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Inflation with hyperbolic potential in the braneworld model

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Abstract. In this paper we study inflationary dynamics with a scalar field in an inverse coshyperbolic potential in the braneworld model. We note that a sufficient inflation may be obtained with the potential considering slow-roll approximation in the high energy limit. We determine the minimum values of the initial inflaton field required to obtain sufficient inflation and also determine the relevant inflationary parameters. The numerical values of spectral index of the scalar perturbation spectrum are determined by varying the number of e-foldings for different initial values of the inflaton field. The result obtained here is in good agreement with the current observational limits.

Keywords. RS braneworld; hyperbolic potential; cosmology.

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1. Introduction

It is now generally believed that at very high energy scale, Einstein's general theory of gravity (henceforth, GTR) fails to predict the early Universe satisfactorily. The high energy perhaps was available shortly after the Big Bang. Consequently, GTR needs to be modified. It is likely that the GTR may be reduced to a limiting case of a more general theory at low energy limit. To accommodate such a theory, one requires a quantized theory of gravity, but a consistent quantum theory of gravity is yet to emerge. The recent successes in superstring/M-theory [1,2] lead us to believe that it is a promising candidate for quantum gravity. These theories require space-time dimensions more than the usual four for their consistent formulation. In recent years there is a paradigm shift in cosmological model building in the higher-dimensional theory from that of the previous approach initiated by Kaluza–Klein [3] and others in cosmology [4]. In modern higher-dimensional scenario, the fields of the standard model are considered to be confined to (3 + 1)-dimensional hypersurface (referred to as 3-brane) embedded in a higher-dimensional space-time but the gravitational field may propagate through the bulk dimension perpendicular to the brane which is referred to as braneworld. According to Randall and Sundrum, although the extra dimension is not compact, four-dimensional Newtonian gravity is recovered in five-dimensional anti-de Sitter space-time (AdS_5) in the low energy limit [5]. The braneworld scenario has interesting cosmological implications. In particular, the prospects of inflation are enhanced on the brane due to the modifications in the Friedmann equation [6]. The high energy corrections of the Friedmann equation is due to the effect of extra dimension projected on the brane and it changes the nature of expansion dynamics of the early Universe [6,7]. According to Randall–Sundrum, braneworld picture [8] is based on a Type IIB D3-brane embedded in a five-dimensional Schwarzschild–anti-de Sitter (AdS_5), where the conventional four-dimensional gravity is recovered in low energy limit even if the extra dimension is non-compact. It was shown that RS-braneworld model is described by a 4d effective gravity induced on the world volume of the D3 brane embedded in 5d Einstein gravity. Consequently, the 5d Planck scale, M_5 is assumed to be considerably smaller than the corresponding 4d effective Planck scale, $M_4 = 1.2 \times 10^{19}$ GeV.

In the literature a large volume of work on cosmological models in the braneworld scenario has been reported, assuming that the brane Universe is dominated by a single minimally coupled scalar field which is rolling in a given potential. The potential must be sufficiently steep at the time of inflation to achieve slow-roll conditions in the very high energy limit. Several authors have also studied inflationary dynamics of the Universe in the Randall–Sundrum (Type II) braneworld model with such relatively steep potentials like exponential and inverse power-law potentials [9–11]. Even Sami and his collaborators [11] have shown that in the case of inverse power law type of potential, the phantom field may successfully drive the current acceleration with an equation of state parameter $\omega < -1$. This model also fits the supernovae data very well, allowing for $-2.4 < \omega_\phi < -1$ at 95% confidence level, where the suffix indicates the phantom field.

In the framework of GTR, the hilltop inflation model is more natural than a very flat potential model as noticed in refs [12,13]. The potential covers a range of possibilities to yield analytic formulas for the structure formation and the prediction of spectral index with the observed value. In this paper we study the inflationary dynamics in the Randall–Sundrum braneworld model with an inflaton field in an inverse coshyperbolic potential which is similar to the potential that was previously considered in GTR [14,15]. The potential was used with a phantom for a viable model with an equation of state $\omega_\phi < -1$ [14]. In the case of a tachyon rolling on the brane such a potential appears in the context of open string field theory for unstable D-brane system in Type II superstring theory [15,16]. Steer and Vernizzi [17] further used the potential to compare single scalar field inflation predictions with those of an inflationary phase driven by a tachyon field. Here, we determine the evolution of the scalar field with the inverse coshyperbolic potential to obtain cosmological solution in braneworld and estimate the allowed range of values for the spectral index of the scalar perturbation spectrum ($n_{\rm s}$). We also analyse the variation of $n_{\rm s}$ with the number of e-foldings (N) for different initial values of the field taking into account the recent observational predictions to determine the permissible values of N here.

The paper is organized as follows: The basic equations of the Randall–Sundrum (Type II) braneworld model are given in §2. In §3 we obtain inflationary dynamics in the braneworld model with inverse coshyperbolic potential and detail properties of this model for physics interests. Last section is devoted to our discussions and conclusions.

2. Basic equations

In the (4+1)-dimensional braneworld scenario inspired by the Randall–Sundrum [8] model, the standard Friedmann equation with Robertson–Walker metric acquires the generalized form [6,18]:

$$H^2 = \frac{\Lambda_4}{3} + \left(\frac{8\pi}{3M_4^2}\right)\rho + \left(\frac{8\pi}{M_5^3}\right)^2\rho^2 + \frac{\varepsilon}{a^4}, \tag{1}$$

where $H(\equiv \dot{a}/a)$ is the Hubble constant, ε is an integration constant which transmits bulk graviton influence onto the brane and Λ_4 is the four-dimensional Planck's constant. The relationship between the four- and five-dimensional Planck masses is given by

$$M_4 = \sqrt{\frac{3}{4\pi}}\left(\frac{M_5^2}{\sqrt{\lambda}}\right)M_5, \tag{2}$$

where λ represents the brane tension. By a suitable choice of brane tension one can set $\Lambda_4 = 0$ [6]. The last term of eq. (1) is termed as the dark radiation term (ε/a^4) which is expected to disappear rapidly once inflation commenced. Hence we effectively get [6,18]

$$H^2 = \frac{8\pi}{3M_4^2}\rho\left[1 + \frac{\rho}{2\lambda}\right]. \tag{3}$$

We now consider a Universe dominated by a single minimally coupled scalar field moving in a potential $V(\phi)$. Thus for homogeneous inflaton field the energy density (ρ) is given by

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi). \tag{4}$$

The equation of motion of the scalar field propagating on the brane is given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{{\rm d}V(\phi)}{{\rm d}\phi} = 0, \tag{5}$$

where dot represents the time derivative.

The set of eqs (3) and (5) are used to obtain cosmological solution for a known potential by employing slow-roll approximation technique. We now invoke the standard assumption that the energy density on the brane can be separated into two parts, where one of them is the ordinary matter component, ρ, and the other contributes when the brane tension, $\lambda > 0$, such that $\sigma = \rho + \lambda$. Steep inflation proceeds in the region of parameter space where $\sigma \approx \rho \gg \lambda$ and naturally comes to an end when $\rho \approx \lambda$. Considering the slow-roll approximation technique, the condition for inflation $\dot{\phi}^2 \ll V$ and $\ddot{\phi} \ll V'$, is used to re-write eqs (3) and (5) as [6,18]:

$$H^2 \simeq \frac{8\pi}{3M_4^2} V \left[1 + \frac{V}{2\lambda}\right], \quad \dot{\phi} \simeq -\frac{V'}{3H}. \tag{6}$$

The factor $[1+\frac{V}{2\lambda}]$ in the above field equation is a modification of the energy density when we compare it to the standard Einstein equation using slow-roll approximation. However, the standard expression in GTR is recovered by taking the brane tension to a limiting value $\lambda \to \infty$. The slow-roll parameters on the brane [6,18] are given below:

$$\varepsilon = -\frac{\dot{H}}{H^2} = \frac{M_4^2}{4\pi}\left(\frac{V'}{V}\right)^2 \left[\frac{\lambda(\lambda + V)}{(2\lambda + V)^2}\right]. \tag{7}$$

$$\eta = \frac{V''}{3H^2} = \frac{M_4^2}{4\pi}\left(\frac{V''}{V}\right)^2 \left[\frac{\lambda}{2\lambda + V}\right]. \tag{8}$$

The number of e-folds is an important quantity which indicates the proper multiplication of the size of the Universe as the inflation ends to get rid of the problems encountered by a standard Big Bang model. Using slow-roll approximation we get

$$N \simeq -\frac{8\pi}{M_4^2}\int_{\phi_\mathrm{i}}^{\phi_\mathrm{f}} \frac{V}{V'}\left(1 + \frac{V}{2\lambda}\right) \mathrm{d}\phi, \tag{9}$$

where ϕ_i and ϕ_f denote the initial and final values of inflaton field respectively.

The scalar amplitude of density perturbations $A_\mathrm{S}^2 = H^4/(25\pi^2\dot{\phi}^2)$ in this case can be written as

$$A_\mathrm{S}^2 = \frac{512\pi}{75M_4^6}\left(\frac{V^3}{V'^2}\right)^2 \left[\frac{2\lambda + V}{2\lambda}\right]^3 \Bigg|_{k=aH}. \tag{10}$$

In the high energy limit, it becomes

$$A_\mathrm{S}^2 \simeq \frac{64\pi}{75\lambda^3 M_4^6}\left(\frac{V^6}{V'^2}\right). \tag{11}$$

In the high energy limit, the contribution of the gravitational wave relative to density perturbation is suppressed and the relative amplitude [18–20] is given by

$$\frac{A_\mathrm{T}^2}{A_\mathrm{S}^2} = 6\varepsilon\frac{\lambda}{\rho} = \frac{3M_4^2}{2\pi}\,\frac{\alpha^2}{M_\mathrm{P}^2\beta^2}\,\left[\frac{\lambda}{V}\right]^2. \tag{12}$$

The spectral index of the scalar spectrum, $n_{\rm s}-1 \equiv ({\rm d}(\ln A_{\rm S}^2)/{\rm d}(\ln k))$ in terms of slow-roll parameters obtained above now can be written as

$$n_{\rm s} = 1 - 6\varepsilon + 2\eta. \tag{13}$$

At high energies, $V/\lambda \to \infty$ leads to the value of the spectral index, $n_{\rm s}$, approaching 1.

In the next section we study the dynamics of an inflaton field in an inverse coshyperbolic potential in the Randall–Sundrum (Type II) braneworld model.

3. Inflationary dynamics with inverse coshyperbolic potential

In the braneworld scenario, at high energy limit where $V \gg \lambda$, the modified terms in the Friedmann equation are important and therefore the field equations are given by

$$H^2 \simeq \left(\frac{4\pi}{3M_5^3}\right) V^2, \quad \dot{\phi} \simeq -\frac{V'}{V}\left(\frac{M_5^3}{4\pi}\right). \tag{14}$$

The slow-roll parameters defined in the previous section become

$$\varepsilon = \frac{M_4^2}{16\pi}\left(\frac{V'}{V}\right)^2\left[\frac{4\lambda}{V}\right], \tag{15}$$

$$\eta = \frac{M_4^2}{8\pi}\left(\frac{V''}{V}\right)\left[\frac{2\lambda}{V}\right], \tag{16}$$

and the number of e-foldings becomes

$$N \simeq -\frac{4\pi}{\lambda M_4^2}\int_{\phi_{\rm i}}^{\phi_{\rm f}} \frac{V^2}{V'}{\rm d}\phi, \tag{17}$$

where $\phi_{\rm i}$ and $\phi_{\rm f}$ denote the initial and final values of inflaton respectively.

We now consider scalar field in an inverse coshyperbolic potential [14–17] which is given by

$$V = V_0\left[\cosh\frac{\alpha\phi}{M_{\rm P}}\right]^{-1} = V_0\,{\rm sech}\,(\gamma\phi), \tag{18}$$

where $M_{\rm P} = M_4/\sqrt{8\pi}$ and $\gamma = \alpha/M_{\rm P}$. The above potential is employed to derive the current accelerating phase of expansion of late Universe in the framework of GTR [13,14]. However, such potential is relevant in the context of phantom field. The motivation of the paper is to explore cosmological model with the potential in the framework of braneworld in the high energy limit and also analyse the variation of the spectral index of the scalar perturbation spectrum ($n_{\rm s}$) with the number of e-foldings (N) to study its suitability. The time evolution of the field is obtained by integrating eq. (14) which is

$$\phi(t) = \frac{1}{\gamma} \sinh^{-1} \exp(Bt + C), \tag{19}$$

where $B = M_5^3 \gamma^2/4\pi$ and $C = \ln[\sinh(\gamma\phi_{\rm i})] - Bt_0$, are constants, and $\phi = \phi_{\rm i}$ when the inflation starts ($t = t_{\rm i}$). We obtain the scale factor by integrating eq. (14), which is given by

$$a(t) = a_{\rm i} {\rm e}^{[\frac{A}{B}(\tanh^{-1} \frac{1}{\sqrt{{\rm e}^{2(Bt+C)}+1}})]}, \tag{20}$$

where $A = \sqrt{4\pi V_0/3M_5^3}$. In the model inflation ends when

$$t_{\rm end} = \frac{\ln\left[\frac{\sqrt{2}\sqrt{A^2+\sqrt{A^4+4B^2A^2}}}{2B}\right] - C}{B}. \tag{21}$$

Thus one can determine the value of the inflaton field at the end of inflation, which is given by

$$\phi_{\rm end} = \frac{1}{\gamma} \sinh^{-1} \left[\frac{\sqrt{2}\sqrt{A^2 + \sqrt{A^4 + 4B^2A^2}}}{2B}\right]. \tag{22}$$

Consequently, one can determine the magnitude of the potential at the end of inflation which is

$$V_{\rm end} = V_0 \,{\rm sech} \left[\sinh^{-1} \left(\frac{\sqrt{2}\sqrt{A^2 + \sqrt{A^4 + 4B^2A^2}}}{2B}\right)\right]. \tag{23}$$

The expression for the number of e-foldings is given by

$$N(t) = \ln \frac{a(t)}{a_0} = \left[\frac{A}{B}\left(\tanh^{-1} \frac{1}{\sqrt{(\exp(Bt + C))^2 + 1}}\right)\right]_{t_{\rm i}}^{t_{\rm end}}. \tag{24}$$

The number of e-foldings is related to the initial and end values of the potential $V_{\rm i}$ and $V_{\rm end}$ respectively which is

$$V_{\rm end} = V_0 \tanh\left[\tanh^{-1}\left(\frac{V_{\rm i}}{V_0}\right) - \left(\frac{B}{A}\right) N\right]. \tag{25}$$

We now derive the analytical expressions of the relevant inflationary parameters as a function of the field. With the inverse coshyperbolic potential given by eq. (18), the slow-roll parameters given by eqs (15) and (16) become

$$\varepsilon = \beta \left(\frac{\tanh^2 x}{{\rm sech}\, x}\right), \tag{26}$$

$$\eta = \beta \left(\frac{\tanh^2 x - {\rm sech}^2 x}{{\rm sech}\, x}\right), \tag{27}$$

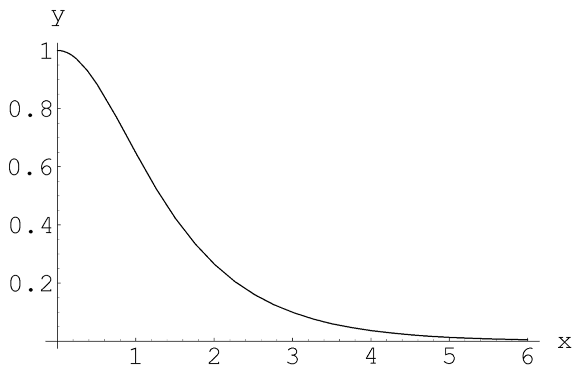

Figure 1. The variation of the potential ($y = V/V_0$) with the field (x).

where $\beta = 3\alpha^2 M_5^6/2\pi M_4^2 V_0$ and $x = \gamma\phi$. At the end of inflation, since $\varepsilon \sim 1$ we determine

$$\beta = \frac{\operatorname{sech} x_{\text{end}}}{\tanh^2 x_{\text{end}}}. \tag{28}$$

The number of e-foldings can be obtained using eq. (16), which is

$$N = \frac{1}{\beta}\left[-\ln\left(\cosh\frac{x}{2}\right) + \ln\left(\sinh\frac{x}{2}\right)\right]_{x_i}^{x_{\text{end}}}, \tag{29}$$

where x_{end} denotes the value of x when inflation ends.

$N \approx 70$ is the minimum number of e-foldings required to solve outstanding problems of cosmology, and using this N, eqs (27) and (28) determine $x_{\text{end}} = 5.10$ and $\beta = 0.012$, assuming $x_{\text{i}} = 0.9$ with a consideration that $\varepsilon \sim 1$ at the end of inflation. We have verified that as the variation of the slope of the potential attains a maximum at $x \approx 0.9$ (figure 2), one obtains a sufficient inflation. This is achieved if the field starts with an initial $x \geq 0.9$ where $V/V_0 \leq 0.7$ (figure 1). It is also evident that the slow-roll parameters, ε and η, remains less than unity for the range $x_{\text{i}} < x < x_{\text{end}}$, and the inequality depends on a chosen initial value x_{i} (figures 3 and 4).

The amplitude of scalar perturbations is given by

$$A_{\text{S}}^2 = \left(\frac{64\pi^2}{45\sqrt{8\pi}}\right)^2 \frac{M_4^2}{M_5^{18}}\frac{V_0^4}{\alpha^2}\left[\frac{\operatorname{sech}^4 x_{\text{COBE}}}{\tanh^2 x_{\text{COBE}}}\right]. \tag{30}$$

For our model, we choose $N_{\text{COBE}} \approx 55$, $x_{\text{i}} = x_{\text{COBE}} = 0.9$ and we get, using eqs (28) and (29),

$$\beta = 0.015 \tag{31}$$

$$x_{\text{end}} = 4.866. \tag{32}$$

Using β, one can determine

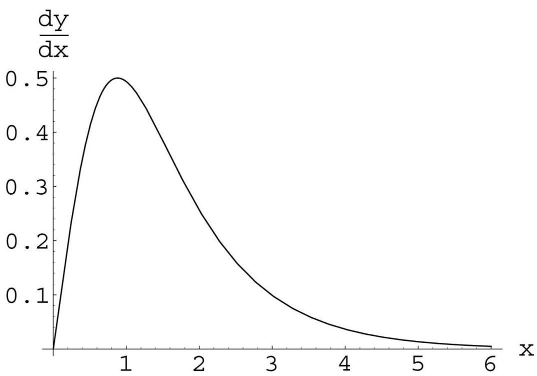

Figure 2. The slope of the potential ($|\mathrm{d}y/\mathrm{d}x|$) with the field ($x$) which is maximum at $x \approx 0.9$.

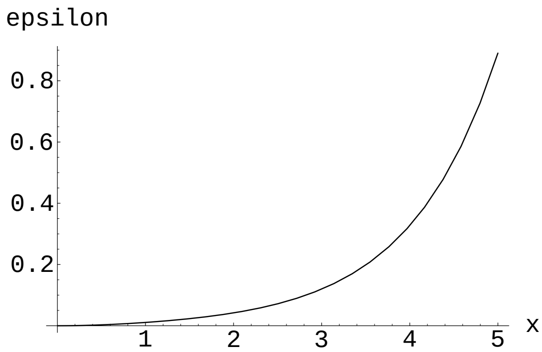

Figure 3. The variation of the slow-roll parameter ε with x.

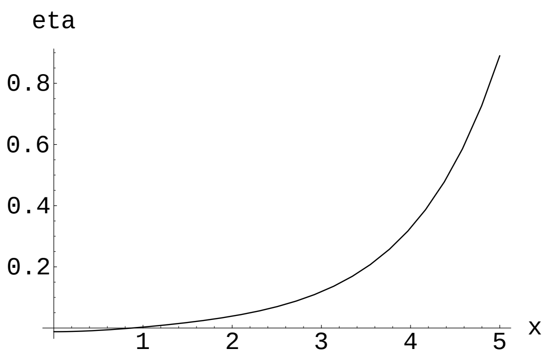

Figure 4. The variation of the slow-roll parameter η with x.

$$V_0 = 31.004\alpha^2 \left(\frac{M_5^6}{M_4^2}\right) \tag{33}$$

which can be determined if α is known. Using COBE normalization, $A_{\mathrm{S}}^2 = 4\times 10^{-10}$,

Table 1. Variation of $x_{\rm f}$, β and $n_{\rm s}$ with N for $x_{\rm i} = 0.9$.

$x_i = 0.9$			
N	$x_{\rm f}$	β	$n_{\rm s}$
10	3.24	0.079	0.657
20	3.89	0.041	0.822
30	4.28	0.028	0.879
40	4.55	0.021	0.909
50	4.77	0.017	0.926
60	4.95	0.014	0.939
70	5.10	0.012	0.947

one determine

$$\alpha = 2.22 \times 10^{-3} \left(\frac{M_4}{M_5}\right) \tag{34}$$

and the corresponding $\phi_{\rm COBE}$ and V_0 become

$$\phi_{\rm COBE} = 80.867 M_5 \tag{35}$$

$$V_0 = 1.53 \times 10^{-4} M_5^4. \tag{36}$$

Since $M_4 > M_5$, we get a limiting value of α from eq. (34) which is

$$\alpha > 2.22 \times 10^{-3}. \tag{37}$$

It is possible to determine the spectral index of the scalar spectrum from eq. (12) using the values of slow-roll parameters for a given number of e-folding. We now take $N = 70$ and obtain

$$n_{\rm s} = 0.947 \tag{38}$$

which lies within the bounds resulting from the latest CMB data [21]:

$$0.8 < n_{\rm s} < 1.05. \tag{39}$$

Repeating the above calculations with $x_{\rm i} = 0.9$, but with different number of e-foldings, N, one can determine the spectral index which are presented in table 1. From table 1 we see that as N increases, β decreases but $x_{\rm f} = x_{\rm end}$ and $n_{\rm s}$ increases. The observation results from CMB puts a limit on N. We get $N > 20$ and $\beta < 0.04$.

Table 2 shows the variation of $x_{\rm f}$, β and $n_{\rm s}$ with $x_{\rm i}$ for $N = 70$. We see that as $x_{\rm i}$ increases from $x_{\rm i} = 0.9$, $x_{\rm f}$ increases but β and $n_{\rm s}$ decrease. It is also found that the spectral index $n_{\rm s}$ is not altered significantly with $x_{\rm i}$.

Repeating the same calculations for other N, we obtain table 3. It is found that for higher N, the variation of $n_{\rm s}$ decreases with $x_{\rm i}$ and the allowed values of spectral index, $n_{\rm s}$, is weakly dependent on N, for $N \geq 50$.

Table 2. Variation of x_{f}, β and n_{s} with x_{i} for $x_{\mathrm{i}} \geq 0.9$ and $N = 70$.

$N = 70$			
x_{i}	x_{f}	β	n_{s}
0.9	5.10	0.012	0.947
1.1	5.32	0.010	0.946
1.3	5.54	0.008	0.946
1.5	5.75	0.006	0.945
1.7	5.95	0.005	0.945
1.9	6.16	0.004	0.944
2.1	6.36	0.003	0.944
2.3	6.56	0.003	0.944
2.5	6.76	0.002	0.944

Table 3. Variation of n_{s} with N for $5.1 \geq x_{\mathrm{i}} \geq 0.9$.

N	10	20	30	40	50	60	70
n_{s}	0.64–0.66	0.81–0.82	0.87–0.88	0.90–0.91	0.92–0.93	0.93–0.94	0.94–0.95

The contribution of gravitational wave relative to density perturbation can now be obtained using the values of the parameters determined above, which is

$$\frac{A_{\mathrm{T}}^2}{A_{\mathrm{S}}^2} \approx 6.6 \times \left(\frac{\lambda}{V}\right)^2. \tag{40}$$

The r-parameter is given by

$$r \approx 4\pi \frac{A_{\mathrm{T}}^2}{A_{\mathrm{S}}^2}. \tag{41}$$

It is evident that (i) for $\lambda/V \sim 1/10$, $r \approx 0.8$, (ii) for $\lambda/V \sim 1/20$, $r \approx 0.2$. The measurement of r-parameter is significant, which however may be verified by the future Planck satellite for the suitability of the model.

4. Conclusions

In this paper we study inflationary dynamics of a scalar field in an inverse coshyperbolic potential in braneworld scenario. We note that sufficient inflation may be obtained in the Randall–Sundrum (Type II) braneworld model at high energy limit with inverse coshyperbolic potential for a restricted domain of the initial values of the scalar field. We obtain cosmological solution in the framework of braneworld in the high energy limit. We determine the allowed range of values of spectral index of the scalar perturbation spectrum and tabulated its variation with the variation of the number of e-foldings. The inflationary parameters are calculated which are in good agreement with the current observational results. We show that the measurement of r-parameter will put an effective limit on the high energy scale in the braneworld. However, it is known [22] that the stabilized RS models have viable cosmology below TeV scale, with interesting new perspective on the mass hierarchy and the cosmological constant. A detailed study on reheating is an important issue to be discussed elsewhere.

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