PRAMANA
— journal of
physics

Vol. 72, No. 4
April 2009
pp. 753–758

Alpha radioactivity for proton-rich even Pb isotopes

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MS received 20 May 2008; revised 17 November 2008; accepted 25 November 2008

Abstract. Half-lives for alpha radioactivity from proton-rich even Pb isotopes in the range $A = 182$–202 have been calculated using the unified fission-like approach. The geometrical shape of the potential barrier is parametrized in terms of a highly versatile, asymmetric and analytically solvable form of potential based on Ginnochio's potential. Good agreement with the experimental data has been obtained with the variation of just one parameter. Half-lives of three unknown alpha emitters in the neutron-deficient Pb chain (^{198}Pb, ^{200}Pb and ^{204}Pb) have been predicted. The exact expression for the transmission coefficient has been compared with those obtained from WKB approximation method for symmetric Eckart potential.

Keywords. Alpha radioactivity; proton-rich nuclei; half-life.

PACS Nos 23.60.+e; 23.90.+w; 27.70.+q; 27.80.+w

1. Introduction

In the chain of neutron-deficient Pb isotopes the alpha branching ratio changes from small value of <1% for ^{202}Pb to ≤100% in ^{182}Pb. The corresponding half-lives of various isotopes exhibit large variation from 1.7×10^{14} s to 5.5×10^{-2} s as the neutron number changes from $N = 120$ to $N = 100$. The proton-rich Pb isotopes thus offer an interesting region to test the validity of existing theories of alpha decay as one moves from the region of naturally occurring alpha radioactive isotopes with $Z/N \cong 0.65$ to the region close to proton drip line with $Z/N \cong 0.82$. The existing theoretical studies can be broadly classified into two categories: the cluster and the fission-like theories [1–3]. In the present work we have studied the systematic for alpha emission from proton-rich even Pb isotopes in the range $A = 182$–202 for ground state to ground state transition using the unified fission-like approach.

Earlier studies have shown that whatever be the approach – cluster theory or unified fission theory, the process of alpha decay reduces to the transmission through a one-dimensional potential barrier which gets modified from its Coulomb plus centrifugal form due to various renormalization effects. In almost all the earlier calculations, the transmission coefficient has been calculated by the WKB approximation method. In the present work we have represented the potential barrier for alpha decay by a highly versatile, asymmetric and analytically solvable form of potential based on Ginnochio's potential developed by Sahu *et al* [4]. The biggest advantage of this potential is that it is highly versatile in nature and also admits exact solution for the transmission coefficient [4]. It is thus possible to parametrize the geometrical shape of the potential barrier and to fold one exact expression of transmission coefficient which has been used in place of the commonly used WKB method.

2. The model

A large number of studies indicate that the effective barrier for the emission of alpha particle gets modified from its simple Coulomb plus centrifugal form due to various physical processes like nuclear forces, proximity force effect, deformation of fragments, shell effects etc.

Thus, in the simplest picture where one visualizes the alpha emission as basically a one-dimensional barrier transmission problem, this notion of modification of the barrier due to various renormalization processes can be incorporated by using a versatile potential barrier developed by Sahu *et al* [4] to describe the phenomenon of the alpha radioactivity. The phenomenon of alpha emission reduced to one-dimensional motion of alpha particle in an effective potential field is described by the Shrödinger equation as

$$-\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2\psi(r)}{\mathrm{d}r^2}+V(r)\psi(r)=E\psi(r), \tag{1}$$

where μ is the reduced mass of the alpha-daughter nucleus system. The potential $V(r)$ has two components, inner barrier in the overlapping region ($r < R_\mathrm{t}$) and outer barrier in the nonoverlapping region ($r > R_\mathrm{t}$), and are given by

$$\begin{aligned} V(r) &= V_{01}[\lambda_1^2\nu_1(\nu_1+1)(1-y_1^2)+\xi_1], \quad \text{if } 0 \le r < R_\mathrm{t} \\ V(r) &= V_{02}[\lambda_2^2\nu_2(\nu_2+1)(1-y_2^2)+\xi_2], \quad \text{if } r > R_\mathrm{t} \end{aligned} \tag{2}$$

and

$$\xi_1 = \frac{1-\lambda_1^2}{4}[5(1-\lambda_1^2)y_1^4-(7-\lambda_1^2)y_1^2+2](1-y_1^2)$$

$$\xi_2 = \frac{1-\lambda_2^2}{4}[5(1-\lambda_2^2)y_2^4-(7-\lambda_2^2)y_2^2+2](1-y_2^2),$$

where V_{01} and V_{02} are the strength of the potential in MeV. The parameters ν and λ describe the height and the shape of the barrier respectively. R_t is the touching configuration of the two nuclei, i.e. $R_\mathrm{t} = R_\mathrm{d} + R_\alpha$. R_d (radius of the daughter nucleus) has been calculated from the droplet model of atomic nuclei [5] and the value of alpha particle radius has been fixed at $R_\alpha = (1.62 \pm 0.01)$ fm as obtained from data on electron scattering from ^{4}He nucleus [6].

The function $y(r)$ is related to the radial variable r by

$$\rho_n = \frac{1}{\lambda_n^2}[\tanh^{-1} y_n - (1-\lambda_n^2)^{1/2} \tanh^{-1}(1-\lambda_n^2)^{1/2} y_n]. \tag{3}$$

The ranges of variation of r, ρ_n and y_n are as follows: In the interior side r varies from 0 to R_t, then ρ_n varies from $-\infty$ to 0 and y_n from -1 to 0 and on the outer side r varies from 0 to external turning point, ρ_n varies from 0 to ∞ and y_n from 0 to 1. The above potential has six parameters such as λ_1, ν_1, V_{01} and λ_2, ν_2, V_{02}, describing the potential on either side of the merger. These parameters are connected to each other through the expressions obtained by equating the two sides of the barrier at $r = R_\mathrm{t}(\rho = 0)$. This composite barrier can become symmetric, asymmetric, more flat or less flat on either side depending on the values of the above parameters. Other mathematical details of the potential are described in ref. [4]. By varying the parameters V_B, ν and λ, the height, range and flatness at the top can be changed and a variety of potentials can be generated. Thus the highly versatile nature of the potential makes it suitable for use in the alpha radioactivity study. The asymmetric potential barrier $V(r)$ is analytically solvable and exact expression for the transmission coefficient across the barrier for $l = 0$ angular momentum state of the alpha particle is taken from ref. [4].

The decay rate has been calculated by $P = P_0 T$. The assault frequency P_0 is calculated from the zero point vibration energy $E_\mathrm{v} = 1/2hP_0$ which in turn is the same as those described in [7]. The half-life time is calculated as $t = \log_\mathrm{e} 2/P$. The transmission coefficient can also be calculated in the WKB method by the expression

$$T = \exp\left[-2\int_{r_1}^{r_2} k(r)\mathrm{d}r\right]$$

with

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2}[V(r) - Q]}.$$

Here $V(r)$ is the potential barrier. The inner and outer turning points r_1 and r_2 respectively are determined using the experimental Q value of the decay in equation $V(r) = Q$.

3. Model parameters

In choosing the model parameters we are guided by the following considerations: In naive calculations barrier height should correspond to Coulomb plus centrifugal potentials, but earlier studies have shown that various renormalization effects tend to lower this value. Royer [8] has modified the alpha decay barrier height by including proximity force effect which lowers the Coulomb barrier to a height between 10 MeV and 17.5 MeV in the region of nuclides $Z = 52$–110 and $A = 108$–269. In the present work we have taken barrier height V_B of ^{182}Pb to be 13.3 MeV for $l = 0$ from the work of Royer. Treating this as reference the barrier heights of other Pb isotopes are calculated as $V'_{\rm B} = (R_{\rm t}/R'_{\rm t})V_{\rm B}$ where $R_{\rm t}$ and $R'_{\rm t}$ are the touching radii. These values lie between 10.8 MeV and 13.4 MeV. As λ increases potential falls off more sharply, whereas the range of the potential increases with ν. Earlier studies have shown that the inner barrier is very narrow and sharply falling [9]. In independent studies it has been shown that the width of the inner barrier decreases with decrease in mass of the emitted particle [9]. In independent studies it has been shown that the width of the inner barrier decreases with decrease in mass of the emitted particle. In view of this we have fixed $\lambda_1 = 3.2$, $\nu_1 = 2.3$ indicative of narrow sharply falling potential. The contribution of the inner barrier to the half-life values are small. As the outer potential barrier simulates the slowly falling long-ranged Coulomb barrier for $l = 0$, we have fixed $\lambda_2 = 1.1$ and varied ν_2 starting from a large base value 30 to 37 to reproduce the experimental data.

4. Results

The half-life of alpha emitters for the even Pb isotopes $^{182-196}$Pb, ^{202}Pb and ^{210}Pb have been calculated and shown in table 1. In all the cases good agreement with experimental results is obtained by varying the parameter ν_2 which controls the range of the potential. The resulting barrier for the limiting and mid-values of ν_2 is shown in figure 1a. The minimum rms deviation of 2.78 is obtained for $\nu_2 = 33$. Half-lives of three unknown proton-rich alpha emitters in the neutron-deficient Pb chain – Pb^{198}, Pb^{200} and Pb^{204} – have been predicted for $\nu_2 = 33$. Also, half-life of doubly closed nucleus Pb^{208} has been calculated for the best fit value of $\nu_2 = 33$. Figures 1b and 1c show the variation of Q_α values and calculated values of logarithm of half-life time as a function of neutron number of the parent isotopes respectively. It is observed that the behaviour of half-life is the perfect inverse

Table 1. Calculated and experimental half-lives of proton-rich even Pb isotopes. The corresponding values of parameter ν_2 which gives best fit with the experimental data are also given.

Nucleus	Assault freq. (P_0) $\times 10^{20}$ s	Q_α (MeV)	ν_2	$\log_{10}(t_{1/2})$ (s) Cal.	$\log_{10}(t_{1/2})$ (s) Exp.
^{182}Pb	3.58	7.06	30.78	−1.26	−1.26
^{184}Pb	3.43	6.77	30.07	−0.25	−0.26
^{186}Pb	3.28	6.47	31.24	1.00	1.00
^{188}Pb	3.09	6.11	31.47	2.04	2.04
^{190}Pb	2.89	5.69	32.02	3.90	3.90
^{192}Pb	2.64	5.22	32.83	6.56	6.56
^{194}Pb	2.40	4.73	33.99	9.99	9.99
^{196}Pb	2.14	4.23	32.98	9.86	9.86
^{202}Pb	1.31	2.59	31.98	14.23	14.23
^{210}Pb*	1.92	3.79	36.05	16.56	16.57

*^{210}Pb is neutron-rich

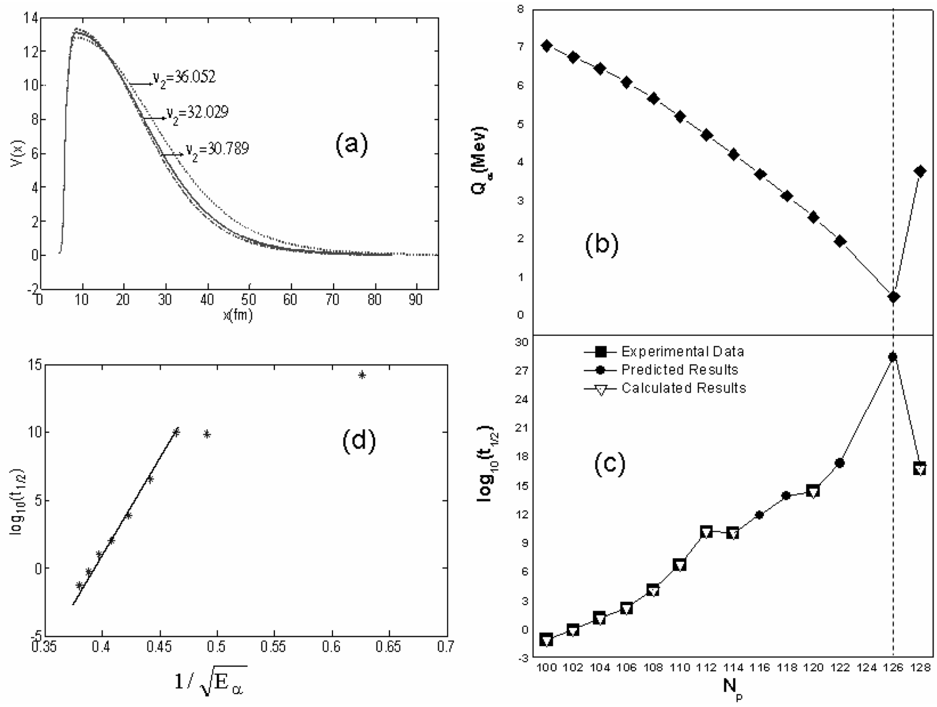

Figure 1. (**a**) Potential plot for alpha emitting nuclei for three values of range parameter ν_2, (**b**) Q_α values of alpha emission vs. neutron number of parent isotopes of Pb chain, (**c**) variation of logarithm of half-life time with neutron number of parent isotopes of Pb chain and (**d**) variation of logarithm of half-life time for alpha emission with $(E_\alpha)^{-1/2}$.

of the Q_α value. The maxima in the half-life and the corresponding minima in the Q_α value at $N = 126$ are manifestations of neutron shell closure at $N = 126$. Thus our model demonstrates the shell closure effect correctly. The energies of alpha particle E_α are calculated from experimental Q_α values using the expression $E_\alpha = (A - 4/A)Q_\alpha$. The variation of logarithm of half-life with $1/\sqrt{E_\alpha}$ is shown in figure 1d. The empirical Geiger–Nuttal law $\log_{10} t_{1/2} = C_1 + C_2 E_\alpha^{1/2}$ is followed up to $A = 194$ with $C_1 = -5$ s and $C_2 = 17.7$ MeV$^{-1/2}$ s.

5. Summary and conclusion

In this work a parametrized form of the potential has been used for detailed analysis of proton-rich even Pb isotopes. The parameters are chosen so as to simulate a narrow tail in the inner region, parabolic shape at the top and a long Coulombic tail in the outer region. The present calculation has three distinct advantages over the previous works employing similar techniques. Firstly, in the present method, it has been possible to parametrize the complete geometrical shape of the barrier. Secondly, we have calculated assault frequency from the zero point vibrational energy

Table 2. Comparison of WKB and exact transmission coefficients for different ranges with $V_0 = 13.3$ MeV and $Q_\alpha = 7.066$ MeV.

$1/a$ (fm^{-1})	$\Delta = \hbar^2/2ma^2$ (MeV)	T_{WKB}	T_{Exact}	$T_{\text{WKB}}/T_{\text{Exact}}$
0.5	1.25	1.38×10^{-5}	1.57×10^{-5}	0.878
1.0	5.0	3.48×10^{-3}	4.48×10^{-3}	0.776
2.0	20	0.058	0.088	0.659
4.0	80	0.200	0.402	0.497
8.0	320.51	0.333	0.754	0.441
16.0	1300.72	0.414	0.927	0.446
32.0	5200	0.419	0.981	0.427

data instead of using a constant value. Thirdly, exact expression for transmission coefficient through the barrier has been used to calculate the half-lives whereas in the earlier studies transmission probabilities through the barrier have been calculated using WKB approximation method. In a recent study, it has been shown that the transmission probabilities calculated using WKB method are insensitive to the tail region of the barrier potential especially for large half-life values [10]. In order to test the accuracy of WKB method we have calculated T_{WKB} and T_{Exact} for Eckart barrier of different ranges. Our potential barrier reduces to the Eckart barrier $V(r) = V_0\ \text{sech}^2(x/2a)$, where $a = 1/2b$ for $\lambda = 1$, $b = (2m/\hbar^2 V_0)^{1/2}$ is the scale parameter in the units of inverse distance. The results are shown in table 2 for different values of a. It is observed that the WKB approximation becomes poorer with increasing a. According to [11] we observe that the parameter $\Delta = \hbar^2/2ma^2$ sets the lower limit on energy above which the WKB approximation would work well provided $V_0 > \Delta$. The condition, $E > \Delta$ comes to the de Broglie wavelength of the incident particle being less than that of the size of the 'obstacle', i.e., when $\lambda_{\text{d}} < 2\pi a$. For $\lambda_{\text{d}} > 2\pi a$ the tunnelling becomes more and more quantal and hence the semiclassical approximation becomes poorer.

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