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Application of FIRE for the calculation of photon matrix elements

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Abstract. The next-to-next-to-leading order (the order $\alpha\alpha_s^2$) corrections to the first moment of the polarized virtual photon structure function $g_1^\gamma(x, Q^2, P^2)$ are studied in perturbative QCD for the kinematical region $\Lambda^2 \ll P^2 \ll Q^2$, where $-Q^2$ $(-P^2)$ is the mass square of the probe (target) photon and Λ is the QCD scale parameter. In order to evaluate the two-loop Feynman diagrams for the photon matrix element of the gluon operator, I apply the recently developed algorithm FIRE which reduces a complicated sum of scalar Feynman integrals to a linear combination of a few master integrals. The details of the calculation are presented.

Keywords. FIRE; polarized photon structure function; axial current.

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1. Introduction

Both theoretically and experimentally, the investigation of the photon structure has been an active field of research in recent years [1]. Also there has been growing interest in the study of the spin structure of photon. Especially, the first moment of the polarized photon structure function g_1^γ has attracted attention due to its relevance to the QED and QCD axial anomalies [2–4]. The polarized photon structure functions can be measured from two-photon processes in the polarized e^+e^- collider experiments as shown in figure 1, where $-Q^2$ $(-P^2)$ is the mass square of the probe (target) photon.

A unique and interesting feature of the photon structure functions is that the target mass squared $-P^2$ is not fixed but can take various values and that the structure functions show different behaviour depending on the values of P^2. In the case of a real photon $(P^2 = 0)$ target, there exists only one spin-dependent structure function $g_1^\gamma(x, Q^2)$ and it satisfies a remarkable sum rule [2–4]

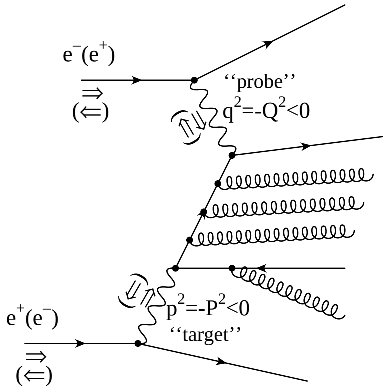

Figure 1. Deep inelastic scattering on a virtual photon in e^+e^- collision.

$$\int_0^1 dx g_1^\gamma(x, Q^2) = 0. \tag{1}$$

In fact, it was shown in ref. [4] that the sum rule (1) holds to all orders in perturbation theory in both QED and QCD.

For a virtual photon target ($P^2 \neq 0$) there appear two spin-dependent structure functions, $g_1^\gamma(x, Q^2, P^2)$ and $g_2^\gamma(x, Q^2, P^2)$. The former has been investigated up to the next-to-leading order (NLO) in QCD [5–7]. In refs [5,6] the structure function $g_1^\gamma(x, Q^2, P^2)$ was analysed in the kinematical region

$$\Lambda^2 \ll P^2 \ll Q^2, \tag{2}$$

where Λ is the QCD scale parameter. The advantage in studying the virtual photon target in this kinematical region is that we can calculate whole structure functions (their shape and magnitude) by the perturbative method without any unknown parameters [8], which is contrasted with the case of the real photon target where in the NLO there exist nonperturbative and, therefore, unknown pieces. When the target photon becomes off-shell, i.e., $P^2 \neq 0$, the first moment of the photon structure function $g_1^\gamma(x, Q^2, P^2)$ does not vanish any more. Indeed the first moment has been calculated for case (2) up to the NLO (the order $\alpha\alpha_s$) as follows [3,5]:

$$\begin{aligned}\int_0^1 \mathrm{d}x g_1^\gamma(x, Q^2, P^2) \\ = -\frac{3\alpha}{\pi}\left[\sum_{i=1}^{n_f} e_i^4 \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)\right. \\ \left. - \frac{2}{\beta_0}\left(\sum_{i=1}^{n_f} e_i^2\right)^2 \left(\frac{\alpha_s(P^2)}{\pi} - \frac{\alpha_s(Q^2)}{\pi}\right)\right],\end{aligned} \tag{3}$$

with β_0 being the one-loop QCD β function. Here $\alpha = e^2/4\pi$ and $\alpha_s(Q^2) = \overline{g}^2(Q^2)/4\pi$ are the QED coupling constant and the QCD running coupling constant, respectively, and e_i is the electromagnetic charge of the active quark (i.e., the massless quark) with flavour i in the unit of proton charge and n_f is the number of active quarks. Note that the r.h.s. of (3) does not involve any unknown parameters.

In this paper we analyse the next-to-next-to-leading order (NNLO) (the order $\alpha\alpha_2^2$) corrections to the first moment sum rule of $g_1^\gamma(x, P^2, Q^2)$ in the kinematical region (2). In fact, the analysis of this sum rule up to the NNLO has appeared already in [9]. For the evaluation of the NNLO corrections we need to calculate the three-loop-level photon matrix element of the flavour singlet quark axial current. It is shown in ref. [9] that when the Adler–Bardeen theorem for the axial anomaly is applied, the calculation reduces to the one in the two-loop level, to be more specific, the calculation of the two-loop diagrams for the photon matrix element of the gluon operator. But still it is a complicated computation. In ref. [9], the two-loop integrals are reduced to the one-loop level by using a reduction relation which is derived from the rule of triangle [10,11], and the program FORM is employed to perform the necessary algebra [12]. We need a confirmation of the correctness of the previous calculation.

Recently Smirnov [13] has developed a powerful algorithm called FIRE which performs the reduction of scalar Feynman integrals to master integrals. I apply this algorithm and re-analyse the NNLO corrections to the first moment of $g_1^\gamma(x, P^2, Q^2)$. Using a projection operator, in this paper, the transformation of the contributions of the two-loop diagrams into the sum of two-loop scalar integrals is described. Then, FIRE is used to express these scalar integrals as a linear combination of the two master integrals and the details of the calculation are presented. In §2 the expression for the first moment of $g_1^\gamma(x, P^2, Q^2)$ is given up to the NNLO corrections. Then, in §3, it is shown in detail how the algorithm FIRE is used for the calculation of the previously unknown ingredient, i.e., the three-loop-level photon matrix element of the flavour singlet quark axial current. Throughout this paper the effect of quark masses is neglected. The same result that was derived in ref. [9] is reached using different methods. The final section is devoted to the conclusion.

2. The NNLO ($\alpha\alpha_s^2$) corrections

The NNLO ($\alpha\alpha_s^2$) corrections to the first moment of $g_1^\gamma(x, Q^2, P^2)$ is analysed in the framework of the operator product expansion (OPE) supplemented by the renormalization group method. For the OPE of two electromagnetic (and thus gauge-invariant) currents, only gauge-invariant operators need to be included with their renormalization basis [14]. Since there is no gauge-invariant twist-two gluon and photon operators with spin one, only the following quark operators, i.e., the flavour singlet $J_{5\mathrm{S}}^\sigma$ and nonsinglet $J_{5\mathrm{NS}}^\sigma$ axial currents, are considered:

$$J_{5\mathrm{S}}^\sigma = \bar{\psi}\gamma^\sigma\gamma_5 1\psi, \quad J_{5\mathrm{NS}}^\sigma = \bar{\psi}\gamma^\sigma\gamma_5(Q_{\mathrm{ch}}^2 - \langle e^2\rangle 1)\psi, \tag{4}$$

where 1 is an $n_f \times n_f$ unit matrix and Q_{ch}^2 is the square of the $n_f \times n_f$ quark-charge matrix so that $\mathrm{Tr}(Q_{\mathrm{ch}}^2 - \langle e^2\rangle 1) = 0$. Writing the photon matrix elements of the quark currents as

$$\langle\gamma(p)|J^{\sigma}_{5i}(\mu^2)|\gamma(p)\rangle = \epsilon^*_\lambda[-2i\epsilon^{\sigma\lambda\tau\rho}p_\rho]\epsilon_\tau\langle\gamma(p)\|J_{5i}(\mu^2)\|\gamma(p)\rangle, \qquad i = \mathrm{S}, \mathrm{NS}, \quad (5)$$

where ϵ^*_μ and ϵ_τ are the polarization vectors of the target photon with momentum p and μ is the the renormalization point. Then the first moment sum rule of $g^\gamma_1(x, Q^2, P^2)$ is expressed as

$$\int_0^1 \mathrm{d}x g^\gamma_1(x, Q^2, P^2) = \langle\gamma(p)\|J_{5\mathrm{S}}(\mu^2)\|\gamma(p)\rangle C_{\mathrm{S}}(Q^2/\mu^2, \bar{g}(\mu^2), \alpha) + \langle\gamma(p)\|J_{5\mathrm{NS}}(\mu^2)\|\gamma(p)\rangle C_{\mathrm{NS}}(Q^2/\mu^2, \bar{g}(\mu^2), \alpha). \quad (6)$$

Here C_{S} and C_{NS} are the coefficient functions corresponding to the currents $J^\sigma_{5\mathrm{S}}$ and $J^\sigma_{5\mathrm{NS}}$, respectively. Putting it more closely, C_{S} and C_{NS} are the $n = 1$ coefficient functions which appear in the OPE of two electromagnetic currents.

We choose the renormalization point μ at $\mu^2 = P^2$. The Q^2 dependence of the coefficient functions C_{S} and C_{NS} is governed by the renormalization group equations. The solutions to these equations are given by

$$C_i(Q^2/P^2, \bar{g}(P^2), \alpha) = \exp\left[\int_{\bar{g}(Q^2)}^{\bar{g}(P^2)} \mathrm{d}g' \frac{\gamma_i(g')}{\beta(g')}\right] C_i(1, \bar{g}(Q^2), \alpha), \qquad i = \mathrm{S}, \mathrm{NS}, \quad (7)$$

where $\gamma_i(g)$ is the anomalous dimension of the quark axial current J^σ_{5i} and $\beta(g)$ is the QCD β-function. The anomalous dimension $\gamma_i(g)$ is expanded in powers of g as

$$\gamma_i(g) = \gamma^{(0)}_i \frac{g^2}{16\pi^2} + \gamma^{(1)}_i \left(\frac{g^2}{16\pi^2}\right)^2 + \gamma^{(2)}_i \left(\frac{g^2}{16\pi^2}\right)^3 + \mathcal{O}(g^8), \quad i = \mathrm{S}, \mathrm{NS}. \quad (8)$$

Since the flavour nonsinglet quark axial current $J^\sigma_{5\mathrm{NS}}$ is conserved in the massless limit, it undergoes no renormalization, and thus we have

$$\gamma^{(0)}_{\mathrm{NS}} = \gamma^{(1)}_{\mathrm{NS}} = \gamma^{(2)}_{\mathrm{NS}} = \cdots = 0. \quad (9)$$

Thus the nonsinglet coefficient function C_{NS} is expressed as

$$C_{\mathrm{NS}}(Q^2/P^2, \bar{g}(P^2), \alpha) = C_{\mathrm{NS}}(1, \bar{g}(Q^2), \alpha). \quad (10)$$

On the other hand, the flavour singlet axial current $J^\sigma_{5\mathrm{S}}$ has a nonvanishing anomalous dimension $\gamma_{\mathrm{S}}(g)$ due to the axial anomaly. At one-loop, we know $\gamma^{(0)}_{\mathrm{S}} = 0$. But at higher loops we have nonzero $\gamma^{(1)}_{\mathrm{S}}$, $\gamma^{(2)}_{\mathrm{S}}$ and so on.

The β function is expanded up to the two-loop level as

$$\mu\frac{\partial g}{\partial\mu} = \beta(g) = -\beta_0\frac{g^3}{16\pi^2} - \beta_1\frac{g^5}{(16\pi^2)^2} + \cdots. \quad (11)$$

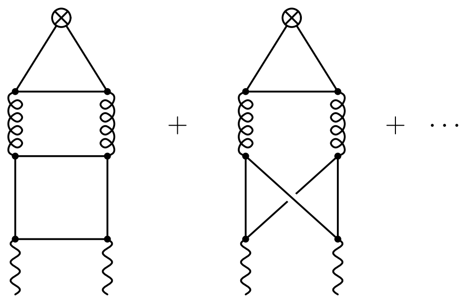

Figure 2. The three-loop diagrams of the gluon–photon scattering type.

Then using eqs (8) and (11), we obtain up to the order α_s^2,

$$\exp\left[\int_{\bar{g}(Q^2)}^{\bar{g}(P^2)} \mathrm{d}g' \frac{\gamma_{\mathrm{S}}(g')}{\beta(g')}\right] = 1 + \frac{\gamma_{\mathrm{S}}^{(1)}}{2\beta_0}\left(\frac{\alpha_s(Q^2)}{4\pi} - \frac{\alpha_s(P^2)}{4\pi}\right) + \frac{1}{4\beta_0}\left(\gamma_{\mathrm{S}}^{(2)} - \gamma_{\mathrm{S}}^{(1)}\frac{\beta_1}{\beta_0}\right) \times \left[\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2 - \left(\frac{\alpha_s(P^2)}{4\pi}\right)^2\right] + \frac{1}{8}\left(\frac{\gamma_{\mathrm{S}}^{(1)}}{\beta_0}\right)^2\left(\frac{\alpha_s(Q^2)}{4\pi} - \frac{\alpha_s(P^2)}{4\pi}\right)^2 + \mathcal{O}(\alpha_s^3). \tag{12}$$

The flavour singlet and nonsinglet quark coefficient functions, $C_{\mathrm{S}}(1,\bar{g}(Q^2),\alpha)$ and $C_{\mathrm{NS}}(1,\bar{g}(Q^2),\alpha)$, are expanded in power of $\alpha_s(Q^2)$ up to the two-loop level as

$$C_{\mathrm{S}}(1,\bar{g}(Q^2),\alpha) = \langle e^2\rangle\left\{1 + B_{\mathrm{S}}^{(1)}\frac{\alpha_s(Q^2)}{4\pi} + B_{\mathrm{S}}^{(2)}\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2 + \cdots\right\}, \tag{13}$$

$$C_{\mathrm{NS}}(1,\bar{g}(Q^2),\alpha) = \left\{1 + B_{\mathrm{NS}}^{(1)}\frac{\alpha_s(Q^2)}{4\pi} + B_{\mathrm{NS}}^{(2)}\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2 + \cdots\right\}. \tag{14}$$

Finally, the photon matrix elements of the axial currents can be calculated perturbatively for $-p^2 = P^2 \gg \Lambda^2$, and they are expressed in the form as

$$\langle\gamma(p)\|J_{5i}(\mu^2 = P^2)\|\gamma(p)\rangle = \frac{\alpha}{4\pi}A_i, \quad i = \mathrm{S}, \mathrm{NS}, \tag{15}$$

with

$$A_i = A_i^{(0)} + \frac{\alpha_s(P^2)}{4\pi}A_i^{(1)} + \left(\frac{\alpha_s(P^2)}{4\pi}\right)^2 A_i^{(2)} + \cdots. \tag{16}$$

The leading terms $A_{\rm S}^{(0)}$ and $A_{\rm NS}^{(0)}$ are connected with Adler–Bell–Jackiw anomaly [15] and are already known. For the NLO and NNLO terms, $A_i^{(1)}$ and $A_i^{(2)}$, we obtain due to the nonrenormalization theorem for the triangle anomaly [16],

$$A_{\rm NS}^{(1)} = A_{\rm NS}^{(2)} = 0, \qquad A_{\rm S}^{(1)} = 0. \tag{17}$$

However, $A_{\rm S}^{(2)}$ has a nonvanishing value. This is because the three-loop diagrams shown in figure 2, in which the quark triangle part is connected to the photon-vertex part by two gluon lines, contribute to $A_{\rm S}^{(2)}$.

Then putting eqs (10) and (12)–(17) into eq. (6), we obtain the expression for the first moment sum rule of $g_1^\gamma(x, Q^2, P^2)$ up to the NNLO ($\alpha\alpha_s^2$) corrections as follows:

$$\begin{aligned}
&\int_0^1 {\rm d}x g_1^\gamma(x, Q^2, P^2)/\left(\frac{\alpha}{4\pi}\right) \\
&= \langle e^2\rangle A_{\rm S}^{(0)} + A_{\rm NS}^{(0)} + \left(\langle e^2\rangle A_{\rm S}^{(0)} B_{\rm S}^{(1)} + A_{\rm NS}^{(0)} B_{\rm NS}^{(1)}\right)\frac{\alpha_s(Q^2)}{4\pi} \\
&\quad + \langle e^2\rangle A_{\rm S}^{(0)} \frac{\gamma_{\rm S}^{(1)}}{2\beta_0}\left[\frac{\alpha_s(Q^2)}{4\pi} - \frac{\alpha_s(P^2)}{4\pi}\right] \\
&\quad + \left(\langle e^2\rangle A_{\rm S}^{(0)} B_{\rm S}^{(2)} + A_{\rm NS}^{(0)} B_{\rm NS}^{(2)}\right)\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2 \\
&\quad + \langle e^2\rangle A_{\rm S}^{(0)} B_{\rm S}^{(1)} \frac{\gamma_{\rm S}^{(1)}}{2\beta_0}\frac{\alpha_s(Q^2)}{4\pi}\left[\frac{\alpha_s(Q^2)}{4\pi} - \frac{\alpha_s(P^2)}{4\pi}\right] \\
&\quad + \langle e^2\rangle A_{\rm S}^{(0)} \frac{1}{4\beta_0}\left(\gamma_{\rm S}^{(2)} - \gamma_{\rm S}^{(1)}\frac{\beta_1}{\beta_0}\right)\left[\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2 - \left(\frac{\alpha_s(P^2)}{4\pi}\right)^2\right] \\
&\quad + \langle e^2\rangle A_{\rm S}^{(0)} \frac{1}{8}\left(\frac{\gamma_{\rm S}^{(1)}}{\beta_0}\right)^2\left[\frac{\alpha_s(Q^2)}{4\pi} - \frac{\alpha_s(P^2)}{4\pi}\right]^2 \\
&\quad + \langle e^2\rangle A_{\rm S}^{(2)}\left(\frac{\alpha_s(P^2)}{4\pi}\right)^2 .
\end{aligned} \tag{18}$$

All the ingredients necessary to evaluate the r.h.s. of eq. (18) have been calculated in the literature and already known, except for $A_{\rm S}^{(2)}$, the photon matrix element of the flavour singlet axial current at three loops. These necessary ingredients except for $A_{\rm S}^{(2)}$ are enumerated in Appendix **??**. The expressions are the ones calculated in the modified minimal subtraction ($\overline{\rm MS}$) scheme [17].

3. Calculation of $A_{\rm S}^{(2)}$

For the calculation of $A_{\rm S}^{(2)}$, we need to evaluate the three-loop Feynman graphs. It was shown [9] recently that when we resort to the Adler–Bardeen (AB) theorem for the QCD axial anomaly,

$$\partial_\mu J^\mu_{5S} = \frac{g^2}{16\pi^2}\frac{n_f}{2}\epsilon^{\mu\nu\rho\kappa}G^a_{\mu\nu}G^a_{\rho\kappa}, \tag{19}$$

where $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$ is the gluonic field-strength tensor, the calculation reduces to the one in the two-loop level and becomes much simpler. Let us define the following amplitude:

$$\begin{aligned} S^{\sigma\lambda\tau}(-p,p) &\equiv \int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{e}^{ip(x_1-x_2)}\langle 0|T \\ &\quad \times [J^\sigma_{5S}(x)A^\lambda(x_1)A^\tau(x_2)]|0\rangle_{\text{Amputated}} \\ &= [-2i\epsilon^{\sigma\lambda\tau\rho}p_\rho]\langle\gamma(p)\|J_{5S}(\mu^2)\|\gamma(p)\rangle, \end{aligned} \tag{20}$$

where A^λ is a photon field. We see that $S^{\sigma\lambda\tau}(-p,p)$ is nothing but the photon matrix element of J^σ_{5S} at zero momentum transfer (see eq. (5)). Then, by using the AB theorem (19), it can be shown [9] that $S^{\sigma\lambda\tau}(-p,p)$ is rewritten up to the order e^2g^4 as,

$$\begin{aligned} S^{\sigma\lambda\tau}(-p,p) &= \frac{g^2}{4\pi^2}\frac{n_f}{2}\int \mathrm{d}^4x_1\mathrm{d}^4x_2\mathrm{e}^{ip(x_1-x_2)}\epsilon^{\sigma\nu\rho\kappa}\langle 0|T \\ &\quad \times \left[A^a_\nu(x)\partial_\rho A^a_\kappa(x)A^\lambda(x_1)A^\tau(x_2)\right]|0\rangle_{\text{Amputated}}. \end{aligned} \tag{21}$$

In order to obtain $A^{(2)}_{\mathrm{S}}$, we calculate $S^{\sigma\lambda\tau}(-p,p)$ in the order of e^2g^4. Instead of evaluating the r.h.s. of eq. (20), which is a three-loop calculation, we compute the r.h.s. of eq. (21). Then the calculation reduces to the two-loop level.

The relevant two-loop diagrams contributing to $S^{\sigma\lambda\tau}(-p,p)$ are shown in figure 3. There are also two counter-term diagrams corresponding to the diagrams (a) and (b). The result for each diagram is invariant under the reversing charge flow, and so each diagram is computed only for one arrow direction and then mutiplied by a factor of 2. A calculation in the $\overline{\mathrm{MS}}$ scheme with $d = 4-2\epsilon$ is performed. The QCD coupling constant in d-dimension is written by

$$\hat{g} = g\mu^\epsilon, \tag{22}$$

where g is the dimensionless coupling constant.

3.1 *Contribution of diagram* (a)

The contribution of the diagram (a), computed in Feynman–'t Hooft gauge and for massless quarks, is given by

$$\begin{aligned} \text{Diagram (a)} &= 2\times\left[\frac{\hat{g}^2}{4\pi^2}\frac{n_f}{2}\right]\epsilon^{\sigma\nu\rho\kappa}\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{\mathrm{d}^dq}{(2\pi)^d}(-iq_\rho)\left(\frac{-i}{q^2}\right)^2 \\ &\quad \times(-)\mathrm{Tr}\left[(-ie\gamma_\lambda)\frac{i}{\not{k}}(i\hat{g}\gamma_\nu T^a)\frac{i}{\not{k}-\not{q}}(i\hat{g}\gamma_\kappa T^a)\right. \\ &\quad \left.\times\frac{i}{\not{k}}(-ie\gamma_\tau)\frac{i}{\not{k}-\not{p}}\right] \\ &\equiv -2i\left[\frac{\hat{g}^2}{4\pi^2}\frac{n_f}{2}\right]e^2\hat{g}^2\ \mathrm{Tr}[T^aT^a]\epsilon^{\sigma\nu\rho\kappa}W^{(a)}_{\lambda\tau\nu\rho\kappa}, \end{aligned} \tag{23}$$

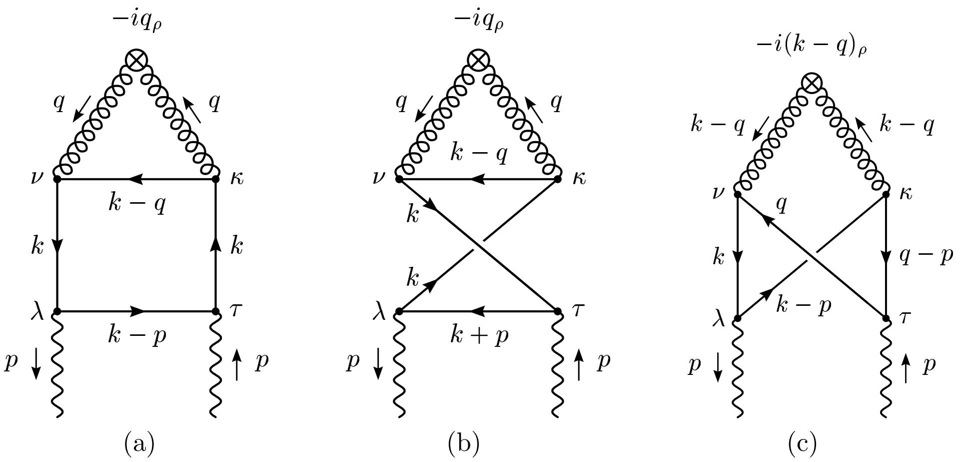

Figure 3. Two-loop diagrams contributing to $S^{\sigma\lambda\tau}(-p,p)$

with

$$W^{(\mathrm{a})}_{\lambda\tau\nu\rho\kappa} = \int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{\mathrm{d}^d q}{(2\pi)^d}\frac{q_\rho}{(k^2)^2(q^2)^2(k-q)^2(k-p)^2} \times \mathrm{Tr}[\gamma_\lambda \not{k}\gamma_\nu(\not{k}-\not{q})\gamma_\kappa \not{k}\gamma_\tau(\not{k}-\not{p})], \tag{24}$$

and T^a is the generator of the colour group $SU(3)_{\mathrm{C}}$.

Due to the levi-civita symbol $\epsilon^{\sigma\nu\rho\kappa}$ in eq. (23), only the part of $W^{(\mathrm{a})}_{\lambda\tau\nu\rho\kappa}$, which is totally antisymmetric in three indices ν,ρ and κ, contributes. So we introduce a projection operator $T_{\lambda\tau\nu\rho\kappa}$ which is made up of the metric tensor $g_{\mu\nu}$ and the momentum p and is linear in p. Furthermore, $T_{\lambda\tau\nu\rho\kappa}$ is totally antisymmetric in three indices ν,ρ and κ:

$$T_{\lambda\tau\nu\rho\kappa} = g_{\lambda\nu}g_{\tau\rho}p_\kappa - g_{\lambda\rho}g_{\tau\nu}p_\kappa + g_{\lambda\rho}g_{\tau\kappa}p_\nu - g_{\lambda\kappa}g_{\tau\rho}p_\nu + g_{\lambda\kappa}g_{\tau\nu}p_\rho - g_{\lambda\nu}g_{\tau\kappa}p_\rho. \tag{25}$$

Note that $T_{\lambda\tau\nu\rho\kappa}$ is also antisymmetric in indices λ and τ. Writing $W^{(\mathrm{a})}_{\lambda\tau\nu\rho\kappa}$ as

$$W^{(\mathrm{a})}_{\lambda\tau\nu\rho\kappa} = T_{\lambda\tau\nu\rho\kappa}S^{(\mathrm{a})} + \cdots, \tag{26}$$

where $\cdots$ expresses irrelevant terms, then inserting eq. (26) into eq. (23) we obtain

$$\text{Diagram (a)} = -2i\left[\frac{\hat{g}^2}{4\pi^2}\frac{n_f}{2}\right]e^2\hat{g}^2\ \mathrm{Tr}[T^aT^a][6\epsilon^{\sigma}_{\ \lambda\tau\rho}p^\rho]S^{(\mathrm{a})}. \tag{27}$$

On the other hand, we find, in d dimension,

$$T^{\lambda\tau\nu\rho\kappa}W^{(\mathrm{a})}_{\lambda\tau\nu\rho\kappa} = 6p^2(d-1)(d-2)S^{(\mathrm{a})}, \tag{28}$$

since $T^{\lambda\tau\nu\rho\kappa}T_{\lambda\tau\nu\rho\kappa} = 6p^2(d-1)(d-2)$.

Now $T^{\lambda\tau\nu\rho\kappa}W^{(\mathrm{a})}_{\lambda\tau\nu\rho\kappa}$ can be expressed as a linear combination of the following scalar integrals $F(\nu_1,\nu_2,\nu_3,\nu_4,\nu_5)$:

$$F(\nu_1,\nu_2,\nu_3,\nu_4,\nu_5) = \int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{\mathrm{d}^d q}{(2\pi)^d}\frac{1}{[k^2]^{\nu_1}[q^2]^{\nu_2}[(k-q)^2]^{\nu_3}[(k-p)^2]^{\nu_4}[(q-p)^2]^{\nu_5}}, \quad (29)$$

and its expression is given in eq. (B1) in Appendix B. Next we apply the algorithm FIRE in order to perform the reduction of the scalar integrals $F(\nu_1,\nu_2,\nu_3,\nu_4,\nu_5)$ to the master integrals. The relevant reduction formulae for the scalar integrals are given in Appendix C. Using these reduction formulae, we find from eq. (B1)

$$T^{\lambda\tau\nu\rho\kappa}W^{(\mathrm{a})}_{\lambda\tau\nu\rho\kappa} = -\frac{4(d-2)^2(d-1)d}{(d-6)}F(1,0,1,0,1), \quad (30)$$

where $F(1,0,1,0,1)$ is one of the master integrals. Then we obtain from eqs (27)–(30),

$$\text{Diagram (a)} = \left[\frac{\hat{g}^2}{4\pi^2}\frac{n_f}{2}\right] e^2\hat{g}^2\mathrm{Tr}[T^aT^a][-2i\epsilon^{\sigma}{}_{\lambda\tau\rho}p^{\rho}] \times\frac{-4(d-2)d}{(d-6)}\frac{1}{p^2}F(1,0,1,0,1). \quad (31)$$

3.2 *Contribution of diagram* (b)

Diagram (b) is obtained from diagram (a) by interchanging Lorentz indices λ and τ and by replacing the external momentum p with $-p$. Then a factor $[-2i\epsilon^{\sigma}_{\lambda\tau\rho}p^{\rho}]$ in eq. (31) will be replaced with $[-2\epsilon^{\sigma}_{\tau\lambda\rho}(-p)^{\rho}]$, which is the same as the former. Therefore, the contributions of diagrams (a) and (b) are the same.

3.3 *Contribution of diagram* (c)

The contribution of the diagram (c) is given by

$$\begin{aligned}\text{Diagram (c)} &= 2\times\left[\frac{\hat{g}^2}{4\pi^2}\frac{n_f}{2}\right]\epsilon^{\sigma\nu\rho\kappa}\int\frac{\mathrm{d}^d k}{(2\pi)^d}\frac{\mathrm{d}^d q}{(2\pi)^d}(-i(k-q)_\rho)\\ &\quad\times\left(\frac{-i}{(k-q)^2}\right)^2(-)\mathrm{Tr}\left[(-ie\gamma_\lambda)\frac{i}{\not k}(i\hat{g}\gamma_\nu T^a)\right.\\ &\quad\left.\times\frac{i}{\not q}(-ie\gamma_\tau)\frac{i}{\not q-\not p}(i\hat{g}\gamma_\kappa T^a)\frac{i}{\not k-\not p}\right]\\ &\equiv -2i\left[\frac{\hat{g}^2}{4\pi^2}\frac{n_f}{2}\right]e^2\hat{g}^2\ \mathrm{Tr}[T^aT^a]\epsilon^{\sigma\nu\rho\kappa}W^{(\mathrm{c})}_{\lambda\tau\nu\rho\kappa}\ ,\end{aligned} \quad (32)$$

with

$$W^{(\mathrm{c})}_{\lambda\tau\nu\rho\kappa} = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d q}{(2\pi)^d} \frac{(k-q)_\rho}{k^2 q^2 (k-p)^2 (q-p)^2 [(k-q)^2]^2} \times \mathrm{Tr}[\gamma_\lambda \not{k} \gamma_\nu \not{q} \gamma_\tau (\not{q} - \not{p}) \gamma_\kappa (\not{k} - \not{p})]. \quad (33)$$

Following the same procedures as we did for the calculation of diagram (a) (see eq. (B2) in Appendix B and the reduction formulae in Appendix C), we find

$$T^{\lambda\tau\nu\rho\kappa} W^{(\mathrm{c})}_{\lambda\tau\nu\rho\kappa} = \frac{8(d-2)^2}{(d-4)^2} [2(d-3) d F(1,0,1,0,1) - (d-4) p^2 F(1,1,0,1,1)], \quad (34)$$

where $F(1,1,0,1,1)$ is another master integral. Then we obtain

$$\text{Diagram (c)} = \left[\frac{\hat{g}^2}{4\pi^2} \frac{n_f}{2}\right] e^2 \hat{g}^2 \mathrm{Tr}[T^a T^a] [-2i\epsilon^\sigma_{\lambda\tau\rho} p^\rho] \frac{8(d-2)}{(d-1)(d-4)^2} \times \left[2(d-3) d \, \frac{1}{p^2} F(1,0,1,0,1) - (d-4) F(1,1,0,1,1)\right]. \quad (35)$$

3.4 *Master integrals*

There appear two master integrals $F(1,0,1,0,1)$ and $F(1,1,0,1,1)$. We evaluate them in the $\overline{\mathrm{MS}}$ scheme with $d = 4 - 2\epsilon$:

$$F(1,0,1,0,1) = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d q}{(2\pi)^d} \frac{1}{k^2 (k-q)^2 (q-p)^2} = \frac{1}{(16\pi^2)^2} S^{2\epsilon} (-p^2)^{1-2\epsilon} \frac{\tilde{\Gamma}(2\epsilon)}{2\epsilon - 1} B(1-\epsilon, 1-\epsilon) \times B(2-2\epsilon, 1-\epsilon), \quad (36)$$

$$F(1,1,0,1,1) = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d q}{(2\pi)^d} \frac{1}{k^2 q^2 (k-p)^2 (q-p)^2} = -\frac{1}{(16\pi^2)^2} S^{2\epsilon} (-p^2)^{-2\epsilon} \left[\tilde{\Gamma}(\epsilon) B(1-\epsilon, 1-\epsilon)\right]^2, \quad (37)$$

where $B(x,y)$ is the beta function and

$$S^\epsilon \equiv (4\pi)^\epsilon \mathrm{e}^{-\epsilon\gamma_{\mathrm{E}}}, \quad \tilde{\Gamma}(\epsilon) \equiv \mathrm{e}^{\epsilon\gamma_{\mathrm{E}}} \Gamma(\epsilon) \quad (38)$$

with γ_{E} and $\Gamma(x)$ being the Euler constant and the gamma function, respectively.

Now we summarize the contributions of diagrams (a), (b) and (c). Here we replace $\mathrm{Tr}[T^a T^a]$ with $3C_F$ and $\hat{g}^2$ with $g^2 \mu^{2\epsilon}$. We find from eqs (31), (35)–(37),

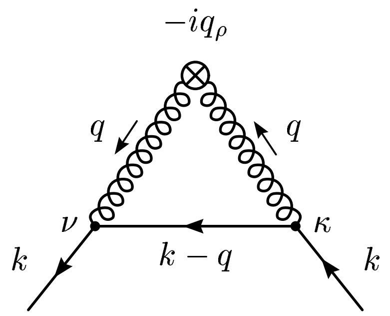

Figure 4. Subdivergence graph of the self-energy type.

$$\begin{aligned}\text{Diagram (a)} &= \text{Diagram (b)} \\ &= \left[\frac{\alpha_s}{\pi}\frac{n_f}{2}\right]\left[\frac{\alpha}{4\pi}\frac{\alpha_s}{4\pi}\right]3C_F S^{2\epsilon}\left(\frac{\mu^2}{-p^2}\right)^{2\epsilon} \\ &\quad \times\left[\frac{4}{\epsilon}+16\right][-2i\epsilon^{\sigma}_{\lambda\tau\rho}p^{\rho}] \end{aligned} \tag{39}$$

$$\begin{aligned}\text{Diagram (c)} &= \left[\frac{\alpha_s}{\pi}\frac{n_f}{2}\right]\left[\frac{\alpha}{4\pi}\frac{\alpha_s}{4\pi}\right]3C_F S^{2\epsilon}\left(\frac{\mu^2}{-p^2}\right)^{2\epsilon} \\ &\quad \times\left[\frac{4}{\epsilon}+\frac{82}{3}-16\zeta_3\right][-2i\epsilon^{\sigma}_{\lambda\tau\rho}p^{\rho}]. \end{aligned} \tag{40}$$

3.5 *Contributions of the counter-term diagrams*

In diagram (a), there appears a subdivergent self-energy part which is (shown in figure 4) given by

$$\begin{aligned} A_{\nu\rho\kappa} &\equiv \int \frac{\mathrm{d}^d q}{(2\pi)^d}(-iq_\rho)\left(\frac{-i}{q^2}\right)^2 (i\hat{g}\gamma_\nu T^a)\frac{i}{\not{k}-\not{q}}(i\hat{g}\gamma_\kappa T^a) \\ &= \hat{g}^2 T^a T^a \int \frac{\mathrm{d}^d q}{(2\pi)^d}\frac{q_\rho}{(q^2)^2\,(k-q)^2}\left[\gamma_\nu(\not{k}-\not{q})\gamma_\kappa\right]. \end{aligned} \tag{41}$$

In the $\overline{\mathrm{MS}}$ scheme, $A_{\nu\rho\kappa}$ is evaluated as

$$A_{\nu\rho\kappa} = \bar{A}_{\nu\rho\kappa}\, S^{\epsilon}\frac{1}{\epsilon} + \text{finite terms}, \tag{42}$$

with

$$\bar{A}_{\nu\rho\kappa} \equiv -i\frac{g^2}{4\pi^2}T^a T^a\frac{1}{4}\gamma_\nu\gamma_\rho\gamma_\kappa. \tag{43}$$

The counter-term diagrams are derived from diagrams (a) and (b), by replacing the self-energy part $A_{\nu\rho\kappa}$ with $(-\bar{A}_{\nu\rho\kappa}S^{\epsilon}\frac{1}{\epsilon})$. No counter-term diagram arises from diagram (c), since there appears no self-energy part in diagram (c). The counter-term (Ct) diagram corresponding to the two-loop diagram (a) is

$$\begin{aligned}\text{CtDiagram (a)} &= 2\times\left[\frac{\hat{g}^2}{4\pi^2}\frac{n_f}{2}\right]\epsilon^{\sigma\nu\rho\kappa}\int\frac{\mathrm{d}^dk}{(2\pi)^d}\\ &\quad\times(-)\mathrm{Tr}\left[(-ie\gamma_\lambda)\frac{i}{\not{k}}(-\bar{A}_{\nu\rho\kappa}\ S^\epsilon\frac{1}{\epsilon})\frac{i}{\not{k}}(-ie\gamma_\tau)\frac{i}{\not{k}-\not{p}}\right]\\ &\equiv 2\left[\frac{\hat{g}^2}{4\pi^2}\frac{n_f}{2}\right]e^2\left(\frac{g^2}{4\pi^2}\ \mathrm{Tr}[T^aT^a]S^\epsilon\frac{1}{\epsilon}\right)\epsilon^{\sigma\nu\rho\kappa}W^{\mathrm{Ct(a)}}_{\lambda\tau\nu\rho\kappa},\end{aligned} \tag{44}$$

with

$$W^{\mathrm{Ct(a)}}_{\lambda\tau\nu\rho\kappa} = \frac{1}{4}\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{1}{(k^2)^2(k-p)^2}\mathrm{Tr}[\gamma_\lambda\not{k}\gamma_\nu\gamma_\rho\gamma_\kappa\not{k}\gamma_\tau(\not{k}-\not{p})]. \tag{45}$$

Writing $W^{\mathrm{Ct(a)}}_{\lambda\tau\nu\rho\kappa}$ as

$$W^{\mathrm{Ct(a)}}_{\lambda\tau\nu\rho\kappa} = T_{\lambda\tau\nu\rho\kappa}S^{\mathrm{Ct(a)}} + \cdots, \tag{46}$$

we find

$$\begin{aligned}\epsilon^{\sigma\nu\rho\kappa}W^{\mathrm{Ct(a)}}_{\lambda\tau\nu\rho\kappa} &= 6[\epsilon^\sigma_{\lambda\tau\rho}p^\rho]S^{\mathrm{Ct(a)}}\\ &= [\epsilon^\sigma_{\lambda\tau\rho}p^\rho]\frac{1}{p^2(d-1)(d-2)}T^{\lambda\tau\nu\rho\kappa}W^{\mathrm{Ct(a)}}_{\lambda\tau\nu\rho\kappa}.\end{aligned} \tag{47}$$

The calculation of $T^{\lambda\tau\nu\rho\kappa}W^{\mathrm{Ct(a)}}_{\lambda\tau\nu\rho\kappa}$ is performed in Appendix D. We obtain

$$\begin{aligned}T^{\lambda\tau\nu\rho\kappa}W^{\mathrm{Ct(a)}}_{\lambda\tau\nu\rho\kappa} &= \frac{i}{16\pi^2}(-p^2)^{1-\epsilon}3(d-1)(d-2)(d-4)\\ &\quad\times S^\epsilon\tilde{\Gamma}(\epsilon)B(1-\epsilon,1-\epsilon).\end{aligned} \tag{48}$$

From eqs (44), (47) and (48), we obtain

$$\begin{aligned}\text{CtDiagram (a)} &= \left[\frac{\alpha_s}{\pi}\frac{n_f}{2}\right]\left[\frac{\alpha}{4\pi}\frac{\alpha_s}{4\pi}\right]3C_FS^{2\epsilon}\left(\frac{\mu^2}{-p^2}\right)^\epsilon\\ &\quad\times\left[-\frac{6}{\epsilon}-12\right][-2i\epsilon^\sigma_{\lambda\tau\rho}p^\rho].\end{aligned} \tag{49}$$

Finally, the contribution of the counter-term diagram corresponding to the two-loop diagram (b) is the same with Ct diagram (a).

3.6 $S^{\sigma\lambda\tau}(-p,p)$ *in the order of* e^2g^4

Summing up all contributions, we obtain in the order of e^2g^4

$$S^{\sigma\lambda\tau}(-p,p) = \frac{\alpha}{4\pi}\left(\frac{\alpha_s}{4\pi}\right)^2 6C_F n_f \left\{\frac{106}{3} - 16\zeta_3 + 12\ln\frac{\mu^2}{(-p^2)}\right\} \times \left[-2i\epsilon_{\alpha\mu\nu\rho}p^\rho\right]. \tag{50}$$

We renormalized the photon matrix element at $\mu^2 = P^2$. We also take care of the quark charge factor and replace α with $\alpha n_f\langle e^2\rangle$. Then we find

$$A_s^{(2)} = 12\langle e^2\rangle C_F n_f^2 \left\{\frac{53}{3} - 8\zeta_3\right\}. \tag{51}$$

4. Conclusions

We have investigated the next-to-next-to-leading order ($\alpha\alpha_s^2$) corrections to the first moment of the polarized virtual photon structure function $g_1^\gamma(x,Q^2,P^2)$ in the kinematic region $Q^2 \gg P^2 \gg \Lambda^2$ in QCD. The use of the Adler–Bardeen theorem for axial current J_5^μ reduces the loop level of the Fynman diagram by one.

We have recalculated the photon matrix element $A_S^{(2)}$ by using reduction method. By using this method, the difficult integral is reduced to the same easy master integrals. The result we have obtained is consistent with the one calculated before by program FORM [12].

We have also re-evaluated eq. (18) and found that the NNLO($\alpha\alpha_s^2$) corrections are about 3% to the sum of the LO(α) and the NLO($\alpha\alpha_s$) in the region $Q^2 = 30$–100 GeV^2 and $P^2 = 3$ GeV^2. It is consistent with the result of ref. [9].

Appendix A: Parameters in the $\overline{\text{MS}}$ scheme

The one-loop and two-loop QCD β functions (see eq. (11)) are given by

$$\beta_0 = 11 - \frac{2}{3}n_f, \quad \beta_1 = 102 - \frac{38}{3}n_f. \tag{A1}$$

The results for the two-loop [18] and three-loop [19] anomalous dimensions (see eq. (8)) are

$$\gamma_S^{(1)}=12C_F n_f, \quad \gamma_S^{(2)}=\left(\frac{284}{3}C_F C_A - 36C_F^2\right)n_f - \frac{8}{3}C_F n_f^2, \tag{A2}$$

with $C_F = \frac{4}{3}$ and $C_A = 3$. The one-loop [20] and two-loop [21–23] coefficient functions (see eqs (13)–(14)) are given by

$$B_{\rm S}^{(1)} = B_{\rm NS}^{(1)} = -3C_F, \tag{A3}$$

$$B_{\rm S}^{(2)} = C_F\left[\frac{21}{2}C_F - 23C_A + \left(8\zeta_3 + \frac{13}{3}\right)n_f\right], \tag{A4}$$

$$B_{\rm NS}^{(2)} = C_F\left[\frac{21}{2}C_F - 23C_A + 4n_f\right], \tag{A5}$$

where ζ_3 is the Riemann zeta-function ($\zeta_3 = 1.202056903\cdots$). The leading terms of the photon matrix elements of the axial currents (see eqs (15) and (16)) are [5,24]

$$A_{\rm S}^{(0)} = -12n_f\langle e^2\rangle, \quad A_{\rm NS}^{(0)} = -12n_f(\langle e^4\rangle - \langle e^2\rangle^2). \tag{A6}$$

Appendix B: $[T^{\lambda\tau\nu\rho\kappa}W^{(a)}_{\lambda\tau\nu\rho\kappa}]$ and $[T^{\lambda\tau\nu\rho\kappa}W^{(c)}_{\lambda\tau\nu\rho\kappa}]$

We write down the expressions of $[T^{\lambda\tau\nu\rho\kappa}W^{(a)}_{\lambda\tau\nu\rho\kappa}]$ and $[T^{\lambda\tau\nu\rho\kappa}W^{(c)}_{\lambda\tau\nu\rho\kappa}]$ as a linear combination of the scalar integrals $F(\nu_1,\nu_2,\nu_3,\nu_4,\nu_5)$.

$$\begin{aligned}
T^{\lambda\tau\nu\rho\kappa}W^{(a)}_{\lambda\tau\nu\rho\kappa} = &-2(d-3)(d-2)(p^2)^2F(2,1,1,1,0)\\
&+2(d-6)(d-3)p^2F(2,1,1,1,-1)\\
&+2(d-3)(d-2)(p^2)^2F(1,2,1,1,0)\\
&-4(d-4)(d-3)p^2F(1,2,1,1,-1)\\
&+2(d-6)(d-3)F(1,2,1,1,-2)\\
&+4(d-4)(d-2)p^2F(1,1,1,1,0)\\
&-4(d-4)(d-3)F(1,1,1,1,-1)\\
&-2(d-5)(d-2)p^2F(0,2,1,1,0)\\
&+2(d-6)(d-3)F(0,2,1,1,-1)\\
&-4(d-2)F(-1,2,1,1,0)\\
&-2(d-5)(d-2)F(0,1,1,1,0)+\cdots
\end{aligned}\tag{B1}$$

$$\begin{aligned}
&T^{\lambda\tau\nu\rho\kappa}W^{(c)}_{\lambda\tau\nu\rho\kappa}\\
&\quad = 8(d-2)(p^2)^2F(1,1,1,1,1) - 6(d-2)p^2F(1,0,1,1,1)\\
&\quad\ +2(d-2)F(1,0,2,-1,1) - 4(d-2)p^2F(1,0,2,0,1)\\
&\quad\ +2(d-2)F(1,-1,2,0,1) - 6(d-2)p^2F(1,1,1,0,1)\\
&\quad\ -4(d-2)F(1,0,1,0,1) - 6(d-2)p^2F(0,1,1,1,1)\\
&\quad\ +4(d-2)F(0,0,1,1,1) - 6(d-2)p^2F(1,1,1,1,0)\\
&\quad\ -4(d-2)F(0,1,1,1,0) + 2(d-2)F(-1,1,2,1,0)\\
&\quad\ -4(d-2)p^2F(0,1,2,1,0) + 2(d-2)F(0,1,2,1,-1)\\
&\quad\ +8(d-2)p^2F(1,1,0,1,1)+\cdots
\end{aligned}\tag{B2}$$

where $\cdots$ means other terms which give no contribution. These terms involve the following scalar integrals which turn out to vanish due to a general property in dimensional regularization [25]:

$$\begin{aligned}
&F(2,0,1,1,0) = F(1,0,1,1,0) = F(2,0,1,0,0)\\
&= F(2,1,0,1,0) = F(2,1,1,0,-1) = F(2,1,1,0,0)\\
&= F(2,2,0,1,-1) = F(2,1,0,0,0) = F(2,2,0,1,0)
\end{aligned}$$

$$
\begin{aligned}
&= F(2,2,0,0,0) = F(2,2,0,1,-1) = F(2,2,0,1,0)\\
&= F(1,0,1,1,0) = F(2,1,1,-1,0) = F(1,1,1,0,0)\\
&= F(1,2,0,1,0) = F(2,2,-1,1,0) = F(1,2,0,1,-1)\\
&= F(1,2,1,0,0) = F(0,2,1,0,0) = F(0,1,2,0,1)\\
&= F(1,0,2,1,0) = F(0,0,1,1,1) = F(0,0,2,0,1)\\
&= F(1,0,2,0,0) = F(0,0,2,1,0) = F(0,1,2,0,0)\\
&= 0. \qquad \text{(B3)}
\end{aligned}
$$

Appendix C: The reduction of $F(\nu_1,\nu_2,\nu_3,\nu_4,\nu_5)$ to master integrals

The scalar integrals $F(\nu_1,\nu_2,\nu_3,\nu_4,\nu_5)$ are expressed in terms of the master integrals, $F(1,0,1,0,1)$ and $F(1,1,0,1,1)$, as follows:

$$(p^2)^2F(2,1,1,1,0) = \frac{(3d-10)(3d-8)}{(d-6)(d-4)}F(1,0,1,0,1) \qquad \text{(C1)}$$

$$p^2F(2,1,1,1,-1) = \frac{(d-2)(3d-8)}{(d-6)(d-4)}F(1,0,1,0,1) \qquad \text{(C2)}$$

$$(p^2)^2B(1,2,1,1,0) = -\frac{(d-3)(3d-10)(3d-8)}{(d-6)(d-4)}F(1,0,1,0,1) \qquad \text{(C3)}$$

$$p^2F(1,2,1,1,-1) = -\frac{(d-2)^2(3d-8)}{(d-6)(d-4)}F(1,0,1,0,1) \qquad \text{(C4)}$$

$$F(1,2,1,1,-2) = -\frac{(d-2)(d-1)d}{(d-6)(d-4)}F(1,0,1,0,1) \qquad \text{(C5)}$$

$$p^2F(1,1,1,1,0) = \frac{3d-8}{d-4}F(1,0,1,0,1) \qquad \text{(C6)}$$

$$F(1,1,1,1,-1) = \frac{d-2}{d-4}F(1,0,1,0,1) \qquad \text{(C7)}$$

$$p^2F(0,2,1,1,0) = -\frac{(d-3)(3d-8)}{d-4}F(1,0,1,0,1) \qquad \text{(C8)}$$

$$F(0,2,1,1,-1) = -\frac{(d-2)^2}{d-4}F(1,0,1,0,1) \qquad \text{(C9)}$$

$$F(-1,2,1,1,0) = -(d-3)F(1,0,1,0,1) \qquad \text{(C10)}$$

$$F(0,1,1,1,0) = F(1,0,1,0,1) \qquad \text{(C11)}$$

$$
\begin{aligned}
(p^2)^2F(1,1,1,1,1) &= \frac{2(3d-10)(3d-8)}{(d-4)^2}F(1,0,1,0,1)\\
&\quad -\frac{2(d-3)}{d-4}p^2F(1,1,0,1,1) \qquad \text{(C12)}
\end{aligned}
$$

$$p^2F(1,0,1,1,1) = \frac{3d-8}{d-4}F(1,0,1,0,1) \qquad \text{(C13)}$$

$$F(1,0,2,-1,1) = -(d-3)F(1,0,1,0,1) \qquad \text{(C14)}$$

$$p^2F(1,0,2,0,1) = -\frac{(d-3)(3d-8)}{d-4}F(1,0,1,0,1) \tag{C15}$$
$$F(1,-1,2,0,1) = -(d-3)F(1,0,1,0,1) \tag{C16}$$
$$p^2F(1,1,1,0,1) = \frac{3d-8}{d-4}F(1,0,1,0,1) \tag{C17}$$
$$p^2F(0,1,1,1,1) = \frac{3d-8}{d-4}F(1,0,1,0,1) \tag{C18}$$
$$F(0,0,1,1,1)) = 0 \tag{C19}$$
$$F(-1,1,2,1,0) = -(d-3)F(1,0,1,0,1) \tag{C20}$$
$$p^2F(0,1,2,1,0) = -\frac{(d-3)(3d-8)}{d-4}F(1,0,1,0,1) \tag{C21}$$
$$F(0,1,2,1,-1) = -(d-3)F(1,0,1,0,1). \tag{C22}$$

Appendix D: $[T^{\lambda\tau\nu\rho\kappa}W^{\mathrm{Ct(a)}}_{\lambda\tau\nu\rho\kappa}]$

First note that $[T^{\lambda\tau\nu\rho\kappa}W^{\mathrm{Ct(a)}}_{\lambda\tau\nu\rho\kappa}]$ is expressed as a linear combination of the scalar integrals $G(\nu_1,\nu_2)$,

$$G(\nu_1,\nu_2) = \int \frac{\mathrm{d}^dk}{(2\pi)^d}\frac{1}{[k^2]^{\nu_1}[(k-p)^2]^{\nu_2}}. \tag{D1}$$

We get

$$T^{\lambda\tau\nu\rho\kappa}W^{\mathrm{Ct(a)}}_{\lambda\tau\nu\rho\kappa} = -3(d-2)(d-5)p^2G(1,1) \\ +3(d-2)(d-3)(p^2)^2G(2,1), \tag{D2}$$

where we have used the fact that when either ν_1 and ν_2 is a non-positive integer, then $G(\nu_1,\nu_2)=0$. The two scalar integrals $G(1,1)$ and $G(2,1)$ are calculated as follows:

$$G(1,1) = \frac{1}{(-p^2)^\epsilon}\tilde{I}, \quad G(2,1) = \frac{d-3}{-p^2}G(1,1) \tag{D3}$$

with

$$\tilde{I} \equiv \frac{i}{16\pi^2}S^\epsilon\tilde{\Gamma}(\epsilon)B(1-\epsilon,1-\epsilon). \tag{D4}$$

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