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Bianchi Type V magnetized string dust cosmological models with Petrov-type degenerate

RAJ BALI* and UMESH K PAREEK
Department of Mathematics, University of Rajasthan, Jaipur 302 004, India
*Corresponding author. E-mail: balirs@yahoo.co.in

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Abstract. Bianchi Type V massive string cosmological models with free gravitational field of Petrov Type degenerate in the presence of magnetic field with variable magnetic permeability are investigated. The magnetic field is due to an electric current produced along the x-axis. The F_{23} is the only non-vanishing component of electromagnetic field tensor F_{ij}. Maxwell's equations $F_{[ij;k]} = 0$ and $F^{ij}_{;j} = 0$ are satisfied by F_{23} = constant. The behaviour of the model in the presence and absence of magnetic field and other physical aspects are also discussed.

Keywords. Bianchi V; magnetized; string dust; Petrov D.

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1. Introduction

The anisotropic and homogeneous cosmological models contribute significantly to the description of the Universe such as formation of galaxies during its early stages of evolution. Even though the present magnetic energy is very small in comparison with the estimated matter density, it might not have been negligible during the early stages of the Universe. It is therefore of considerable interest to construct cosmological models with magnetic field to represent the early Universe. The breakdown of isotropy is also due to the magnetic field. A detailed discussion of the primordial magnetic field in the case of Bianchi Type I space-time has been given by Thorne [1]. Jacobs [2,3] investigated Bianchi Type I cosmological model satisfying barotropic equation of state in the presence of magnetic field. Collins [4] gave a qualitative analysis of Bianchi Type I models in the presence of magnetic field. Roy and Prakash [5] have investigated a plane symmetric cosmological model with an incident magnetic field for perfect fluid distribution. Homogeneous cosmological models representing matter and electromagnetic field have been discussed by Vaijk and Eltgroth [6], Damiao Soares and Assad [7], Dunn and Tupper [8], Lorentz

[9–11]. Roy and Singh [12] have investigated LRS Bianchi Type V cosmological models filled with matter and radiation. Bali [13] has investigated a magnetized perfect fluid cosmological model in which expansion (θ) is proportional to σ_1^1, the eigenvalue of shear tensor σ_i^j. The large-scale intergalactic magnetic field is speculated by Asseo and Sol [14]. Roy and Banerjee [15] have investigated Bianchi Type II cosmological model of Petrov Type D representing an imperfect fluid with a source-free magnetic field. It is believed that cosmic strings give rise to density perturbation which leads to the formation of galaxies [16]. These strings possess stress energy and are coupled to gravitational field. The gravitational effects of such strings are investigated by Vilenkin [17]. Letelier [18,19] and Stachel [20] developed the relativistic treatment of the strings. Melvin [21] pointed out that during the evolution of the Universe, the matter was in highly ionized state and is smoothly coupled with the field and forms neutral matter as a result of the expansion of Universe. Therefore, the presence of magnetic field in a string dust Universe is not unrealistic. Banerjee *et al* [22] investigated an axially symmetric Bianchi Type I string dust cosmological model in the presence of magnetic field. The string cosmological models with magnetic field are also investigated by Chakraborty [23], Tikekar and Patel [24,25], Patel and Maharaj [26], Bali and Anjali [27].

In this paper, we have investigated some Bianchi Type V massive string cosmological models with free gravitational field of Petrov Type degenerate in the presence of magnetic field with variable magnetic permeability. The behaviour of the models in the presence and absence of magnetic field and singularities in the models are discussed. The physical aspects of the models are also discussed.

2. The metric and field equations and solutions

We consider the Bianchi Type V metric in the form given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2\mathrm{d}x^2 + B^2\mathrm{e}^{2x}\mathrm{d}y^2 + C^2\mathrm{e}^{2x}\mathrm{d}z^2, \tag{1}$$

where A, B, C are functions of t alone.

Einstein's field equation is given by

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi T_i^j, \tag{2}$$

where

$$T_i^j = \varepsilon v_i v^j - \lambda x_i x^j + E_i^j \tag{3}$$

with

$$v_i v^i = -x_i x^i = -1 \tag{4}$$

$$v^i x_i = 0 \tag{5}$$

$$x_1 \neq 0, \quad x_2 = 0, \quad x_3 = 0, \quad x_4 = 0, \tag{6}$$

where ε is the energy density, v^i the velocity flow vector, λ the string tension density, x_i the direction of string, E_i^j the electro-magnetic field given by Lichnerowicz [28] as

$$E_i^j = \bar{\mu}\left[|h|^2\left(v_i v^j + \frac{1}{2}g_i^j\right) - h_i h^j\right], \tag{7}$$

where h_i is the magnetic flux vector given by

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}}\varepsilon_{ijk\ell}F^{k\ell}v^j. \tag{8}$$

Here $\bar{\mu}$ is the magnetic permeability and $\varepsilon_{ijk\ell}$ the Levi–Civita tensor. We assume that current is flowing along x-axis. Thus F_{23} is the only non-vanishing component of F_{ij}. Maxwell's equations

$$F[ij;k] = 0 \tag{9}$$

and

$$F^{ij}_{;j} = 0 \tag{10}$$

are satisfied by

$$F_{23} = \text{constant} \; = H(\text{say}). \tag{11}$$

Thus

$$h_1 \neq 0, \quad h_2 = 0 = h_3 = h_4. \tag{12}$$

Equation (8) leads to

$$h_1 = \frac{AH}{\bar{\mu}BC\mathrm{e}^{2x}}. \tag{13}$$

$F_{14} = 0 = F_{24} = F_{34}$ due to assumption of infinite electrical conductivity (Maartens [29]). We assume that magnetic permeability ($\bar{\mu}$) is a variable and consider $\bar{\mu} = \mathrm{e}^{-4x}$, i.e. when $x \to \infty$, then $\bar{\mu} \to 0$. Thus eqs (7) and (13) lead to

$$E_1^1 = -\frac{H^2}{2B^2C^2}, \tag{14}$$

$$E_2^2 = \frac{H^2}{2B^2C^2}, \tag{15}$$

$$E_3^3 = \frac{H^2}{2B^2C^2}, \tag{16}$$

$$E_4^4 = -\frac{H^2}{2B^2C^2}. \tag{17}$$

We also assume coordinates to be co-moving so that

$$v^1 = 0 = v^2 = v^3, \quad v^4 = 1.$$

The Einstein field equation (2) for the line element (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{1}{A^2} = 8\pi \left(\frac{H^2}{2B^2 C^2} + \lambda \right), \tag{18}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = -\frac{8\pi H^2}{2B^2 C^2}, \tag{19}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} = -\frac{8\pi H^2}{2B^2 C^2}, \tag{20}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} - \frac{3}{A^2} = 8\pi \left(\varepsilon + \frac{H^2}{2B^2 C^2} \right), \tag{21}$$

$$\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} = 0. \tag{22}$$

Equation (22) leads to

$$A = L\sqrt{BC}, \tag{23}$$

where L is the constant of integration.

The conformal curvature tensor C_{hijk} and its physical component $C_{(abcd)}$ are related by $C_{(abcd)} = C_{hijk}\lambda^h_{(a)}\lambda^i_{(b)}\lambda^j_{(c)}\lambda^k_{(d)}$ where $\lambda^i_{(a)} (a = 1, 2, 3, 4)$ is the set of four mutually orthogonal unit vectors. The non-vanishing physical components $C_{(abcd)}$ of conformal curvature tensor C_{hijk} are given by

$$\begin{aligned} C_{(2323)} &= -C_{(1414)} \\ &= \frac{1}{6}\left[\frac{2A_{44}}{A} - \frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{A_4 B_4}{AB} + \frac{2B_4 C_4}{BC} - \frac{A_4 C_4}{AC} \right], \end{aligned} \tag{24}$$

$$\begin{aligned} C_{(1313)} &= -C_{(2424)} \\ &= \frac{1}{6}\left[-\frac{A_{44}}{A} + \frac{2B_{44}}{B} - \frac{C_{44}}{C} - \frac{A_4 B_4}{AB} - \frac{B_4 C_4}{BC} + \frac{2A_4 C_4}{AC} \right], \end{aligned} \tag{25}$$

$$\begin{aligned} C_{(1212)} &= -C_{(3434)} \\ &= \frac{1}{6}\left[-\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{2C_{44}}{C} + \frac{2A_4 B_4}{AB} - \frac{B_4 C_4}{BC} - \frac{A_4 C_4}{AC} \right], \end{aligned} \tag{26}$$

$$C_{(1224)} = -C_{(1334)} = \frac{1}{2A}\left[\frac{B_4}{B} - \frac{C_4}{C}\right]. \tag{27}$$

To get the deterministic model of the Universe, we assume that the free gravitational field is of Petrov Type I degenerate. Thus Petrov Type ID condition leads to

$$C_{(1212)} = C_{(1313)}. \tag{28}$$

For Bianchi Type V metric (1), the above condition leads to

$$\frac{1}{6}\left[-\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{2C_{44}}{C} + \frac{2A_4B_4}{AB} - \frac{B_4C_4}{BC} - \frac{A_4C_4}{AC}\right]$$
$$= \frac{1}{6}\left[-\frac{A_{44}}{A} + \frac{2B_{44}}{B} - \frac{C_{44}}{C} - \frac{A_4B_4}{AB} - \frac{B_4C_4}{BC} + \frac{2A_4C_4}{AC}\right].$$

Thus, we have

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{A_4}{A}\left[\frac{B_4}{B} - \frac{C_4}{C}\right] = 0. \tag{29}$$

Equation (29) after using (22) leads to

$$\frac{(CB_4 - BC_4)_4}{(CB_4 - BC_4)} = \frac{A_4}{A} = \frac{1}{2}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) \tag{30}$$

which leads to

$$\frac{\nu_4}{\nu} = \frac{KL}{\sqrt{\mu}}, \tag{31}$$

where $BC = \mu, B/C = \nu$, K being the constant of integration.

Equations (19) and (20) lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} - \frac{B_4}{B}\frac{C_4}{C} - \frac{1}{LBC} = -\frac{K}{2B^2C^2}, \tag{32}$$

where

$$K = 4\pi H^2. \tag{33}$$

Equation (32) after using $BC = \mu, B/C = \nu$ leads to

$$\frac{\mu_{44}}{\mu} - \frac{1}{4}\frac{\mu_4^2}{\mu^2} + \frac{1}{4}\frac{\nu_4^2}{\nu^2} = \frac{1}{L\mu} - \frac{K}{2\mu^2}. \tag{34}$$

Using (31) in eq. (34), we have

$$\frac{\mu_{44}}{\mu} - \frac{1}{4}\frac{\mu_4^2}{\mu^2} + \frac{1}{4}\frac{K^2L^2}{\mu} = \frac{1}{L\mu} - \frac{K}{2\mu^2}$$

which leads to

$$2\mu_{44} - \frac{1}{2\mu}\mu_4^2 = \gamma - \frac{K}{\mu}, \tag{35}$$

where

$$\gamma = \left[\frac{2}{L} - \frac{K^2L^2}{2}\right]. \tag{36}$$

Let $\mu_4 = f(\mu)$. Thus $\mu_{44} = ff'$, where $f' = \mathrm{d}f/\mathrm{d}\mu$.

From eq. (35), we have

$$2ff' - \frac{1}{2}f^2/\mu = \gamma - K/\mu$$

which implies that

$$\frac{\mathrm{d}}{\mathrm{d}\mu}(f^2) - \frac{1}{2\mu}(f^2) = \gamma - \frac{K}{\mu}. \tag{37}$$

Equation (37) leads to

$$f^2 = \left(\frac{\mathrm{d}\mu}{\mathrm{d}t}\right)^2 = 2\gamma\mu + N\sqrt{\mu} + 2K, \tag{38}$$

where N is the constant of integration.

Equation (38) leads to

$$\sqrt{\left(\sqrt{\mu} + \frac{N}{4\gamma}\right)^2 + \beta^2} - \frac{N}{2\gamma}\sinh^{-1}\left[\frac{\sqrt{\mu} + \frac{N}{4\gamma}}{\beta}\right] = at + b \tag{39}$$

which determines the value of μ and ν is determined by (31) as

$$\frac{\mathrm{d}\nu}{\nu} = \frac{KL}{\sqrt{\mu}}\frac{\mathrm{d}t}{\mathrm{d}\mu}\mathrm{d}\mu = \frac{KL}{\sqrt{\mu}}\frac{\mathrm{d}\mu}{\sqrt{2\gamma\mu + N\sqrt{\mu} + 2K}} \tag{40}$$

which leads to

$$\nu = M\exp\left[\frac{\sqrt{2}KL}{\sqrt{\gamma}}\sinh^{-1}\frac{\left(\sqrt{\mu} + \frac{N}{4\gamma}\right)}{\ell}\right], \tag{41}$$

where $\ell = \sqrt{\frac{K}{\gamma} - \frac{N^2}{16\gamma^2}}$ and M is the constant of integration.

Thus the metric (1) reduces to the form

$$\mathrm{d}s^2 = -\frac{\mathrm{d}T^2}{2\gamma T + N\sqrt{T} + 2K} + LT\mathrm{d}X^2 + T\nu\mathrm{e}^{2x}\mathrm{d}Y^2 + T\nu^{-1}\mathrm{e}^{2x}\mathrm{d}Z^2, \tag{42}$$

where

$$\mu = T, \quad x = X, \quad y = Y, \quad z = Z.$$

ν is determined by eq. (40) as

$$\nu = M \exp\left[\frac{\sqrt{2}KL}{\sqrt{\gamma}} \sinh^{-1}\left[\frac{\left(T + \frac{N}{4\gamma}\right)}{\ell}\right]\right]. \tag{43}$$

In the absence of magnetic field, the metric (41) reduces to the form

$$\mathrm{d}s^2 = -\frac{\mathrm{d}T^2}{N\sqrt{T} + 2\gamma T} + LT\mathrm{d}X^2 + T\nu \mathrm{e}^{2x}\mathrm{d}Y^2 + \mathrm{e}^{2x}T\nu^{-1}\mathrm{d}Z^2. \tag{44}$$

3. Discussion

The energy density (ε), string tension density (λ) and the particle density (ε_{p}) for the model (42) are given by

$$8\pi\varepsilon = \frac{3}{4}\left[\frac{2\gamma T + N\sqrt{T} + 2K}{T^2}\right] - \frac{3}{L^2 T} - \frac{K}{T^2} - \frac{1}{4}\frac{K^2L^2}{T}, \tag{45}$$

$$8\pi\lambda = \frac{\gamma}{2T} - \frac{3K}{T^2} + \frac{1}{4}\frac{K^2L^2}{T} - \frac{1}{L^2T}. \tag{46}$$

Now $8\pi\varepsilon_{\mathrm{p}} = 8\pi(\varepsilon - \lambda)$.

$$8\pi\varepsilon_{\mathrm{p}} = \frac{\gamma}{T} + \frac{3N}{4T^{3/2}} + \frac{2K}{T^2} - \frac{2}{L^2T} - \frac{1}{2}\frac{K^2L^2}{T}. \tag{47}$$

The expansion (θ) and the shear (σ) are given by

$$\theta = \frac{3}{2}\frac{\sqrt{2\gamma T + N\sqrt{T} + 2K}}{T} \tag{48}$$

and

$$\sigma^2 = \frac{1}{2}[(\sigma_1^1)^2 + (\sigma_2^2)^2 + (\sigma_3^3)^2 + (\sigma_4^4)^2] = \frac{K^2L^2}{8T} + \frac{K^2L^2}{8T}.$$

Thus

$$\sigma = \frac{KL}{2\sqrt{T}}. \tag{49}$$

In particular, if we choose $N = 0$, then from eq. (38), we have $\frac{\mathrm{d}\mu}{\sqrt{2\gamma\mu + K}} = \mathrm{d}t$ which leads to

$$\mu = (at+b)^2 - \frac{K}{2\gamma}. \tag{50}$$

From eq. (31), we have

$$\frac{\mathrm{d}\nu}{\nu} = \frac{KL}{\sqrt{(at+b)^2 - \frac{K}{2\gamma}}}\mathrm{d}t$$

which leads to

$$\nu = S\exp\left[\frac{KL}{a}\cosh^{-1}\left\{\sqrt{\frac{2\gamma}{K}}(at+b)\right\}\right], \tag{51}$$

where $a = \sqrt{\gamma/2}$ and S is the constant of integration.

Hence, the metric (1) reduces to the form

$$\begin{aligned}\mathrm{d}s^2 = {} & -\frac{\mathrm{d}T^2}{a^2} + \left(T^2 - \frac{K}{2\gamma}\right)\mathrm{d}X^2 + \left(T^2 - \frac{K}{2\gamma}\right)S \\ & \times\exp\left[\frac{KL}{a}\cosh^{-1}\left[\sqrt{\frac{2\gamma}{K}}T\right]\mathrm{e}^{2x}\mathrm{d}Y^2\right] \\ & + \frac{\left[T^2 - \frac{K}{2\gamma}\right]\mathrm{e}^{2x}\mathrm{d}Z^2}{S\exp\left[\frac{KL}{a}\cosh^{-1}\sqrt{\frac{2\gamma}{K}}T\right]},\end{aligned} \tag{52}$$

where $x = X, y = Y, z = Z$, $at + b = T$.

In the absence of magnetic field, i.e. when $K = 0$ then the metric (52) reduces to

$$\mathrm{d}s^2 = -\frac{\mathrm{d}T^2}{a^2} + T^2\mathrm{d}X^2 + ST^2\mathrm{e}^{2X}\mathrm{d}Y^2 + \frac{T^2}{S}\mathrm{e}^{2X}\mathrm{d}Z^2. \tag{53}$$

The energy density (ε), string tension density (λ), the particle density (ε_{p}) for the model (52) are given by

$$8\pi\varepsilon = \frac{3a^2T^2}{\left[T^2 - \frac{K}{2\gamma}\right]^2} - \frac{3}{L^2\left[T^2 - \frac{K}{2\gamma}\right]} - \frac{K}{\left[T^2 - \frac{K}{2\gamma}\right]} - \frac{K^2L^2}{4\left[T^2 - \frac{K}{2\gamma}\right]} \tag{54}$$

$$\begin{aligned}8\pi\lambda = {} & \frac{2a^2}{\left[T^2 - \frac{K}{2\gamma}\right]} - \frac{a^2T^2}{\left[T^2 - \frac{K}{2\gamma}\right]^2} + \frac{1}{4}\frac{K^2L^2}{\left[T^2 - \frac{K}{2\gamma}\right]} - \frac{1}{L^2\left[T^2 - \frac{K}{2\gamma}\right]} \\ & - \frac{K}{\left[T^2 - \frac{K}{2\gamma}\right]^2},\end{aligned} \tag{55}$$

$$8\pi\varepsilon_{\rm p} = 8\pi\varepsilon - 8\pi\lambda = \frac{4a^2T^2}{\left[T^2 - \frac{K}{2\gamma}\right]^2} - \frac{2a^2}{\left[T^2 - \frac{K}{2\gamma}\right]} - \frac{2}{L^2\left[T^2 - \frac{K}{2\gamma}\right]} - \frac{1}{2}\frac{K^2L^2}{\left[T^2 - \frac{K}{2\gamma}\right]}. \tag{56}$$

The expansion (θ) and shear (σ) for the model (52) are given by

$$\theta = \frac{3aT}{\left[T^2 - \frac{K}{2\gamma}\right]}, \tag{57}$$

$$\sigma = \frac{KL}{2\sqrt{T^2 - \frac{K}{2\gamma}}}. \tag{58}$$

For the model (52), the energy density $\varepsilon \to \infty$ when $T \to 0$ and $\varepsilon \to 0$ when $T \to \infty$. The energy condition $\varepsilon \geq 0$ leads to

$$0 < T \leq \sqrt{\frac{\frac{3K}{2L^2\gamma} + \frac{K^2}{2\gamma} + \frac{K^3L^2}{8\gamma}}{\frac{K^2L^2}{4} + K + \frac{3}{L^2} - 3a^2}}. \tag{59}$$

In the presence of magnetic field, the model (52) has singular origin at $T = \sqrt{\frac{K}{2\gamma}}$ [30] and the rate of expansion slows down and drops to zero as $T \to \infty$. The energy density ε becomes negligible for large values of T. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} = \frac{KL}{6a} \neq 0$, the model does not isotropize for large values of T in the presence of magnetic field. However, for small K, the model is quasi-isotropic, i.e. $(\sigma/\theta) \sim 0$.

For the model (53), in the absence of magnetic field, the energy condition $\varepsilon \geq 0$ leads to $L^2a^2 \geq 1$. The energy density (ε), the string tension (λ) and the particle density ($\varepsilon_{\rm p}$) tend to zero when $T \to \infty$. The model (53) has singular origin at $T = 0$ and the rate of expansion slows down and drops to zero as $T \to \infty$. The energy density ε, string tension density λ and the particle density $\varepsilon_{\rm p}$ become negligible for large values of T. Since $\varepsilon, \varepsilon_{\rm p}, \lambda, \theta$ tend to infinity and spatial volume (R^3) tends to zero at $T = 0$, the model (53) in the absence of magnetic field has line singularity [22]. In the absence of magnetic field, the model (53) represents an isotropic Universe.

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