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Complementarity controversy in wave–particle duality revisited

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Abstract. On the face of some recent experiments claiming the simultaneous presence of both 'sharp interference' and 'highly reliable *which way* information' and some others casting light on the origin of complementarity in quantum interferometric experiments, the whole issue is reviewed on the basis of our earlier precise formulation of Bohr's complementarity principle. It is pointed out that contradicting the principle (in this specific formulation) is impossible without contradicting quantum mechanics and a lack of general consensus regarding the origin of the mutual exclusiveness is at the root of the controversy and confusions.

Keywords. Bohr's complementarity principle; quantum mechanics; mutual exclusiveness; quantum entanglement; uncertainty principle.

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1. Introduction

Interests and bouts of controversy regarding Bohr's complementary principle (BCP) or more precisely concerning the validity of the so-called wave–particle complementarity appear to breakout intermittently mainly due to its imprecise original formulation. Bohr [1] formulated the complementarity principle to eliminate the paradoxical consequences of wave–particle dualism without addressing the basic principle of quantum mechanics (QM) which enforces the mutual exclusiveness (ME). The discomfiture with this pleonastic formulation is explicitly documented in Einstein's complaint: "despite much effort which I have expended on it, I have been unable to achieve the sharp formulation of Bohr's principle of complementarity" [2].

In two earlier papers [3,4], a definitive formulation of BCP consistent with the principles of QM was developed to analyse a number of experiments [5–8] (both suggested and actually performed) which appeared to confront BCP. It was also pointed out that Bohr's intuitive formulation of the notion of complementarity may be placed into two distinct classes on the basis of the origin of ME and violation of BCP without violating the principles of QM is not possible.

2. Precise formulation of BCP

A pair of variables obtained from a Fourier transform of the quantum mechanical state function and a pair of dynamical variables for which the corresponding operators (quantum mechanical) do not commute are the complementary variables (Class I) in QM. In addition to the canonically conjugate dynamical variables, these include radiation field vectors $\vec{E}$ and $\vec{H}$ satisfying field noncommutation relation. Claim for more esoteric complementary variables may also be found in the existing literature [3] from experiments on elementary particles. This complementarity is referred to as 'kinematic–dynamic' by Murdoch [9]. The assertion by Scully *et al* [10] that "For each degree of freedom the dynamical variables are a pair of complementary observables" is quite alright but the characterization of complementarity even for the dynamical variables (Class I) remains incomplete. Pair of operators corresponding to the components of angular momentum such as J_x and J_y or J_x and J_z etc. do not commute and are complementary observables. Moreover, even though time is not an observable, it forms a complementary pair with energy of a quantum mechanical system. The complementarity and time–energy uncertainty relation in this case may be traced [11] from the noncommutation of an observable with the Hamiltonian of the system and the (quantum mechanical) equation of motion for the observable. The existence of complementary domains for the description of physical processes was elegantly demonstrated in the interference experiments performed by Zou *et al* [12] and Rauch [13]. These experiments show that if the interference is lost due to lack of coherence (spatial or temporal) in ordinary space-time, it may be revived in the complementary domain of momentum or frequency! Agarwal [14] has discussed the theoretical formulation encompassing interference in complementary domains with reference to some relevant experimental works.

On the other hand, in all interference experiments, interference arising from the superposition of two (or more) states and the property associated with the *which path* (*which state* in general) information form a pair of complementary properties (Class II – the so-called 'wave–particle complementarity'). The entanglement of the interfering wave function branches with orthogonal detector states (necessary for the *which state* detection) is responsible for the loss of interference pattern and makes the pair of complementary properties mutually exclusive. The entangled wave function gets irreversibly collapsed to either of the two wave function branches when the information is read out from the detector. This may be regarded as the physical content of the quantum mechanical collapse hypothesis. The term entanglement ('verschränkung' in German) was coined by Schrödinger to describe the correlation which develops between two or more interacting systems according to the quantum equation of motion. The 'entangled' combined system form a single quantum state which cannot be expressed as a product of the state functions of the individual single systems. In his two-part article [15], Schrödinger examined and extended the famous arguments of Einstein, Podolsky and Rosen (EPR) [16] where a weird revelation of quantum entanglement first appeared. According to Schrödinger: "When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (the quantum states) have become entangled."

In classical physics, energy is transported either by particles or by waves – the two ultimate categories of physical entities having entirely different mathematical description and there is no mix-up. But in quantum mechanics the basic difference in mathematical representation of the two distinct classical entities disappears and we are in for the typical wave–particle duality. With an extremely weak beam of incoming single electrons in a double-slit type arrangement, the brick by brick build up of the interference pattern was vividly demonstrated by Tonomura *et al* [17]. In this classic experiment the electrons were detected one by one on the interference screen (or array of detectors) as localized particles leading to the formation of an interference pattern after a large number of accumulation. The authors' claim of an unambiguous demonstration of wave–particle duality of electrons is, therefore, thoroughly justified. Similar claim was also made by Grangier *et al* [18] in their experiments with ideal single photon states of radiation field which entail particle-like detection (on the interference screen) as the probability of joint detection of more than one photon is exactly zero. However, it is the incompatibility between the appearance of a sharp interference pattern and the precise *which path* knowledge in quantum interferometry that was highlighted by Bohr in his complementarity principle.

A correlation between the two types of complementarity was debated at great length [19–21] with diverse interpretations of Bohr's not-so-very clear standpoint. In view of the distinctly different basis of the origin of ME, we concur with Murdoch [9] and Faye and Folse [22] that Class I and Class II complementarities are independent notions.

It is very important to note that QM does not guarantee the claim of complementarity between arbitrarily chosen 'wave' and 'particle' properties and in the first place, liberal use of the term 'wave–particle complementarity' in literature and textbooks by several authors is at the root of much controversies [3,4]. It may be noted that, *Bohr has always remained confined in his discussions of complementarity aspect of wave–particle duality to interference experiments*. Perhaps this is due to Bohr's intuitive recognition that quantum mechanical formalism imply only this type of complementarity. Moreover, the quantum interference effect is much more general than the classical wave-like interference in ordinary position space. When the interference is among wave function branches other than those in position space, the *which path* information (in position space) does not require entanglement with the interfering parts of the wave function and the interference pattern remains unaffected. Ray and Home [23] have analysed an experiment using nuclear heavy-ion reaction where interference in angular correlations between emitted gamma pulses and the *which path* information coexist because the detection of *which path* in 'configuration space' does not involve entanglement with the interfering nuclear angular momentum eigenfunctions. Therefore, with the general quantum mechanical interference, the *which path* question (in position space) cannot be meaningful in every case. In the precise formulation [3,4] we have proposed to replace the term 'wave–particle complementarity' with *which state*-interference complementarity. Holladay

[24] has subsequently termed this as 'which value-interference' complementarity and proposed 'a restricted version of particle–wave complementarity' without clarifying the origin of ME.

The complementarity principle always implies sharp measurement. Experiments [25,26] which furnish partial interference with partial knowledge of the *which path* do not invalidate BCP. The 'trade-off' is consistent with the quantum formalism and expressed in terms of the well-known Englert–Greenberger duality relation [27–29]. An inequality, derived involving suitably defined visibility V of the interference fringes and distinguishability D between the two interfering wave function branches (also referred to as wave–particle duality relation), is given by

$$D^2 + V^2 \leq 1, \tag{1}$$

– the equality holds for pure quantum states.

Some well-referred biprism experiments, proposed [5,8] (one of which was actually performed [6]) to confront the 'orthodox tenet' of 'wave–particle complementarity', are also analysed by Kar *et al* [30] following Busch and Lahti [31]. It is shown that in all these experiments the observables corresponding to the so-called 'wave' and 'particle' properties, when described in the appropriate Hilbert space, are commuting and hence their arrangements are noncomplementary. It is also possible to have higher-order interference, i.e. spatial modulation of intensity correlations that are higher (fourth-order, say) than the ordinary second-order in field amplitudes. Quantum mechanical predictions are fully vindicated by the experimental observations [32], showing nonclassical effects and consistency with the complementarity principle. BCP is also shown to be consistent with the novel idea of 'quantum eraser', invented by Scully and Drühl [33]. This important concept has subsequently been realized in practice and found useful applications in quantum computing and quantum cryptography.

3. Confronting the complementarity principle – Some recent claims

Recently, Afshar *et al* [34] claimed to have obtained violation of BCP in their double-slit type welcher weg experiment. Reitzner [35] has performed quantum mechanical numerical simulations of the experimental arrangement and confirmed that Afshar *et al*'s experimental results are in accordance with the prediction of standard quantum mechanics. It implies that, if we accept the definitive formulation of BCP recounted in §2, there cannot be any contradiction. When they (Afshar *et al*) prepare the photon in a superposed state at the plane of the two pinholes, its detection 'further downstream through the known imaging capabilities of the lens system' at the final screen position (in the overlap region) in their experiment can never really give a *which way* information. It cannot tell us which pinhole the photon actually went through and the authors have in fact inferred the 'path' of the interfering photons in their experiment without any detection measurement. Qureshi [36] has elaborated this point by analysing an equivalent gedanken experiment with a spin system. Taking 'the reverse approach' he argues that the existence of interference (in the overlap region) implies loss of the *which way* or *which initial-state* information and 'complementarity is robust' in any variant of

Young's interference experiment. The claim of Afshar *et al* for the violation of Englert–Greenberger duality relation is also contested by Steuernagel [37] from a quantitative analysis in a similar set-up.

In another proposal using some modifications to earlier Aharonov–Bohm quantum interferometer set-up, Bandyopadhyay [38] comprehended a rigorous test of BCP. However, we insist that contradicting *which state*-interference complementarity in any interferometric experiment amounts to contradicting a specific quantum mechanical principle which is responsible for the complementarity. The issue will be clarified further with the following discussions.

4. Controversy regarding the origin of ME in *which state*-interference complementarity

We now argue how the controversy is rooted to a lack of general consensus regarding the origin of ME in *which state*-interference complementarity. According to the precise formulation of BCP [3,4] we note that, for the Class I type of complementarity, ME follows from the noncommutation of operators representing dynamical variables of a system. The quantum mechanical uncertainty relation also follows from noncommutation and is defined in terms of variances σ_A and σ_B of two noncommuting observables $\hat{A}$ and $\hat{B}$ (called as incompatible observables) invoking the Cauchy–Schwarz inequality for the inner product of two vectors:

$$\sigma_A \sigma_B \geq \frac{1}{2i}\langle[\hat{A},\hat{B}]\rangle, \tag{2}$$

where

$$\sigma_A^2 = \langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2 \text{ etc.}$$

It should be noted here that the nonstatistical notion of Heisenberg's uncertainty principle (HUP) involving simultaneous measurement of canonically conjugate dynamical variables pertaining to a single particle (the essence of Heisenberg's famous gedanken experiment) cannot be accommodated in the standard interpretation of QM [39]. However, a tentative indeterminacy interpretation of QM is presented in the formal scheme for approximate joint measurement of noncommuting observables proposed by Busch *et al* [40] in terms of positive operator valued measures (POVMs). In this formulation, the state of a quantum particle is defined by unsharply localized position x and momentum p (the centres of the respective wavepackets) with corresponding variances representing the 'measurement imprecisions' Δx and Δp. But, the confusion regarding a proper understanding of the mechanism of ME in Class II type of complementarity is much more serious. In the archetypal gedanken experiments of Einstein's recoiling slit [41] or Feynman's light microscope [42], the disappearance of the interference pattern due to welcher weg detection is explained using Heisenberg's position–momentum uncertainty relation. The Bohr–Feynman explanations [41,42] which conveniently use a mixture of classical and quantum languages, are indeed semiclassical.

In QM, interference is the existence of bright and dark fringes described by the density distribution according to the superposed state function. The possibility of assigning suitable pair of operators corresponding to the complementary properties (Class II) has been explored in [4,43] and found to be state-dependent. It was pointed out that the commutation relation of such suggested pair of state-dependent operators does not lead to any uncertainty type relation [4]. In order to provide a connection between the Englert–Greenberger duality relation and quantum mechanical position–momentum uncertainty relation Marzlin *et al* [44] observed that "Interference experiments do not directly reveal information about the momentum but rather about the modular momentum" (introduced by Aharonov and Rohrlich [45]). They have further noted that "for non-overlapping atomic beams, the uncertainty relation between position and modular momentum can be derived using the duality relation. Whereas both inequalities are related, they are not equivalent ... The complementarity inequality appears stronger than the Heisenberg's position–momentum uncertainty relation." In this context it may be recalled that the quantum mechanical formalism represent a more stringent condition than the uncertainty inequality. The quantum mechanical wave function cannot have a compact support in both position and momentum representation due to the in-built Fourier correlation between the coordinate and momentum representations [46,47].

The quantum beat experiments of Hellmuth *et al* [48] illustrate that beats were absent whenever there exist the information about the decay mode and no uncertainty argument is required to account for the absence of interference. Also, from an analysis of their experimental results, Wang and co-workers [49] have pointed out that when the two possible photon paths are distinguishable, the interference pattern disappears – uncertainty argument is redundant. It is also evident from their analysis that the mere possibility of having the *which way* information, complete or partial, will be reflected as complete or partial loss of the fringe pattern.

In quantum mechanical description, interference is basically a property of the characteristic wave function. Any explanation of the disappearance of the interference due to a measurement without clarifying how the wave function is altered by the measurement, seems incomplete [4,47,50]. This was analysed in our study by tracing the changes in the state vector at different stages. It is concluded that the decoherence arising from entanglement [4,47] with orthogonal detector states in the expanded Hilbert space of the combined system can only provide an unambiguous description of the disappearance of the interference pattern in a *which state* measurement. The mathematical description is given [47] in terms of the initial superposed state

$$\Psi \sim \psi_1 + \psi_2 \tag{3}$$

of the two interfering branches ψ_1 and ψ_2 which after entanglement (with the detector) in the larger Hilbert space becomes

$$\Psi \sim \psi_1 \otimes |D_1\rangle + \psi_2 \otimes |D_2\rangle. \tag{4}$$

For nonorthogonal detector states, partial or complete interference phenomenon is observed with partial or no *which state* information. However, with some special types of detectors, one can have the so-called *ein weg* knowledge [7,51]. Like the experiments which furnish partial interference and partial *which state* knowledge, *ein weg* experiments do not contradict BCP and in fact challenge the orthodox Copenhagen interpretation of QM. But for an unambiguous *which state*/*welcher* *weg* detection, the detector state D_i's are required to be mutually orthogonal and the entangled state of eq. (4) will show no interference effect – the mere possibility of obtaining sharp *which state* information ensures that the component wave functions (of the superposed state) will not interfere. The entanglement with orthogonal detector states represents 'potentiality' from which one can either get the sharp *which state* information by a measurement on the detector states (by projecting the state of the system on to D_1 or D_2) and no interference pattern, or can retrieve the interference fringes with the help of 'quantum eraser' which (for some special type of microdetector [10]) projects the state of the system on to the symmetric/antisymmetric combination $D_1 \pm D_2$. In the first case the system irreversibly collapses to either ψ_1 or ψ_2, whereas in the second instance we get back the superposed state of eq. (3), capable of producing interference fringes. The interference pattern can be retrieved if it is possible to 'erase', even partially, the information in the entangled wave function which could reveal the 'path' or state. The above analysis is adequate and consistent with the complementarity principle which asserts that one cannot have sharp *which state* information and interference fringes simultaneously.

Scully *et al* [10] first proposed a gedanken experiment to devise a *welcher weg* detection devoid of any momentum transfer and thereby claimed that the consequent destruction of interference pattern in their experimental arrangement cannot be related to position–momentum uncertainty relation. On the contrary, Bhandari [52] and Tan and Walls [53] maintained that, in principle there is no difference between this scheme and Einstein's recoiling slit. Subsequently, Eichmann *et al* [54] and Schulman [55] have also discussed specific experimental arrangements where the uncertainty principle cannot be invoked as the enforcing mechanism of ME. But Storey and co-workers [56] have argued that whenever interference is destroyed due to a *welcher weg* detection in a double slit experiment, transverse momentum is transferred according to the uncertainty principle and 'the principle of complementarity is a consequence of the Heisenberg uncertainty relation'. Joining issue with the prolonged debate between Storey *et al* and Scully *et al* [56–58], Wiseman and co-workers [59] have shown that if the momentum transfer function (quantum mechanical) is defined correctly in terms of 'convolution of the particles momentum wave function with a set of momentum transfer amplitude distribution', the 'nonlocal' momentum transfer function must be nonzero for some momentum $p > p_m(= \pi\hbar/2d)$, d being the slit separation, consistent with the Heisenberg's uncertainty relation. At the same time, for an unambiguous *welcher weg* detection, it has been shown [47,59] that the momentum dispersion is the same for the superposed and the collapsed state. Recently, Mir *et al* [60] have reported an experimental work on the 'momentum disturbance by a *which way* measurement in a double-slit apparatus'. Using a set-up 'akin to that proposed by Scully *et al*' they have measured a weak-valued probability distribution for momentum transfer which spreads well beyond the minimum value required by HUP, 'but nevertheless has a variance consistent with zero'. With these observations they have concluded that the experimental result 'reflects both sides of the debate'. The quantum mechanical uncertainty relation is defined in terms of variances and it should be noted here that after defining the 'quantum momentum kick distribution' the authors finally resort to HUP to assert that ME in Class II complementarity is a consequence of the uncertainty principle. Therefore, any explanation of the washing-out of the interference pattern in a *welcher weg* detection in terms of momentum kick distribution – 'classical' [41,42] (obtained using the classical notion of convolution of the momentum probability distribution) or 'quantum' [56,59] is semiclassical and not quantum mechanical.

The mental picture emerging from the analysis in terms of HUP is that each individual microparticle produces its own pattern and if we have the *welcher weg* information, 'shifted interference patterns add together, washing out the fringes' [56]. This implies the unacceptable conclusion that superposition is not lost even after the exact *which path* detection. The gradual build-up of the distribution pattern in interference experiments with a series of incoming single microentity one at a time confirms that each registration of the microentity on the screen conforms to one of the two patterns (either a continuous distribution or interference fringes) depending on whether the experimental set-up is providing the *which slit* information or not. In the shifted fringe interpretation even after an exact *which slit* detection the only change in the original Ψ-function is believed to be a change in the wavelength due to momentum transfer and the superposed structure (i.e. $\Psi \sim \psi_1 + \psi_2$) remains unaltered. The analogy appears to be with the double-slit experiment using white light. The fallacy of this argument becomes apparent because in this interpretation the central bright band is always present and there is no washing out of the entire fringe system. But in the case of an entanglement with the orthogonal detector states, the entire fringe pattern disappears and one gets a continuous distribution.

Dürr *et al* [61] in 1998 have performed an ingenious experiment using a more general arrangement ('atom interferometer') and concluded that the disappearance of the interference pattern is due to 'correlation between the *which way* detector and the atomic motion, rather than to the uncertainty principle'. Commenting on this experiment, Mir *et al* [60] have observed that this experiment is not relevant to their analysis as no double-slit was used and no position measurement was performed. It implies that the make-shift uncertainty argument is not even applicable for all types of *which state*-interference complementarity. Bertet *et al* [62] from an analysis of their more recent experiment on complementarity concluded emphatically that the 'entanglement approach is more precise' and appropriate while the usual explanation given in terms of uncertainty principle is 'superficial'. They have also noted that the latter approach describes the blurring away of quantum interference in terms of a deceptive 'irreversible noisy perturbation' whereas 'the coding of the *which path* information' in their experiment can be shown as 'a fully reversible process'.

Apart from all these examples, lending support to the theoretical assertion [3,4,47,50], there are obvious instances where HUP cannot be dragged for an explanation. For example, in the double slit experiment if we put a 100% efficient detector close to slit 1 and record on the screen only those events for which the detector does not click, the result does not show any interference. In this case the disappearance of interference takes place without any exchange of energy or momentum between the detector and the particle. Also, when the interference is not among wave functions in position space, the 'momentum kick' argument loses any significance whatsoever and entanglement of the superposed wave function with the orthogonal detector states can only offer a comprehensive explanation for the disappearance of interference.

5. Summary

The importance of BCP, as it forcefully illustrates the impossibility of a classical-like unambiguous description in QM, cannot be ignored. A precise formulation of the complementarity principle, in the spirit of Bohr and consistent with the principles of QM, is possible. Two distinct classes of complementarity appear on the basis of the origin of ME. In the case of complementarity of quantum mechanical observables (Class I) it is the noncommutation of operators corresponding to dynamical variables which ensures ME. Complementarity between the Fourier pair time (t) and energy (E) may be also included in this class. In quantum interferometric *which state*-interference complementarity (Class II), ME is enforced by the entanglement of the interfering wave function branches with orthogonal detector states.

Entanglement which implies nonlocal connections between different parts of the same entity, has been repeatedly demonstrated through experimentation. Although a complete understanding of the entire mechanism is still lacking, quantum entanglement is increasingly appreciated as a ubiquitous feature of the real world phenomena. In recent times it has emerged as a very active research area and contributed some dramatic practical applications in quantum information and communication, quantum cryptography, quantum-state teleportation and quantum computing.

The semiclassical explanation using HUP provides only a naive, deceptive and inadequate understanding of the origin of ME in complementarity problems. It is unable to address the more general example where no double slit is used or where no position measurement takes place (as in the experimental arrangement of Dürr *et al*). Also it lacks a proper description of the retrieval of interference pattern with the help of a 'quantum eraser'.

Finally, in the light of the formulation discussed here, it may be reiterated that violation of BCP without violating the principles of QM is not possible.

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