimal performance of the Carnot heat pump with heat resistance and heat leakage ($m \neq 0, n \neq 0, q \neq 0, \Phi = 1$), and the optimal performance of the irreversible Carnot heat pump with generalized radiative heat transfer law $q \propto (\Delta T^n)$ ($m = 1, n \neq 0$) and generalized convective heat transfer law $q \propto (\Delta T)^m$ ($m \neq 0, n = 1$). This paper is a synthesis of the finite-time thermodynamics analysis results of Carnot-type theoretical heat pump cycle and reflects the universal optimal performance of generalized irreversible Carnot heat pump with complex heat transfer law. The heating load vs. COP characteristic of a generalized irreversible Carnot heat pump is a parabolic curve, which is consistent with the experimental result of thermoelectric heat pump. The obtained results indicated that the analysis results of the generalized irreversible Carnot heat pump are more suitable for practical engineering devices than those of the endoreversible Carnot heat pump.

Acknowledgements

This work is supported by the Program for New Century Excellent Talents in University of P. R. China (Project No. 20041006) and The Foundation for the

Author of National Excellent Doctoral Dissertation of P. R. China (Project No. 200136). The authors wish to thank the reviewers for their careful, unbiased and constructive suggestions, which improved the quality of this paper.

References

[1] B Andresen, *Finite time thermodynamics*, Physics Laboratory II, University of Copenhagen, 1983
[2] D C Agrawal and V J Menon, *Eur. J. Phys.* **11(5)**, 305 (1990)
[3] D C Agrawal and V J Menon, *J. Appl. Phys.* **74(4)**, 2153 (1993)
[4] D C Agrawal, J M Gordon and M Huleihil, *Indian J. Engng. Mater. Sci.* **1**, 195 (1994)
[5] A Bejan, *J. Appl. Phys.* **79(3)**, 1191 (1996)
[6] M Feidt, *Thermodynamique et Optimisation Energetique des Systems et Procedes*, 2nd edn (Technique et Documentation, Lavoisier, Paris, 1996)
[7] L Chen, C Wu and F Sun, *J. Non-Equilib. Thermodyn.* **24(4)**, 327 (1999)
[8] R S Berry, V A Kazakov, S Sieniutycz, Z Szwast and A M Tsirlin, *Thermodynamic optimization of finite time processes* (Wiley, Chichester, 1999)
[9] S K Tyagi, G Lin, S C Kaushik and J Chen, *Int. J. Refrigeration* **27(6)**, 924 (2004)
[10] A Durmayaz, O S Sogut, B Sahin and H Yavuz, *Progress Energy & Combustion Science* **30(2)**, 175 (2004)
[11] L Chen and F Sun, *Advances in finite time thermodynamics: Analysis and optimization* (Nova Science Publishers, New York, 2004)
[12] S K Tyagi, J Chen and S C Kaushik, *Int. J. Ambient Energy* **26(3)**, 155 (2005)
[13] L Chen, *Finite-time thermodynamic analysis of irreversible processes and cycles* (Higher Education Press, Beijing, 2005) (in Chinese)
[14] S K Tyagi, G M Chen, Q Wang and S C Kaushik, *Int. J. Thermal Sci.* **45(8)**, 829 (2006)
[15] S K Tyagi, G M Chen, Q Wang and S C Kaushik, *Int. J. Refrigeration* **29(7)**, 1167 (2006)
[16] S K Tyagi, S W Wang, H Chandra, G M Chen, Q Wang and C Wu, *Int. J. Exergy* **4(1)**, 98 (2007)
[17] S Bhattacharyya and J Sarkar, *Energy Convers. Manage.* **48(3)**, 803 (2007)
[18] B H Li, Y R Zhao and J C Chen, *Pramana – J. Phys.* **70(5)**, 779 (2008)
[19] C H Blanchard, *J. Appl. Phys.* **51(5)**, 2471 (1980)
[20] F L Curzon and B Ahlborn, *Am. J. Phys.* **43(1)**, 22 (1975)
[21] Y Goth and M Feidt, *C. R. Acad. Sci. Paris* **303(1)**, 19 (1986)
[22] M Feidt, Finite time thermodynamics applied to optimization of heat pumps and refrigerating machine cycles, *12th IMACS World Congress on Scientific Computation* (Paris, France, 1988)
[23] I Philippi and M Feidt, Finite time thermodynamics applied to inverse cycle machine, *XVIII Int. Congress on Refrigeration* (Montreal, Canada, 1991)
[24] M Feidt, *Entropie* **205(1)**, 53 (1997)
[25] W Chen, F Sun and L Chen, *Chin. Sci. Bull.* **35(19)**, 1670 (1990)
[26] C Wu, *Int. J. Ambient Energy* **14(1)**, 25 (1993)
[27] L Chen, C Wu and F Sun, *Appl. Thermal Engng.* **17(1)**, 103 (1997)
[28] C Wu, L Chen and F Sun, *Energy Convers. Manage.* **39(5/6)**, 445 (1998)
[29] B Sahin and A Kodal, *Energy Convers. Manage.* **40(9)**, 951 (1999)
[30] L Chen, F Sun and W Chen, *J. Engng. Thermal Energy Pow.* **9(2)**, 121 (1994) (in Chinese)

[31] L Chen, C Wu and F Sun, *Int. J. Ambient Energy* **18(3)**, 129 (1997)
[32] L Chen and F Sun, *Sci. Tech. Bullet.* **11(2)**, 126 (1995) (in Chinese)
[33] C Wu and W Schulden, *Energy Convers. Manage.* **35(6)**, 459 (1994)
[34] F Sun, W Chen and L Chen, *Chin. J. Engng. Thermophys.* **12(4)**, 357 (1991) (in Chinese)
[35] M A Ait-Ali, *J. Phys. D: Appl. Phys.* **29(4)**, 975 (1996)
[36] C Cheng and C Chen, *J. Phys. D: Appl. Phys.* **28(12)**, 2451 (1995)
[37] L Chen and F Sun, *J. Naval Acad. Engng.* (4): 49 (1995) (in Chinese)
[38] L Chen and F Sun, *J. Engng. Thermophys.* **18(1)**, 25 (1997)
[39] L Chen, X Zhu, F Sun and C Wu, *Appl. Energy* **84(1)**, 78 (2007)
[40] A Kodal, B Sahin and T Yilmaz, *Energy Convers. Manage.* **41(6)**, 607 (2000)
[41] L Chen, F Sun and W Chen, *J. Engng. Thermal Energy Pow.* **5(3)**, 48 (1990) (in Chinese)
[42] X Zhu, L Chen, F Sun and C Wu, *Open System & Information Dyn.* **9(3)**, 251 (2002)
[43] F Sun, W Chen, L Chen and C Wu, *Energy Convers. Manage.* **38(14)**, 1439 (1997)
[44] W Chen, F Sun, S Cheng and L Chen, *Int. J. Energy Res.* **19(9)**, 751 (1995)
[45] M Feidt, Thermodynamics and optimization of reverse cycle machines, in: *Thermodynamic optimization of complex energy systems* edited by A Bejan and E Mamut (Kluwer Academic Press, Dordrecht, 1999) pp. 385–401
[46] J Li, L Chen and F Sun, *Appl. Energy* **85(2–3)**, 96 (2008)
[47] N Ni, L Chen, F Sun and Wu C, *J. Institute Energy* **72(491)**, 64 (1999)
[48] A Kodal, Heating rate maximization for an irreversible heat pump with a general heat transfer law, in: *Recent advances in finite time thermodynamics* edited by C Wu, L Chen and J Chen (Nova Science Publishers, New York, 1999) pp. 299–306
[49] X Zhu, L Chen, F Sun and C Wu, *Phys. Scr.* **64(6)**, 584 (2001)
[50] X Zhu, L Chen, F Sun and C Wu, *Int. J. Exergy* **2(4)**, 423 (2005)
[51] X Zhu, L Chen, F Sun and C Wu, *J. Energy Institute* **78(1)**, 5 (2005)
[52] L Chen, N Ni, C Wu and F Sun, *Int. J. Pow. Energy Systems* **21(2)**, 105 (2001)
[53] L Chen, N Ni, F Sun and C Wu, *Int. J. Pow. Energy Systems* **19(3)**, 231 (1999)
[54] Y Bi, L Chen, C Wu and S Wang, *J. Non-Equilib. Thermodyn.* **26(1)**, 41 (2001)
[55] L Chen, J Li, F Sun and C Wu, *Int. J. Ambient Energy* **28(4)**, 189 (2007)
[56] L Chen, J Li, F Sun and C Wu, *Appl. Energy* **85(7)**, 641 (2008)
[57] L Chen, F Meng and F Sun, *Proc. IMechE, Part A: J. Power & Energy* **223(A4)**, 329 (2009)
[58] A J Mortlock, *Am. J. Phys.* **33(8)**, 813 (1965)