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Stability of electrostatic ion cyclotron waves in a multi-ion plasma†

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Abstract. We have studied the stability of the electrostatic ion cyclotron wave in a plasma consisting of isotropic hydrogen ions (H^+) and temperature-anisotropic positively (O^+) and negatively (O^-) charged oxygen ions, with the electrons drifting parallel to the magnetic field. Analytical expressions have been derived for the frequency and growth/damping rate of ion cyclotron waves around the first harmonic of both hydrogen and oxygen ion gyrofrequencies. We find that the frequencies and growth/damping rates are dependent on the densities and temperatures of all species of ions. A detailed numerical study, for parameters relevant to comet Halley, shows that the growth rate is dependent on the magnitude of the frequency. The ion cyclotron waves are driven by the electron drift parallel to the magnetic field; the temperature anisotropy of the oxygen ions only slightly enhance the growth rates for small values of temperature anisotropies. A simple explanation, in terms of wave exponentiation times, is offered for the absence of electrostatic ion cyclotron waves in the multi-ion plasma of comet Halley.

Keywords. Ion cyclotron waves; multi-ions; negative ions; stability.

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1. Introduction

The electrostatic ion cyclotron instability (EICI) is a low-frequency, field-aligned current-driven instability of interest because it has one of the lowest threshold drift velocities among current-driven instabilities in an isothermal plasma. The EIC wave

†CVG and GR dedicate this paper to the memory of their revered teacher, the late Prof. K S Viswanathan.

propagates nearly perpendicular to the ambient magnetic field B_0 and has a small, but finite, wave number along B_0, so that it can be destabilized by electrons drifting along the magnetic field [1]. Early work on the EICI was done by Drummond and Rosenbluth [2] and reviewed by Kindel and Kennel [3] and Rasmussen and Schrittwieser [1]. Studies on these waves in multi-component plasmas include a fluid analysis of EIC waves in a negative ion plasma by D'Angelo and Merlino [4] and a kinetic analysis by Chow and Rosenberg [5]. More recently, Jalori and Gwal [6] investigated the role of magnetic shear on the electrostatic current-driven instability in the presence of a parallel electric field while Rosenberg and Shukla [7] studied the stability of obliquely propagating dust waves in a strongly magnetized collisional plasma.

Low-frequency electrostatic and electromagnetic turbulence have been observed in the vicinity of Giacobini–Ziner [8] and Halley comets [9]. The observed electrostatic turbulence, in the frequency range of 0–300 Hz [10], has been suggested as being due to lower hybrid waves [11]. Continuous observations of ion acoustic waves have also been made by the satellite ICE when the spacecraft was about 2×10^6 km away from Giacobini–Ziner comet [12]. Electromagnetic ion cyclotron waves around the gyrofrequencies of O^+ (or H_2O^+) were also observed by the satellite Sakigake [13]. However, there were no reports of EIC waves.

It is well-known that cometary plasmas contain different species of ions. Thus Giotto observations of the inner coma of comet Halley revealed a composition of cometary neutral gas and dust, thermal ions and electrons, fast cometary pick up ions such as O^+, H_2O^+, etc. These cometary new born ions had large perpendicular energies [14]. In addition to these positive ions, negatively charged oxygen ions were also unambiguously identified [15].

We have therefore, in this paper, studied the stability of electrostatic ion cyclotron waves in a plasma containing isotropic hydrogen ions (denoted as H^+) and temperature-anisotropic positively and negatively charged oxygen ions (denoted respectively by O^+ and O^-) with the electrons drifting with a velocity V_{de} parallel to the background magnetic field to find out the role played by temperature anisotropy of the heavy ions in exciting EIC waves. Expressions were derived for the growth/damping rates of both the light (H^+) and the heavy (O^+) ion cyclotron waves (ICWs). We find that, in general, the electron drift parallel to the magnetic field is the source of instability for both the light and heavy ion cyclotron waves rather than the temperature anisotropy of the oxygen ions.

2. Dispersion relation

We consider a four-component collisionless plasma consisting of electrons (e), hydrogen (H^+), positively charged oxygen (O^+) and negatively charged oxygen (O^-) ions. The equilibrium configuration has a uniform magnetic field, B_0, in the z-direction along which the electrons drift with a velocity V_{de}, relative to the ions. The electron distribution is given by

$$f_{0\mathrm{e}} = n_\mathrm{e}\left(\frac{m_\mathrm{e}}{2\pi k_\mathrm{B} T_\mathrm{e}}\right)^{3/2} \exp\left[-\frac{m_\mathrm{e}}{2k_\mathrm{B}T_\mathrm{e}}(v_\perp^2 + (v_\parallel - V_\mathrm{de})^2)\right] \tag{1}$$

while the temperature anisotropic ion component

$$f_{0j} = n_j \left(\frac{m_j}{2\pi k_{\mathrm{B}} T_{\perp j}}\right) \left(\frac{m_j}{2\pi k_{\mathrm{B}} T_{\parallel j}}\right)^{1/2} \times \exp\left[-\frac{m_j}{2k_{\mathrm{B}} T_{\perp j}} v_\perp^2 - \frac{m_j}{2k_{\mathrm{B}} T_{\parallel j}} v_\parallel^2\right], \tag{2}$$

where $j = \mathrm{O}^+$ or O^-. The isotropic component describing H^+ is obtained from (2) by putting $j = \mathrm{H}^+$ and $T_{\parallel j} = T_{\perp j} = T_{\mathrm{H}^+}$. In (1) and (2) all symbols have their standard meanings.

We consider electrostatic ion cyclotron waves with wave vectors $k_\perp$ and $k_\parallel$ respectively perpendicular and parallel to the ambient magnetic field. The dispersion relation for these waves is then given by

$$D(\omega, \mathbf{k}) = 1.0 + \sum_\alpha \chi_\alpha = 0, \tag{3}$$

where $\alpha = \mathrm{e}$, H^+, O^+ or O^- refers to each species of the particle. For frequencies comparable to the ion gyrofrequencies and much less than the electron gyrofrequency, the electron contribution, χ_{e}, is given by [5]

$$\chi_{\mathrm{e}} \approx \frac{1}{k^2 \lambda_{\mathrm{e}}^2} \left[1 + \Gamma_0(\mu_{\mathrm{e}}) \frac{\omega - k_\parallel V_{\mathrm{de}}}{\sqrt{2} k_\parallel V_{\mathrm{e}}} Z\left(\frac{\omega - k_\parallel V_{\mathrm{de}}}{\sqrt{2} k_\parallel V_{\mathrm{e}}}\right)\right] \tag{4a}$$

while

$$\chi_{\mathrm{H}^+} = \frac{1}{k^2 \lambda_{\mathrm{H}^+}^2} \left[1 + \sum_{n=-\infty}^{n=+\infty} \Gamma_n(\mu_{\mathrm{H}^+}) \frac{\omega}{\sqrt{2} k_\parallel V_{\mathrm{H}^+}} Z\left(\frac{\omega - n\Omega_{\mathrm{H}^+}}{\sqrt{2} k_\parallel V_{\mathrm{H}^+}}\right)\right] \tag{4b}$$

and

$$\chi_j = \frac{1}{k^2 \lambda_{\perp j}^2} \left\{\sum_{n=-\infty}^{n=+\infty} \Gamma_n(\mu_{\perp j}) \times \left[\frac{T_{\perp j}}{T_{\parallel j}} + \left(A_{tj} \frac{n\Omega_j}{\sqrt{2} k_\parallel V_j} + \frac{T_{\perp j}}{T_{\parallel j}} \frac{\omega}{\sqrt{2} k_\parallel V_j}\right) Z\left(\frac{\omega - n\Omega_j}{\sqrt{2} k_\parallel V_j}\right)\right]\right\}, \tag{4c}$$

where

$$\Gamma_n(\mu_{\perp\alpha}) = \exp(-\mu_{\perp\alpha}) I_n(\mu_{\perp\alpha}) \tag{4d}$$

with

$$\mu_{\perp\alpha} = k_\perp^2 r_{\perp\alpha}^2. \tag{4e}$$

In (4a)–(4c), Z is the plasma dispersion function, $\lambda_{\perp\alpha} = (k_{\mathrm{B}} T_{\perp\alpha}/4\pi n_\alpha q_\alpha^2)^{1/2}$, $V_{\parallel(\perp)\alpha} = (k_{\mathrm{B}} T_{\parallel(\perp)\alpha}/m_\alpha)^{1/2}$, $\Omega_\alpha = q_\alpha B/(m_\alpha c)$ and $r_{\perp\alpha} = V_{\perp\alpha}/\Omega_\alpha$ represent respectively the Debye lengths, thermal velocities parallel (perpendicular) to the magnetic field, the (signed) gyrofrequency and the gyroradius of the various species. I_n

is the modified Bessel function of order n, while q_α is the charge of species $\alpha(=+\mathrm{e}$ or $-\mathrm{e})$ and c is the velocity of light. Throughout this paper we shall use the unsigned gyrofrequency $\Omega_\alpha = |\Omega_\alpha|$. It should be emphasized that both electrons and hydrogen ions are temperature isotropic, while both O^+ and O^- are temperature anisotropic and $A_{tj} = 1 - (T_{\perp j}/T_{\parallel j})$, $j = O^+$ or O^-. For the electron term in (4a) we use the fact that $\mu_\mathrm{e} \ll 1$ (small electron gyroradius) and retain only the $n = 0$ term in the summation since $\Gamma_0(0) = 1$ and $\Gamma_n(0) = 0$, $n \neq 0$.

2.1 *Analytical results for the light ion EICI*

As mentioned above, in this paper we are interested in the EICI instability in a plasma containing drifting electrons, temperature-isotropic hydrogen ions and two temperature-anisotropic species of oxygen ions (O^+ and O^-). In this section, we discuss the dispersion relation and growth/damping rate for frequencies around the first harmonic of the light ion (hydrogen) gyrofrequency. We thus need the power series expansion for Z for the electrons and its asymptotic expansion for the ions. We thus find that the $n = 0$ and 1 terms of hydrogen ions and the $n = 0$ electron term contribute to the dispersion relation. The $n = 0$ and the $n = p$ (pth harmonic) terms of O^+ and O^- also contribute. Collecting all these terms and solving eq. (3) we find, after a lengthy simplification, that the real frequency can be written as

$$\omega_r = \Omega_{\mathrm{H}^+}(1 + \Delta), \tag{5}$$

where

$$\Delta = \frac{a_{\mathrm{H}^+} + a_{\mathrm{O}^+} + a_{\mathrm{O}^-}}{1 + k^2\lambda_\mathrm{e}^2 + b_{\mathrm{H}^+}} \tag{6}$$

with

$$a_{\mathrm{H}^+} = \frac{T_\mathrm{e}}{T_{\mathrm{H}^+}}\delta\Gamma_1(\mu_{\mathrm{H}^+}) \tag{7a}$$

$$a_{\mathrm{O}^+} = \frac{T_\mathrm{e}}{T_{\perp\mathrm{O}^+}}\delta\varepsilon^+ c_{\mathrm{O}^+}\Gamma_p(\mu_{\perp\mathrm{O}^+}) \tag{7b}$$

$$a_{\mathrm{O}^-} = \frac{T_\mathrm{e}}{T_{\perp\mathrm{O}^-}}\delta\varepsilon^- c_{\mathrm{O}^-}\Gamma_p(\mu_{\perp\mathrm{O}^-}) \tag{7c}$$

and

$$b_{\mathrm{H}^+} = \frac{T_\mathrm{e}}{T_{\mathrm{H}^+}}\delta(1 - \Gamma_0(\mu_{\mathrm{H}^+}) - \Gamma_1(\mu_{\mathrm{H}^+})). \tag{7d}$$

In (7a)–(7d)

$$\delta = \frac{n_{\mathrm{H}^+}}{n_\mathrm{e}} \tag{7e}$$

$$\varepsilon^{+} = c_{\mathrm{O}^{+}} \frac{n_{\mathrm{O}^{+}}}{n_{\mathrm{H}^{+}}} \tag{7f}$$

$$\varepsilon^{-} = c_{\mathrm{O}^{-}} \frac{n_{\mathrm{O}^{-}}}{n_{\mathrm{H}^{+}}}. \tag{7g}$$

In (7e)–(7g), n_α denotes the species density while c_{O^+} and c_{O^-} denote the charge numbers on O^+ and O^- respectively.

Putting $\omega = \omega_r + i\omega_i$ and using the imaginary part of Z and following the same procedure as was used in deriving (5), we can write the final expression for the growth/damping rate as

$$\frac{\omega_i}{\Omega_{\mathrm{H}^+}} = -\sqrt{\frac{\pi}{2}}\Delta^2 \frac{d_{\mathrm{e}} + d_{\mathrm{H}^+} + d_{\mathrm{O}^+} + d_{\mathrm{O}^-}}{e_{\mathrm{H}^+} + e_{\mathrm{O}^+} + e_{\mathrm{O}^-}}, \tag{8}$$

where

$$d_{\mathrm{e}} = (\omega_r - k_\parallel V_{\mathrm{de}})/k_\parallel V_{\mathrm{e}} \tag{9a}$$

$$d_{\mathrm{H}^+} = \frac{T_{\mathrm{e}}}{T_{\mathrm{H}^+}} \delta \frac{\omega_r}{k_\parallel V_{\mathrm{H}^+}} \Gamma_1(\mu_{\mathrm{H}^+}) \exp\left[-\left(\frac{\omega_r - \Omega_{\mathrm{H}^+}}{\sqrt{2}k_\parallel V_{\mathrm{H}^+}}\right)^2\right] \tag{9b}$$

$$d_{\mathrm{O}^+} = \frac{T_{\mathrm{e}}}{T_{\perp\mathrm{O}^+}} \varepsilon^{+} c_{\mathrm{O}^+} \delta \frac{\Omega_{\mathrm{H}^+}}{k_\parallel V_{\parallel\mathrm{O}^+}} \Gamma_p(\mu_{\perp\mathrm{O}^+}) \times \exp\left[-\left(\frac{\omega_r - p\Omega_{\mathrm{O}^+}}{\sqrt{2}k_\parallel V_{\parallel\mathrm{O}^+}}\right)^2\right]\left[\frac{T_{\perp\mathrm{O}^+}}{T_{\parallel\mathrm{O}^+}}\Delta + 1\right] \tag{9c}$$

$$d_{\mathrm{O}^-} = \frac{T_{\mathrm{e}}}{T_{\perp\mathrm{O}^-}} \varepsilon^{-} c_{\mathrm{O}^-} \delta \frac{\Omega_{\mathrm{H}^+}}{k_\parallel V_{\parallel\mathrm{O}^-}} \Gamma_p(\mu_{\perp\mathrm{O}^-}) \times \exp\left[-\left(\frac{\omega_r - p\Omega_{\mathrm{O}^-}}{\sqrt{2}k_\parallel V_{\parallel\mathrm{O}^-}}\right)^2\right]\left[\frac{T_{\perp\mathrm{O}^-}}{T_{\parallel\mathrm{O}^-}}\Delta + 1\right] \tag{9d}$$

$$e_{\mathrm{H}^+} = \frac{T_{\mathrm{e}}}{T_{\mathrm{H}^+}} \delta \Gamma_1(\mu_{\mathrm{H}^+}) \tag{10a}$$

$$e_{\mathrm{O}^+} = \frac{T_{\mathrm{e}}}{T_{\perp\mathrm{O}^+}} \varepsilon^{+} c_{\mathrm{O}^+} \delta \Gamma_p(\mu_{\perp\mathrm{O}^+}) \tag{10b}$$

and

$$e_{\mathrm{O}^-} = \frac{T_{\mathrm{e}}}{T_{\perp\mathrm{O}^-}} \varepsilon^{-} c_{\mathrm{O}^-} \delta \Gamma_p(\mu_{\perp\mathrm{O}^-}). \tag{10c}$$

2.1.1 *Discussion*

We now discuss relations (5) and (8), the expressions for the frequency and the growth/damping rate for EIC waves around the first harmonic of the hydrogen ion gyrofrequency. From (5) we find that as Δ is positive, the waves have a frequency above Ω_{H^+} and the addition of the heavy ions raises the frequency above Ω_{H^+}. Also from (9c) and (9d), we find that both O^+ and O^- contribute equally to the growth/damping rate of the wave; this contribution increases with increasing deviation from Ω_{H^+}.

2.2 *Analytical results for the heavy ion EICI*

In this section, we derive expressions for the wave frequency and the growth/damping rate for waves around the first harmonic of the heavy ion (oxygen) gyrofrequency. Again we use the power series expansion of Z for electrons and it's asymptotic expansion for the ions. Thus the electron contribution remains the same. However, since we are considering waves around the oxygen gyrofrequency, only the $n = 0$ hydrogen ion term contributes. The O^+ contribution is restricted to the $n = +1$ term; the same for O^- is from the $n = -1$ term.

Collecting all the terms and simplifying, we find that the expression for the wave frequency, around the first harmonic of the O^+ ion gyrofrequency, is given by

$$\omega_r = \Omega_{\mathrm{O}^+}(1 + \Delta_{\mathrm{O}^+}), \tag{11}$$

where

$$\Delta_{\mathrm{O}^+} = \frac{a'_{\mathrm{O}^+} + a'_{\mathrm{O}^-}}{1 + k^2\lambda^2_{\mathrm{H}^+} + b'_{\mathrm{H}^+}} \tag{12}$$

with

$$a'_{\mathrm{O}^+} = \frac{T_{\mathrm{H}^+}}{T_{\perp\mathrm{O}^+}}\varepsilon^+ c_{\mathrm{O}^+}\Gamma_1(\mu_{\perp\mathrm{O}^+}) \tag{13a}$$

$$a'_{\mathrm{O}^-} = \frac{T_{\mathrm{H}^+}}{T_{\perp\mathrm{O}^-}}\varepsilon^- c_{\mathrm{O}^-}\Gamma_1(\mu_{\perp\mathrm{O}^-}) \tag{13b}$$

and

$$b'_{\mathrm{H}^+} = \frac{T_{\mathrm{H}^+}}{T_{\mathrm{e}}}(1 + \varepsilon^+ - \varepsilon^-). \tag{13c}$$

Finally, adopting the same procedure as in §2.1, the expression for the growth/damping rate for waves around the oxygen ion gyrofrequency can be written as

$$\frac{\omega_i}{\Omega_{\mathrm{O}^+}} = -\sqrt{\frac{\pi}{2}}\Delta^2_{\mathrm{O}^+}\frac{d'_{\mathrm{e}} + d'_{\mathrm{H}^+} + d'_{\mathrm{O}^+} + d'_{\mathrm{O}^-}}{e'_{\mathrm{O}^+} + e'_{\mathrm{O}^-}}, \tag{14}$$

where

$$d'_{\mathrm{e}} = \frac{T_{\mathrm{H}^+}}{T_{\mathrm{e}}}(1+\varepsilon^+ - \varepsilon^-)(\omega_r - k_\parallel V_{\mathrm{de}})/k_\parallel V_{\mathrm{e}} \tag{15a}$$

$$d'_{\mathrm{H}^+} = \frac{\omega_r}{k_\parallel V_{\mathrm{H}^+}}\Gamma_0(\mu_{\mathrm{H}^+})\exp\left[-\left(\frac{\omega_r}{\sqrt{2}k_\parallel V_{\mathrm{H}^+}}\right)^2\right] \tag{15b}$$

$$\begin{aligned} d'_{\mathrm{O}^+} = {} & \frac{T_{\mathrm{H}^+}}{T_{\perp\mathrm{O}^+}}\varepsilon^+ c_{\mathrm{O}^+}\frac{\Omega_{\mathrm{O}^+}}{k_\parallel V_{\parallel\mathrm{O}^+}}\Gamma_1(\mu_{\perp\mathrm{O}^+}) \\ & \times\exp\left[-\left(\frac{\omega_r - \Omega_{\mathrm{O}^+}}{\sqrt{2}k_\parallel V_{\parallel\mathrm{O}^+}}\right)^2\right]\left[\frac{T_{\perp\mathrm{O}^+}}{T_{\parallel\mathrm{O}^+}}\Delta_{\mathrm{O}^+} + 1\right] \end{aligned} \tag{15c}$$

$$\begin{aligned} d'_{\mathrm{O}^-} = {} & \frac{T_{\mathrm{H}^+}}{T_{\perp\mathrm{O}^-}}\varepsilon^- c_{\mathrm{O}^-}\frac{\Omega_{\mathrm{O}^-}}{k_\parallel V_{\parallel\mathrm{O}^-}}\Gamma_1(\mu_{\perp\mathrm{O}^-}) \\ & \times\exp\left[-\left(\frac{\omega_r - \Omega_{\mathrm{O}^-}}{\sqrt{2}k_\parallel V_{\parallel\mathrm{O}^-}}\right)^2\right]\left[\frac{T_{\perp\mathrm{O}^-}}{T_{\perp\mathrm{O}^-}}\Delta_{\mathrm{O}^+} + 1\right] \end{aligned} \tag{15d}$$

$$e'_{\mathrm{O}^+} = \frac{T_{\mathrm{H}^+}}{T_{\perp\mathrm{O}^+}}\varepsilon^+ c_{\mathrm{O}^+}\Gamma_1(\mu_{\perp\mathrm{O}^+}) \tag{15e}$$

and

$$e'_{\mathrm{O}^-} = \frac{T_{\mathrm{H}^+}}{T_{\perp\mathrm{O}^-}}\varepsilon^- c_{\mathrm{O}^-}\Gamma_1(\mu_{\perp\mathrm{O}^-}). \tag{15f}$$

2.2.1 *Discussion*

We now discuss relations (11) and (14), the expressions for the frequency and the growth/damping rate of the wave. A comparison of (6) and (12) shows that the deviation of the ion cyclotron wave around Ω_{O^+} is mainly dependent on the parameters of the O^+ and O^- ions, unlike in the previous case where it was dependent on all three species of ions. Similarly, the expression for the growth/damping rate is also dependent on the parameters of the O^+ and O^- ions with only a small contribution from the hydrogen ions.

3. Results

Lower hybrid waves were observed by plasma wave instruments aboard the satellite VEGA [16]. More interestingly electrostatic turbulence in the broad frequency range of 0–300 Hz was observed in the vicinity of comet Halley [10]. Similarly, continuous observations of strong ion-acoustic waves by the spacecraft ICE, when it was about 2×10^6 km away from the nucleus of the Giacobini–Ziner comet, was also reported [12]. The presence of electrostatic waves and turbulence in cometary plasma environments has thus been firmly established.

The Giotto spacecraft observed negative ions in three broad mass peaks at 7–19, 22–65 and 85–110 amu with densities ≥ 1, 5×10^{-2} and 4×10^{-2} cm^{-3} respectively near the coma of comet Halley. Their energies ranged between 0.03 eV and 3.0 eV, with a background energy of 1.0 keV. Of the many ionic species, O^- was unambiguously identified [15].

The expressions for the growth/damping rates, namely (8) and (14), were derived using a power series expansion of the plasma dispersion function Z for the electrons and an asymptotic expansion of the same for all the ions. However, since it is necessary to retain higher harmonics for more accurate results, it was decided to solve numerically the dispersion relation as given by (3) [5]. The output of the routine for the computation of the plasma dispersion function [17] was compared with the standard values [18] and the routines for the modified Bessel function were obtained from Press *et al* [19]. The dispersion relation was then set up and solved by the standard routine ZANALY which is based on Muller's method [19].

The hydrogen ion density was therefore set at 3.0 cm^{-3} with an energy of 1.0 keV; the energy of the electrons was set at 10.0 keV. The background magnetic field was assumed to be 10γ. These assignments of densities and temperatures are in good agreement with the observed values. The dispersion relation (3) was then solved numerically by varying the propagation angle (k_z/k_y), the heavy ion densities and temperature anisotropies and the drift velocity of the electrons.

We first consider EIC waves, around the gyrofrequency of oxygen. Figure 1 is thus a plot of the growth/damping rate vs. the normalized wave vector $k_\perp r_{\mathrm{Le}}$ in a plasma containing O^+ and O^- (the parameters of these ions being $c_{O^+} = c_{O^-} = 1$; $n_{O^+} = 0.4$ cm^{-3}, $n_{O^-} = 0.1$ cm^{-3}, $n_{H^+} = 0$, $T_{O^+} = T_{O^-} = 350.0$ eV, $(T_{\perp O^+}/T_{\| O^+}) = (T_{\perp O^-}/T_{\| O^-}) = 5.0$) and electrons, which have a normalized drift velocity $V_{\mathrm{de}}/V_{\mathrm{e}} = 1.75$, as a function of k_z/k_y (= 0.06, curve (a); = 0.09, curve (b) and = 0.12, curve (c)). We find that in all the cases that the wave growths increase with increasing $k_\perp r_{Le}$ reaches a maximum at $k_\perp r_{Le} = 0.025$ and then gradually decreases as $k_\perp r_{Le}$ increases. Wave growth occurs when the drift velocity of the electrons V_{de} is greater than the phase velocity ω_r/k_z of the wave; or, in other words, when the kinetic energy of the streaming electrons is greater than the wave energy so that there is a net transfer of energy from the particle to the wave and the wave grows. The electron drift velocity is thus the source of instability though the temperature anisotropy of O^+ and O^- also contributes marginally.

To verify our assertion that the electron drift velocity is the source of instability, the density of n_{O^-} was increased. Thus the heavy ion parameter values were $c_{O^+} = c_{O^-} = 1$; $n_{O^+} = 0.4$ cm^{-3}; $n_{O^-} = 0.3$ cm^{-3}; the temperatures of O^+ and O^- and the temperature anisotropy values being the same as in figure 1 with

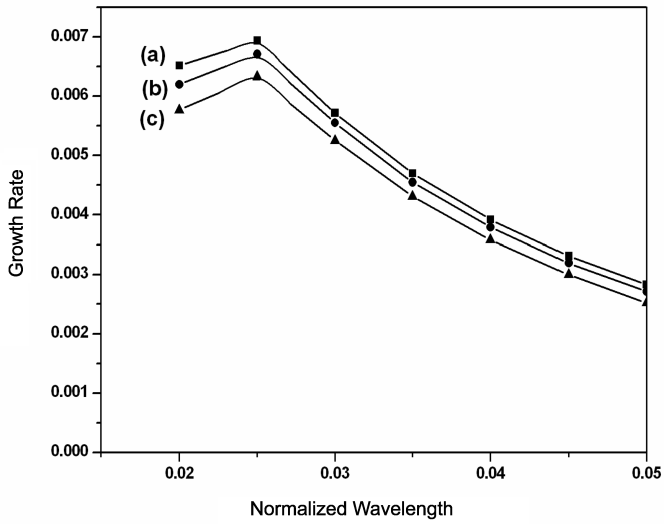

Figure 1. Plot of the growth rate vs. the normalized wave vector $k_\perp r_{\rm Le}$ in a plasma containing only O^+ and O^- (the parameters of these ions being $c_{O^+} = c_{O^-} = 1$; $n_{O^+} = 0.4$ cm^{-3}, $n_{O^-} = 0.1$ cm^{-3}, $n_{\rm H} = 0$, $T_{O^+} = T_{O^-} = 350.0$ eV, $(T_{\perp O^+}/T_{\parallel O^+}) = (T_{\perp O^-}/T_{\parallel O^-}) = 5.0$) and electrons which have a normalized drift velocity $V_{\rm de}/V_{\rm e} = 1.75$ as a function of k_z/k_y (= 0.06, curve (a); = 0.09, curve (b) and = 0.12, curve (c)).

$k_z/k_y = 0.09$ and $V_{\rm de}/V_{\rm e} = 1.75$. We find the growth rate in the latter case ($n_{O^-} = 0.3$ cm^{-3}) to be consistently smaller than the former case ($n_{O^-} = 0.1$ cm^{-3}). This is because as the negative oxygen density increases, the electron density (required to maintain charge neutrality) decreases and hence we have a weaker mechanism to drive the instability.

To further confirm our assumption, the computations were carried out keeping the negative ion density a constant ($c_{O^+} = c_{O^-} = 1$, $n_{O^+} = 0.4$ cm^{-3}, $n_{O^+} = 0.3$ cm^{-3}, $k_z/k_y = 0.09$, the oxygen ion temperatures and anisotropies being the same as in figure 1) but as a function of $V_{\rm de}/V_{\rm e}$ (= 1.25 and 1.75). We find the growth rate is larger when $V_{\rm de}/V_{\rm e} = 1.75$ as compared to $V_{\rm de}/V_{\rm e} = 1.25$; thus reaffirming our conclusion that the electron drift is the driving agent of the instability.

The growth rate was also studied as a function of the O^+ and O^- anisotropy. The parameters that were held constant were $c_{O^+} = c_{O^-} = 1$, $n_{O^+} = 0.4$ cm^{-3}, $n_{O^-} = 0.3$ cm^{-3}, $k_z/k_y = 0.06$ and $V_{\rm de}/V_{\rm e} = 1.5$ while $T_\perp/T_\parallel$ for the oxygen ions were varied from a large value of 0.03 to a nominal value of 3.0. We find the growth rate to be slightly larger when the oxygen ions have a large anisotropy ($T_{\perp O^+}/T_{\parallel O^+} = 0.03$ and $T_{\perp O^-}/T_{\parallel O^-} = 0.03$) as compared to a nominal value of $T_{\perp O^+}/T_{\parallel O^+} = 3.0$ and $T_{\perp O^-}/T_{\parallel O^-} = 3.0$.

We next consider EIC waves around the first harmonic of the hydrogen ion gyrofrequency. Figure 2 is thus a plot of the wave growth rate vs. the normalized wave vector $k_\perp r_{\rm Le}$ for waves around the hydrogen ion gyrofrequency Ω_{H^+}. The composition of the plasma was as follows: $n_{O^+} = 0.4$ cm^{-3}, $n_{O^-} = 0.1$ cm^{-3},

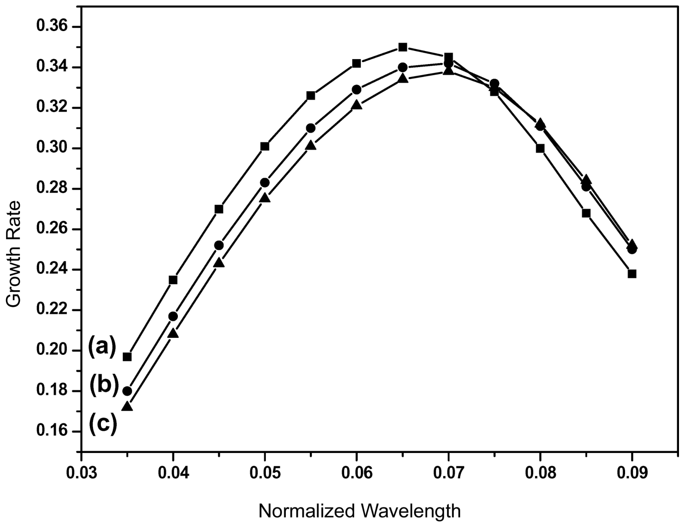

Figure 2. Plot of the growth rate vs. the normalized wave vector $k_\perp r_{\rm Le}$ for waves around the hydrogen ion gyrofrequency $\Omega_{\rm H^+}$. The composition of the plasma is: $n_{\rm O^+} = 0.4$ cm^{-3}, $n_{\rm O^-} = 0.1$ cm^{-3}, $n_{\rm H^+} = 3.0$ cm^{-3} with $c_{\rm O^+} = c_{\rm O^-} = 1$. The temperatures of both O^+ and O^- are 350.0 eV; their temperature anisotropy = 3.0 and $V_{\rm de}/V_{\rm e} = 1.25$. Curve (a) is for $k_z/k_y = 0.06$; curve (b) for $k_z/k_y = 0.09$ and curve (c) for $k_z/k_y = 0.12$.

$n_{\rm H^+} = 3.0$ cm^{-3} with $c_{\rm O^+} = c_{\rm O^-} = 1$. The temperatures of both O^+ and O^- were 350.0 eV; the temperature anisotropy of both species of oxygen ions was 3.0 and $V_{\rm de}/V_{\rm e} = 1.25$. Curve (a) is for $k_z/k_y = 0.06$; curve (b) for $k_z/k_y = 0.09$ and curve (c) for $k_z/k_y = 0.12$. We find that, as in the case of the heavy-ion EIC waves, the growth rate decreases with increasing k_z/k_y . However, in this case the growth rates start from low values, reach a maximum and then gradually decrease. Since the hydrogen ions are temperature isotropic, the source driving the instability is again the drift velocity of the electrons parallel to the magnetic field. An interesting feature is the growth rate when compared to figure 1; which allows us to conclude that the growth rate increases with increasing frequency. It may also be noted here that the addition of O^+ ions reduces the critical streaming velocity of the electrons needed for the onset of the instability. This critical velocity is, however, independent of the O^+ anisotropy.

Finally in figure 3, we again plot the growth rate vs. $k_\perp r_{Le}$ as a function of $V_{\rm de}/V_{\rm e}$ (=1.25, curve (a) and 1.5, curve (b)); the other parameters being $c_{\rm O^+} = c_{\rm O^-} = 1$, $k_z/k_y = 0.06$, $n_{\rm O^+} = 0.4$ cm^{-3}, $n_{\rm O^-} = 0.1$ cm^{-3}, $n_{\rm H^+} = 3.0$ cm^{-3}; $T_{\perp \rm O^+}/T_{\| \rm O^+} = T_{\perp \rm O^-}/T_{\| \rm O^-} = 3.0$. We find that the growth rate for $V_{\rm de}/V_{\rm e} = 1.5$ is lower than that for $V_{\rm de}/V_{\rm e} = 1.25$, in apparent contradiction to our conclusion that the driving mechanism behind the instability is the electron drift velocity $V_{\rm e}$. However, in the region where the growth rate is lower, we find the frequency to be also lower as compared to the situation for curve (a) ($V_{\rm de}/V_{\rm e} = 1.25$). This is in agreement with our conclusion that the growth rate increases with increasing frequency of the waves.

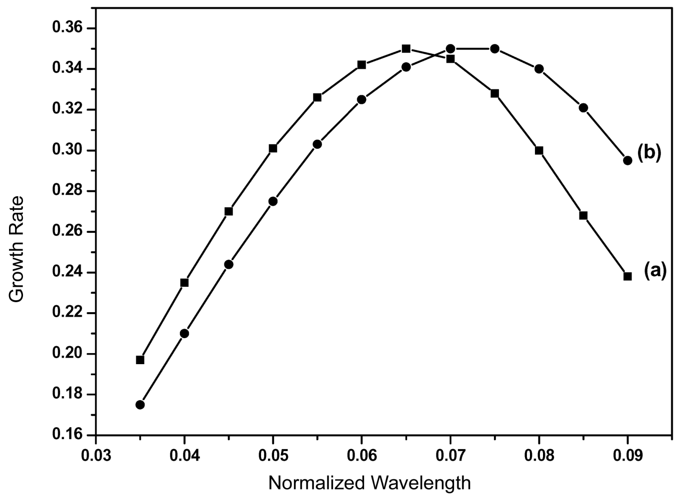

Figure 3. Plot of the growth rate vs. $k_\perp r_{\mathrm{Le}}$ as a function of $V_{\mathrm{de}}/V_{\mathrm{e}}$ (= 1.25, curve (a), = 1.5, curve (b)); the other parameters being $c_{\mathrm{O}^+} = c_{\mathrm{O}^-} = 1$, $n_{\mathrm{O}^+} = 0.4$ cm^{-3}, $n_{\mathrm{O}^-} = 0.1$ cm^{-3}, $n_{\mathrm{H}^+} = 3.0$ cm^{-3}; $k_z/k_y = 0.06$, $T_{\perp\mathrm{O}^+}/T_{\parallel\mathrm{O}^+} = T_{\perp\mathrm{O}^-}/T_{\parallel\mathrm{O}^-} = 3.0$. The temperatures of both O^+ and O^- are 350.0 eV.

For the maximum value of the growth rate around the O^+ ion gyrofrequency, the wave exponentiation time is about 144.1 s, while for the magnetic field under consideration the O^+ gyroperiod is 104.7 s. In the case of H^+, the corresponding times are respectively 4.8 s and 6.5 s. Thus, the waves cannot grow to appreciable amplitudes before they are convected away by the solar wind [13]. This could be the reason why electrostatic ion cyclotron waves have not been observed in the multi-ion plasma environment of comet Halley.

4. Conclusions

We have studied, in this paper, the stability of the electrostatic ion cyclotron wave in a plasma consisting of isotropic hydrogen ions (H^+) and temperature anisotropic positively (O^+) and negatively (O^-) charged oxygen ions, with the electrons drifting parallel to the magnetic field. Analytical expressions for the frequency and growth/damping rate of ion cyclotron waves around the first harmonic of both hydrogen and oxygen ion gyrofrequencies reveal that the frequencies and growth/damping rates are dependent on the densities and temperatures of all species of ions. A detailed numerical study, for parameters relevant to comet Halley, shows that the growth rate is dependent on the magnitude of the frequency. The ion cyclotron waves are driven by the electron drift parallel to the magnetic field; the temperature anisotropy of the oxygen ions only slightly enhances the growth rates for small values of temperature anisotropies. A simple explanation, in terms of wave exponentiation times, is offered for the non-observation of electrostatic ion cyclotron waves in the multi-ion plasma of comet Halley.

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