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On the discovery of the Greisen–Zatzepin–Kuzmin cut-off

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Abstract. The recent claim of the '5 sigma' observation of the Greisen–Zatzepin–Kuzmin cut-off by the HiRes group based on their nine years data is a significant step towards the eventual solution of one of the most intriguing questions in physics for more than 40 years. Recent results from Pierre Auger Observatory seem to confirm the statement. However, the word 'significance' is used in the mentioned paper in a sense which is not quite obvious. In the present paper we argue that this claim is a little premature.

Keywords. Statistical inference; Bayesian confidence level; cosmic rays.

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1. Introduction

According to the usual practice, the '5 sigma' confidence level (the chance probability of occurrence of about 1:10000000) is the level of discovery in physics (an observation of something which by chance can appear so rarely gives us the right to believe that there is some cause, yet unknown, but worth being studied further). A discussion of the recent 'GZK cut-off discovery' is the subject of the present paper.

After almost 100 years of research, the origin of cosmic rays is still an open question. The CR energy spectrum exhibits little structure and is approximated by broken power laws. The first break, called the 'knee', appears at an energy $E \approx 4 \times 10^{15}$ eV, and the differential flux of particles steepens from a power law index of about 2.7 to one of index 3.0. The bulk of the CR flux up to at least this energy is believed to originate within the Galaxy. The spectrum continues with a further steepening to $\sim$3.3 at $E \approx 4 \times 10^{17}$ eV, sometimes called the 'second knee'. There are some indications (ref. [1]), that the mass composition changes from light at the first knee to heavy (dominated by iron nuclei) at the second knee. This is expected if acceleration and propagation are due to magnetic fields only and depend on particle rigidity.

The dip (or 'ankle') at $E \approx 5 \times 10^{18}$ eV could be related to a transition from the galactic cosmic ray flux below to an extragalactic one above. Some argue that this change takes place earlier [2], and some argue that the change takes place after [3] the dip. In any event the spectrum then flattens again to a power law with an index of $\sim$2.8 forming the 'ankle'.

The possibility of another change of the CR composition at the ankle is the subject of extensive experimental studies. If there is a proton-dominated extragalactic cosmic ray flux, then, according to the works of Greisen [4], and Zatsepin and Kuzmin [5] (GZK) published in 1966 just after the cosmic microwave background (CMB) had been discovered, a suppression of the cosmic-ray flux beyond certain energy is inevitable. The mechanism of this cut-off is that protons travelling intergalactic distances would interact with the CMB photons and lose energy, producing the Δ^+ resonance. An energy threshold for this process was predicted in 1966 at $\sim 6 \times 10^{19}$ eV.

The observation of a sharp cut-off of the CR flux around this energy is an evidence of the proton-dominant composition of the ultrahigh-energy cosmic ray (UHECR).

On the other hand, the observation of the absence of the cut-off is no less meaningful. It means that the particles there, because there should be no protons, ought to be heavy nuclei or something even more exotic. The heavy nuclei would doubtlessly disprove any top-down mechanisms of UHECR particle creation.

Of course, there is still a possibility that all extragalactic UHECR (including protons above the GZK threshold) are coming from distances less than $\sim$50 Mpc which is very close in cosmological scales, in which case there would be no cut off.

The HiRes group claimed recently that the idea of Greisen–Zatzepin–Kuzmin was confirmed with the statistical significance of five standard deviations.

From the experimental point of view the determination of particle energy is based on the fact, known for more than 60 years, that a particle entering the atmosphere initiates a cascade of secondary particles created in the chain of subsequent interactions. This cascade, called an extensive air shower (EAS), consists of a huge number of charged particles (mostly electrons and positrons) which lose energy by exciting the atmospheric atoms. They in turn emit light, mainly in the UV region, and this light can be, in principle, registered. Registration of the scintillation light flashes is a way of 'counting' the number of charged particles in an EAS. Another one is to count the particles reaching the ground with the help of detectors spread over a wide area. The number of particles, the 'shower size', is related to the total primary energy of the UHECR particle. The question of how to transform the EAS size in each individual case to the primary particle energy, or the whole measured size spectrum to the primary energy spectrum, is the subject of extensive simulation studies. It is believed that our knowledge allows such transformation with a reasonable accuracy (of the order of about 20%).

The HiRes project uses the fluorescent technique (see [6,7]). The experiment consists of two detector stations (HiRes-I and HiRes-II) located in the Dugway Desert in Utah, US, 12.6 km apart. Each station is assembled from telescope modules (22 at HiRes-I and 42 at HiRes-II) pointing at different parts of the sky, covering nearly 360° in azimuth, and 3°–17° (HiRes-I), and 3°–31° (HiRes-II) in elevation. Each telescope module collects light from air showers using a spherical mirror of about

4 m^2 area. The camera for each telescope is a cluster of photomultipliers of the field of view of a 1° diameter cone on the sky [8].

The data of HiRes consist of three sets: two of them are the monocular data collected by HiRes-I and HiRes-II detector stations, and the third is the set of events registered simultaneously by both stations. The smaller statistics of the last one is related to the high-energy threshold for the showers to be seen by both stations, as well as the limited geometry and thus effective collection area. However, the stereo-observation makes the energy estimation much more accurate and gives the possibility of checking the mono-eye reconstruction procedures for systematic biases, at least in the limited sample of stereoevents.

The conclusions in ref. [9] are based only on the monocular HiRes data from both stations (the showers seen by both were excluded from the HiRes-I sample to preserve the statistical independence of both sets of measurements). We would like to have a look as well at the stereodata which can be found, e.g., in ref. [10].

We do not want to discuss here all the details concerning energy determination procedures, sources of uncertainties (including possible systematics), as well as the complicated question of the aperture estimation. These questions were the subject of extensive analysis by the HiRes team for many years. The published statements make the procedures as trustworthy as one can get, taking into account the facts that we still do not know the very high energy interaction mechanism, the fluorescence yield, and the status of the atmosphere in every particular case. Possible uncertainties, oversimplifications of the simulation, reconstruction programs, etc. are a few possible sources of experimental difficulties.

2. The probability

The paper [9] can be used as an illustration of a general problem with the concept of probability.

There are at least two (main) interpretations of the probability itself. One, called *classical*, is well-known and obvious, but it should be remembered that it has not been accepted for very long. Approximately, as classical as 'classical' is the special theory of relativity (since the times of Pearson), or even as old as quantum physics (since the Neyman or Fisher milestone papers). Sir Francis Galton was the pioneer of the statistical treatment of data, about 100 years ago. The frequentist meaning of the *probability* is given in many textbooks in the common form of the limit of a fraction of successes in an infinitely long sequence of identical trials. (The 'identity' should be understood as the requirement that the 'probability' in question remains constant during the sequence of trials. This *circulus in definiendo* is one of the nightmares of the frequentists.)

Another way of thinking about the probability comes from Thomas Bayes and it is more than 100 years old; however, in its modern form as 'Bayesianism' it has been used only since 1950. The Bayesian point of view stands for the probability as a 'rate of rational bet', a degree of belief.

The Bayesian treatment of the probability makes it closer to common sense. People do not need the big (infinite) sample of results of the repeated experiment (in exactly the same conditions etc.) to say something about, e.g., the Higgs boson mass, or the appearance of the Sun the next day morning.

The Bayesian definition of probability is 'by definition' subjective. The probability itself does not exist, in a sense [11]. Most important for the problem to be discussed here is the fact that the Bayesian probability of the event is defined in a certain moment in time. As time passes, the value of the probability (the rational bet) may change. This change can be caused by the increase or decrease (we will come back to this last intriguing possibility in §5.1) of the information about the subject gathered in the meantime.

This situation is common in physics. The progress of our understanding of the Universe is expected to be related to new experiences, observations, measurements. All of them change a background which makes the base of the estimation of our rational bets on reality (to be made, e.g., for further experiment outputs).

The Bayesian analysis is based on his famous theorem, which can be expressed as: $P(H|E) \sim P(H)P(E|H)$, where the proportionality constant is determined by the normalization. The first factor $P(H)$ is the probability of the hypothesis H being true prior to the experiment output E being known. $P(E|H)$ is the likelihood of the output E if the hypothesis H is true. The left side of the equation is the improved new probability of the hypothesis H being true if we know the result E.

The question of the existence of the GZK cut-off can be answered in terms of probability: The proposition 'there is a sharp cut-off of the very high energy cosmic ray spectrum' and the opposite 'no such cut-off exists'. These are statements about reality and of course only one of them can be true. To judge this in a scientific ('classical') way one has to test the GZK cut-off hypothesis statistically. The standard, Fisher or frequentist, answer to the test question can be only that at a given confidence level there are no observational constraints to the GZK hypothesis, or the hypothesis should be, according to the performed observations, rejected at this confidence level. The Bayesian answer can be that there is a given probability, estimated according to all the knowledge we have, that the GZK hypothesis is true (or false, if one wishes).

We will discuss this difference.

3. The prior

The question as to whether the cosmic ray energy power-law spectrum extends continuously with more or less constant index was afloat already before the famous Greisen, and Zatsepin and Kuzmin papers predicting a sharp end to this spectrum around few (6) times 10^{19} eV appeared. The GZK cut-off as a result of interaction of UHECR protons with CMB photons, could not be proposed before the CMB radiation itself was discovered in 1965. But even UHECR were intriguing due to the fact that they, according to their great magnetic rigidity, could not be confined within the Galaxy or in other known galactic object.

The sharpness of the CR energy spectrum requires EAS arrays of big areas to register the highest energy particles. The first really great one dedicated for the UHECR domain was the Volcano Ranch array. It has been running since 1960. It consists of 19 3.3-m^2 scintillator counters distributed over about 10 km^2 in Dugway. This experiment, relatively simple and small in comparison to contemporary projects, registered in February 1962 event No. 2-4834 [12] is shown in figure 1a.

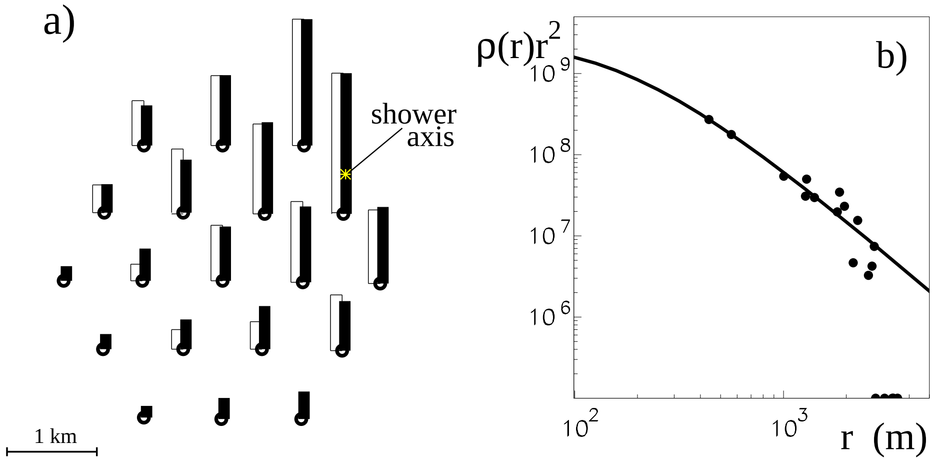

Figure 1. The density map (**a**) of the Volcano Ranch super-GZK event No. 2-4834 of energy estimated as 10^{20} eV. The positions of detectors are shown by circles and the blank bars beside represent the respective registered density (its logarithm) while the filled bars show densities obtained with the 'best fit' lateral shower particle distribution (the NKG formula) shown as a function of the distance to the shower axis in (**b**). The adjusted position of the axis is shown in (**a**).

11 out of the 19 detectors registered the particles. The highest signal was estimated to be equivalent to about 1400 minimum ionization particles per m^2. The lateral distribution of the shower particle was found using the NKG-function. It is well established experimentally and it is close to the shower particle distribution measured in different experiments. The surface integration of the NKG distribution gives the total number of charged particles in the shower. In the Volcano Range super-GZK event it is equal to 5×10^{10}. The registered densities (their logarithms) in comparison with the values of the fit are shown in figure 1a as vertical bars along each detector position. It is seen in figure 1b that the particle distribution describes the points very well. The tests with other distributions used in different experiments do not make more than 20% difference in the total particle number.

As an additional test we tried to eliminate from the fitting procedure the stronger signal detector arguing that it could be made by some internal cascading, thus not representative of the shower particles. Again no significant change was noticed.

All these are mentioned here to explain the statement given by Linsley in ref. [13]: "The first observation of the spectrum above 10^{19} eV, at Volcano Ranch, showed that the spectrum extends to 10^{20} eV without any sign of cut-off".

Before the CMB discovery another really very big array was constructed in the Southern Hemisphere. The Sydney University Giant Airshower Recorder (SUGAR) consists of more than 50 stations, each containing two 6-m^2 scintillator detectors buried underground and spread over the surface of about 100 km^2 area. The detectors of SUGAR were able to register only EAS muons of energies greater than about 1 GeV. The spectrum obtained this way, published in 1973 in refs [14,15], is shown in figure 2.

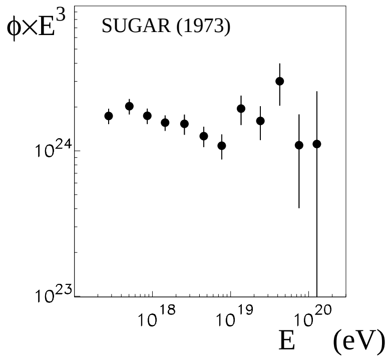

Figure 2. SUGAR 1973 spectrum from [14].

The paper [14] concluded with the statement: "It appears likely that the primary energy spectrum extends beyond 10^{20} eV with no significant features".

The interest in UHECR increased in the meantime when CMB was discovered and famous papers announcing the GZK cut-off were published.

The Northern sky was then monitored by the EAS array built in Haverah Park near Leeds, UK. The array reached the size of that at the Volcano Ranch at about 1968. Different types of detectors (water Čerenkov tanks) and special detectors for muon shower component were installed there. About 30 years ago the results on the UHECR energy spectra were announced. We show in figure 3 the three spectra published by the Haverah Park team from those first published at the end of 1970s (refs [16,17]) to the final 1991 spectrum [18]. A kind of evolution is seen. It will be discussed later on.

In the beginning of 1970s the Yakutsk array in the USSR started collecting data (it is still in operation). The first data concerning the size spectra was published in ref. [19]. It is shown in figure 4 together with the recent Yakutsk result published in ref. [20] in 2003.

The most controversial (at present) spectrum from the big experiment AGASA [21] is shown in figure 5. There are about a dozen of events (the last recorded in 2002) exceeding the GZK limit.

The clear signal presented in the figure is extreme in the sense that it evolved with time (mostly due to adjustments of the energy estimation procedures) becoming less and less pronounced. It should be remembered, however, that the AGASA result always contradicts the existence of the GZK cut-off, more or less strongly. It is worth mentioning that, in 1993, the AGASA array recorded a very well measured almost vertical air shower with an estimated energy of about 2×10^{20} eV [22]. Later, in 2001, an event of energy of about 2.5×10^{20} eV was seen in Japan. However, the world record of the highest UHECR particle energy belongs to the event measured by the precursor of the 'HiRes', the Fly's Eye experiment in 1991. The value to overtake since then is 3.2×10^{20} eV [23].

The Fly's Eye detector began to collect data in 1981, first as single Eye monitoring the scintillation flashes produced by extremely big EAS. In 1985 the second Eye joined the first, completing the apparatus. The result was a significant increase in the geometrical reconstruction accuracy and thus the particle energy determination.

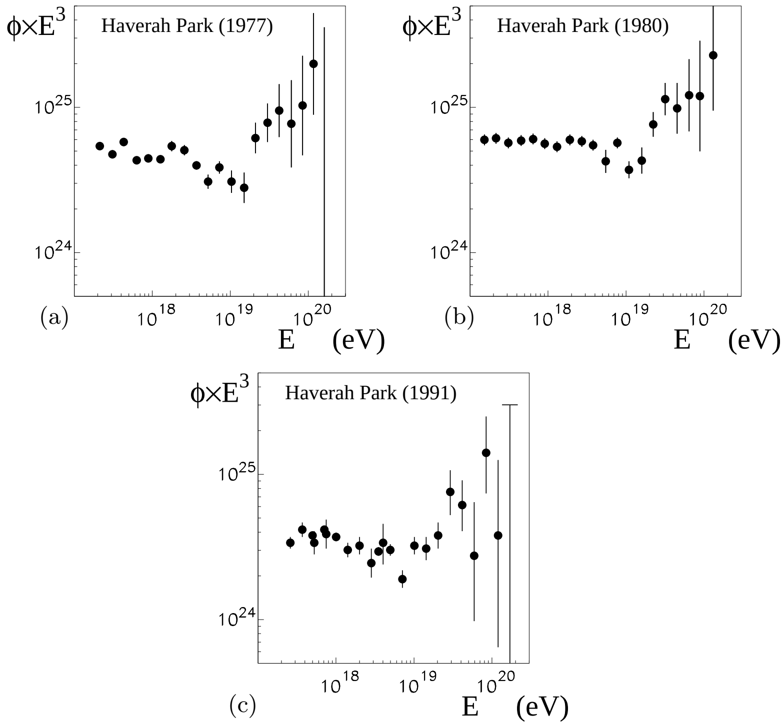

Figure 3. Haverah Park spectra from (**a**) [16], (**b**) [17] and (**c**) [18].

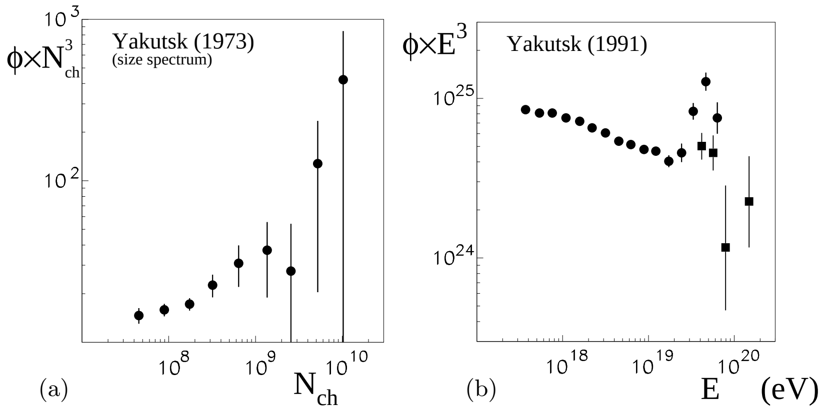

Figure 4. Yakutsk size spectrum from [19] (**a**) and the recent energy spectrum [20] (**b**).

The statistics of events collected by the First Fly's Eye detector, the monocular data set, is much larger than the stereo data set. Both spectra are shown in figure 6 [24]. The mentioned single event of energy 3.2×10^{20} eV is seen as a separated single point in figure 6b.

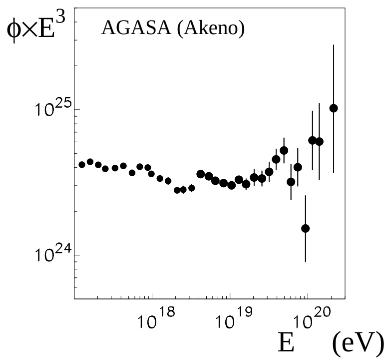

Figure 5. Akeno and AGASA experiment spectra.

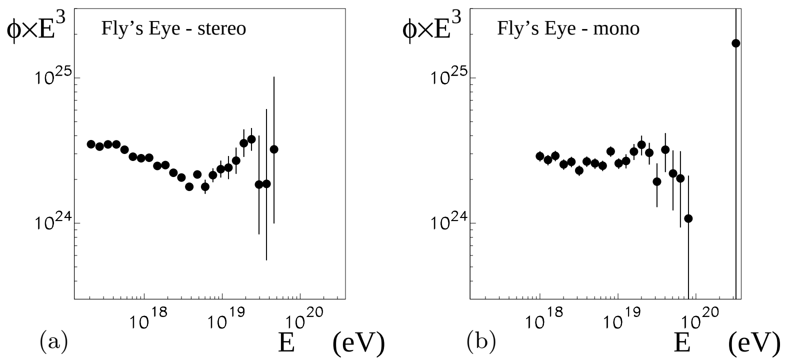

Figure 6. Fly's Eye (**a**) stereo and (**b**) monospectra.

It is well-known that the spectra from the different experiments are displaced, in absolute energy calibration, as well as in the normalization of the total measured UHECR flux. The procedure to correct the data of each experiment for its individual energy resolution and to adjust them using the 'ankle' structure for an absolute energy 'calibration' was developed in ref. [25]. In our opinion the 'ankle' feature is related to the change from the galactic to extragalactic CR flux component. When it is done, the spread of the points in the different energy bins is Gaussian, the individual measurement points can be averaged, and the observed spread can be used to determine the error of the average.

This procedure was applied to all data, with two exceptions: One is the Pierre Auger Observatory (PAO) recent spectrum [26] published after the result of HiRes was published in ref. [27]. The PAO spectrum will be discussed separately in §5. The second exception is the HiRes spectra which will be discussed in detail in the next section. In figure 7 the result of the summary are shown.

The 'world average' presented in figure 7b forms exactly the *prior* needed in Bayesian reasoning treatment of probability. The world record event from Fly's Eye mono and the Volcano Ranch first UHECR showers are not included in the shown *prior*.

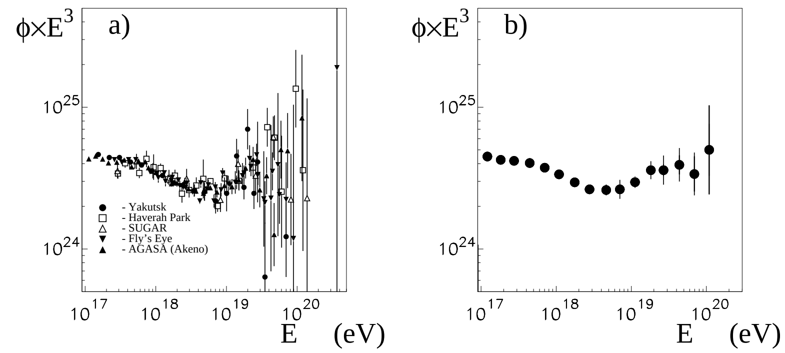

Figure 7. (**a**) Spectra from Haverah Park, SUGAR, AGASA, Akeno, Fly's Eye and Yakutsk adjusted to the same shape at the dip below 1019 eV (the procedure is described in [25]) and (**b**) the sum of all listed data – the *prior*.

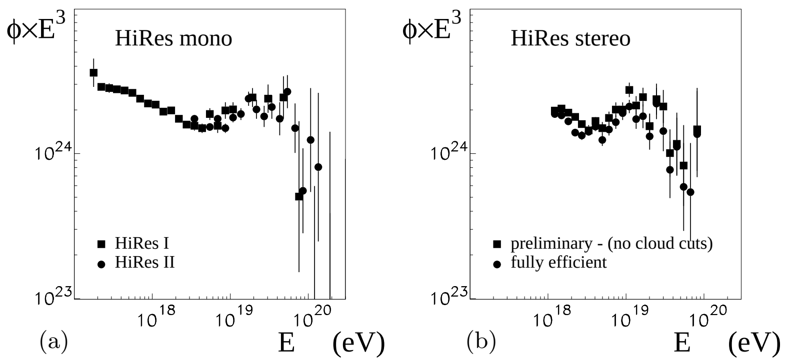

Figure 8. HiRes (**a**) mono and (**b**) stereospectra.

4. The likelihood

The factor next to the *prior* in a Bayes formula is the likelihood. It describes the increase of our knowledge related to the new measurement, in our case the recent HiRes experiment spectrum [9] shown in figure 8. The left-hand panel shows results of two 'mono' spectra of events registered separately by the Eye I and II. The right figure shows the more accurate, but statistically poorer, 'stereo' spectrum obtained from events seen by both Eyes.

Let us first discuss the '5 sigma' statement published in ref. [9]. It is based on figure 8a and it was obtained by comparing two numbers: 43.2 expected and 13 observed events above the expected GZK limit $10^{19.75} = 5.6 \times 10^{19}$ eV. The comparison was done using the Poisson distribution with the average, expected value of 43.2, and indeed the probability of observing 13 or less events is on the level of 7×10^{-8}.

The value of 43.2 expected events is obtained extrapolating the spectrum with the same index as adjusted to the data between the energies of about $10^{18.5}$ eV and the point of abrupt GZK break announced at $10^{19.75}$ eV. The accuracy of this index reported is equal to 0.03 [9]. The change of the index from 2.81 to 2.81+0.03 increases the likelihood $P(E|H)$ about twice, which is not large.

The change of the estimated position of the GZK break in the spectrum (19.75 ± 0.04 [9]) from 19.75 to 19.79 decreases the expectations from 43.2 to about 36.5, decreasing the likelihood significantly. To preserve its level of 7×10^{-8} the number of observed events should change from 13 to 9 (1/3 of the events observed above the GZK cut-off have to have energies not more than 10% higher than the cut-off energy of 5.6×10^{19}). It is still possible within the accuracy of the method of energy determination by the experiment. So it is hard to estimate the shape of the spectrum above the GZK with the actual measured sample of 13 events.

All these details can change the 'chance' probability, but we are still close to the level of '5 sigma'. The data collected by HiRes monoexperiments produce a likelihood of about 10^{-7}.

5. The improvement

According to the conventional wisdom and, formally, to the Bayes formula, any new measurement improves our knowledge. The procedure of averaging UHECR spectra used to obtain the prior (figure 7b) can be used also for all experiments, including HiRes mono and stereo, as well as PAO. The result is presented in figure 9.

Figure 9a shows the result of combining the HiRes monospectra [9] and the *prior*. The prior is moved to match the dip structure where it is seen well in the HiRes data. We assume that the HiRes energy and flux calibration are correct.

As has been said in the previous section, the HiRes monodata itself gives '5 sigma' confidence. From the Bayesian point of view this statement represents the situation when nothing else about the UHECR flux is known. As was shown in §3 we already know quite a lot about the real spectrum. Accordingly, the GZK significance estimation after the HiRes (mono) measurement, can be estimated from the posterior spectrum shown in figure 9a. To derive some numbers we follow the procedure used by the HiRes group in ref. [9] and discussed in the previous section. First we get the index of the UHECR particle spectrum above the ankle where it seems to be stable and where there is no signs of any cut-off. We used the same energy interval $18.5 \leq \log_{10} E \leq 19.75$ as was used in ref. [9]. The value of the index is found to be 2.87 ± 0.08.

We estimate the probability that, when there is no GZK cut-off, the measured flux at the very tail of the spectrum is at least as low as our improved UHECR flux. The 'absence of the GZK cut-off' means that the UHECR flux above $\log_{10} E = 19.75$ continues the trend found at the ankle below this energy. This probability can be estimated with the help of the χ^2 statistics. The obtained value is $\chi^2/NDF = 8.7/4$ giving the chance probability of about 3×10^{-2}.

Introducing the HiRes stereodata gives the posterior flux shown in figure 9b. The estimated index below $\log_{10} E = 19.75$ remains unchanged. The complete HiRes (mono+stereo) gives $\chi^2/NDF = 7.4/4$, and the chance probability is even bigger: 6×10^{-2}. We can say that if the HiRes data are combined with the *prior* then the existence of the GZK cut-off is observed with a significance below 2σ level.

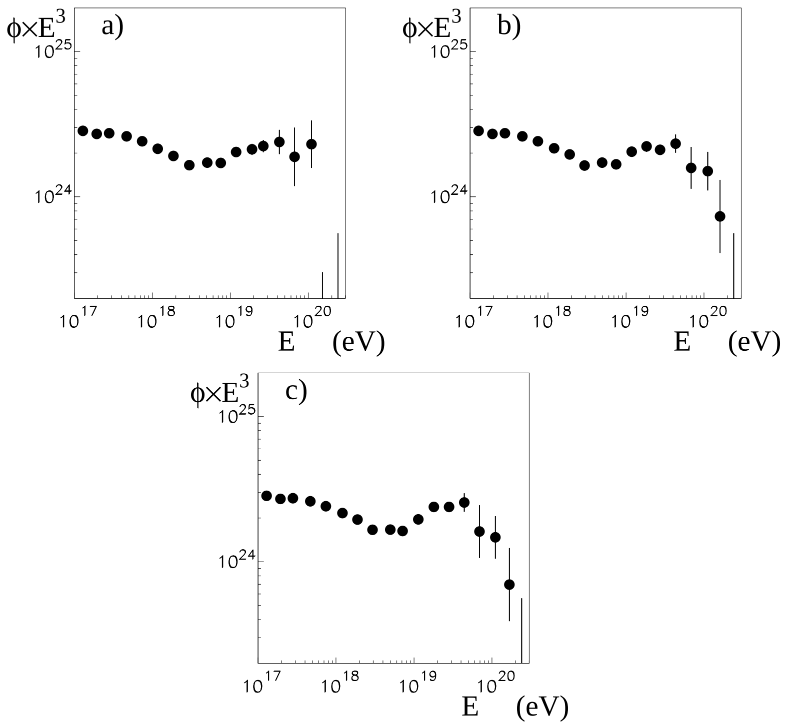

Figure 9. UHECR spectra (**a**) combined HiRes monospectra and the *prior* from 7b, (**b**) HiRes mono and stereospectra and the *prior*, and (**c**) HiRes and Auger and the *prior* – the 'world average spectrum'.

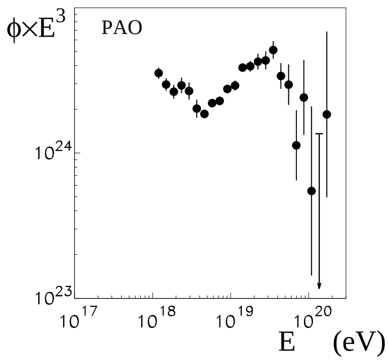

Figure 10. Pierre Auger Observatory UHECR spectrum [26].

There is, as mentioned above, one more data set on the UHECR spectrum, available for some time. This is the Pierre Auger Observatory result [26] shown in figure 10. The PAO spectrum exhibits also the cut-off in general accordance with HiRes result, in spite of its energy calibration and absolute flux normalization. An enhancement of the probability in favour of the GZK picture is expected.

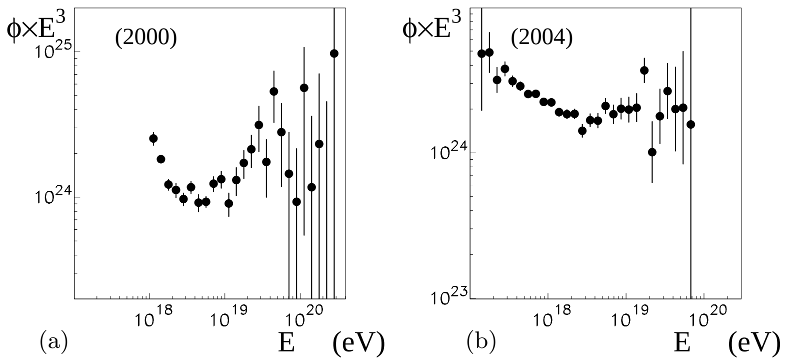

Figure 11. (**a**) HiRes spectrum collected in 1997–1999 [29] and (**b**) the spectrum published in [30].

In fact, it is not very substantial as can be seen in figure 9c, where the combination of all measured spectra is shown. The index before the GZK threshold energy is slightly changed to 2.86±0.13, and the chance probability changes to 1.4×10^{-2} and the significance level of the GZK cut-off discovery, expressed in sigmas, exceeds 2σ.

We should also mention about the existence of the very high energy events not included in the present analysis. The first ever super-GZK Volcano Ranch event discussed in §3, the mentioned 'world record' Fly's Eye event, and the one registered by PAO just on the edge (but slightly outside) of the working part of the array, additionally reduce the probability of existence of the GZK cut-off.

5.1 *Observation of an evolutionary effect*

It is interesting to note the evolution of the UHECR flux results measured by different experiments. It is in general nothing extraordinary. It is well known, e.g., in the lasting case of gravitational constant. Another example is given in the first one of the Particle Data Group history plots: the neutron lifetime. The error boxes of the results measured before 1970 are far outside the value accepted nowadays [28].

The measurement history of the flux at Haverah Park is given in figure 3 and for Yakutsk in figure 4. Both exhibit a similar effect. The initially published spectra do not follow the GZK hypothesis, just opposite, they continue gradually.

The spectra from the HiRes telescopes shown in figure 8 are the recent version showing a '5 sigma' deficit of super-GZK events. At the end of the previous century the HiRes spectrum looked quite different. In figure 11a the spectrum of HiRes I (BigH) detector is shown as it was published in ref. [29]. There are 13 events observed between May 1997 and June 1999 with energy exceeding 6×10^{19} eV and 7 events with energy greater 10^{20} eV! All the details of these super-GZK events are published in ref. [29].

The HiRes data a few years ago (but in the 21st century) looked also slightly different. Figure 11b presents the HiRes II spectrum obtained from the data collected between December 1999 and September 2001 [30].

The probability of the GZK cut-off is increasing in the last years, not only according to the new measurements but also confirmed by the re-analysis of the old data.

The 'evolution' of the UHECR spectra is sustained by another interesting fact. The knowledge gathered by the experimental groups not so long ago, even in 1980, recently vanishes very rapidly, in a sense. This possibility was mentioned in §2. The contemporary new and big experiments, much bigger than the old ones, and hundreds of people working there draw their conclusions as though there was nothing before them! But the super-GZK events seen some time ago remain the super-GZK still, even if one does not like them.

Similar conclusion can be found in the recent paper analysing the UHECR data by Glushkov and Pravdin [31].

6. Summary and conclusions

To conclude, to analyse the problem of the existence of the GZK cut-off of the ultra high energy cosmic ray flux we have to take into account non-physical factors, which for frequentists sound like an insult, and it is neutral for Bayesians: the factor of *belief*.

The belief in the GZK (thus belief in extragalactic protons) acts as an additional *prior* and influences the reasoning resulting in conclusions which are (or, in general, could be) wrong. Wrong, from the 'frequentist', physical point of view, which they (the frequentists) believe, should be free of any beliefs.

If one would bet 10000000 to 1 for the GZK cut-off existence (thus the pure proton flux in the UHECR domain) than this additional, 'non-physical' belief in GZK, is on the level of 10000 to 1. This is the Bayesian conclusion of the simple calculations presented above. The '5 sigma' effect combined with the prior (and additionally with the PAO result) turns out to be only little above '2 sigma'.

In conclusion we have shown that the GZK cut-off, if it exists, has an overall significance of about '2 sigma', far less than '5 sigma'. Thus, the claim that the Greisen–Zatzepin–Kuzmin effect has been established experimentally is a little premature.

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