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Chaos in the solar wind flow near Earth

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Abstract. We have done a time series analysis of daily average data of solar wind velocity, density and temperature at 1 AU measured by ACE spacecraft for a period of nine years. We have used the raw data without filtering to give a faithful representation of the nonlinear behaviour of the solar wind flow which is a novel one. The sensitivity of the results on filtering is highlighted. The attractor dimension is estimated for every parameter of the solar wind and it is found that they differ substantially. Hence a chaotic picture for the problem from different angles have been obtained. The calculated Kolmogorov entropies and Lyapunov exponents are positive showing evidences that the complex solar wind near the Earth is most likely a deterministic chaotic system.

Keywords. Solar wind plasma and sources of solar wind; time series analysis; nonlinear dynamics and chaos.

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1. Introduction

In this paper we focus our studies on time series analysis of daily average solar wind velocity, density and temperature at 1 AU measured by the ACE spacecraft for a period of nine years. The time series analysis is the only method available to study these data for the testing of chaos. We took the daily average data of the solar wind parameters by taking into account both fast and slow solar winds. Due to the mixing of both kinds of solar winds we expect a chaotic nature for the problem and this has been proven here by the existence of fractional correlation dimensions for the chaotic attractor. The existence of such a strange attractor is a necessary and sufficient condition for chaos. At a distance of 1 AU the terrestrial magnetospheric fluctuations give rise to interaction between solar wind particles and low frequency Alfven waves which leads to nonlinear behaviour and chaos. There is a high frequency component in the data, which may not be significant for a daily averaged data. However, its significance cannot be ruled out in view of the general objective of the problem. In addition to the fractional correlation dimensions, we have calculated the Kolmogorov entropy and largest Lyapunov exponent using this data. Finite and positive values of these provide further evidences for the chaotic nature of the solar wind flow at this location.

The solar wind plasma flowing supersonically away from the Sun has two forms: slow wind, with velocity less than 400 km/s and fast wind, with velocity greater than 400 km/s. The fast wind is associated with coronal holes and slow wind is associated with coronal streamers. Burlaga [1,2] has done a detailed analysis of the solar wind data obtained from various spacecrafts and he found some signatures of chaos (multifractals, intermittence and turbulence) in the solar wind. Buti [3] showed that the chaotic fields generated in the solar wind can lead to anomalously large plasma heating and acceleration.

Time series analysis of the observed solar radio pulsations has been done by Kurth and Herzel [4] and they suggested that there must be a low-dimensional attractor in solar radio pulsations. Macek and Redaelli [5] studied the dimension of chaotic attractor in low speed stream of solar wind at 0.3 AU and have shown that the Kolmogorov entropy of the low speed solar wind attractor is positive and finite and it holds a chaotic system. The correlation dimension of the reconstructed attractor has been calculated by Macek [6]; and has provided tests for nonlinearity in the solar wind data. The chaos in low speed stream of solar wind plasma including Alfvenic fluctuations measured by Helios spacecraft in the inner heliosphere has been analysed by Macek and Redaelli [7]. The Kolmogorov entropy of the Alfvenic fluctuations in the low speed stream of solar wind has been studied by Redaelli and Macek [8]. The existence of chaos in the evolution of pressure in coronal loops has been investigated by Sasidharan *et al* [9] by studying the power spectrum of the data generated by the solution of the MHD equations and by evaluating the invariant dimension D_2 of the system.

We took the daily average unfiltered data of the solar wind parameters near Earth by taking into account both fast and slow solar winds which have more complex behaviour. We have calculated the dimensions of the chaotic attractors in the solar wind along with Kolmogorov entropies and largest Lyapunov exponents using these data. Finite and positive Kolmogorov entropies and positive Lyapunov exponents which we have obtained ensure that the complex solar wind is most likely a deterministic chaotic system.

The paper is organized as follows: Section 2 of this paper contains a brief introduction to the methods used for the calculation of various chaotic parameters. Section 3 describes the data analysis in detail and in §4 results and discussion are given. Then §5 gives the conclusion.

2. Methods

2.1 *Autocorrelation function*

In this paper our goal is to test the existence of deterministic chaos in the solar wind flow and we have employed the commonly used techniques of time series analysis. Firstly an autocorrelation analysis is done for determining a proper time delay in the computation of correlation integrals. By means of the shape of the autocorrelation function, a preliminary detection of deterministic chaotic dynamics in time series for this problem has been done. If the autocorrelation function falls quickly to zero, then the time series is probably purely stochastic and there is no determinism in the series, the value at each point being independent of all other values in the series. Random Gaussian white noise is an example of such a system. On the other hand, the autocorrelation function of the signal which is governed by the deterministic chaotic dynamics decreases more slowly – even slowly than in an exponential manner, because the points are not independent of each other and a self-similarity is present in the process. This shows that the system has a memory. In this paper we have done the autocorrelation analysis for velocity, density and temperature in the solar wind data. In all the three cases we were able to determine the relevant autocorrelation functions which indicate the presence of chaos in the problem.

The autocorrelation time t_{a} is given by

$$\frac{\langle V(t)V(t+t_{\mathrm{a}})\rangle - \langle V(t)\rangle^2}{\sigma^2} = e^{-1}, \tag{1}$$

where σ is the standard deviation. We took the time delay for each parameters slightly greater than the first zero of the autocorrelation function as in figures 1a–c.

2.2 *Correlation integrals and correlation dimensions*

There are several ways to detect chaotic patterns in the time series. The correlation dimension method, discussed by Grassberger and Procaccia [10,11], Lyapunov exponent method [12] and the Kolmogorov entropy method by Grassberger and Procaccia [13] are the most common ones for problems like this. In this study we have applied all the methods and positive results of chaos were obtained. The novelty of our study is that we have used unfiltered data for the study and employed all the above-mentioned techniques together. We have obtained clear-cut symptoms of deterministic chaos in our present study which other authors have never seen. Our main intention in this paper is to show that the system would appear differently when looked through different windows.

Let $x_1, x_2, \ldots, x_N$ where $x_i \in R$ is the measurement of some state variables such as temperature, velocity etc. taken as time $t_i = t_0 + i\Delta t$. Out of this sequence of one-dimensional variables create a set of m-dimensional vectors whose components are just the time-delayed values of the variables.

$X^m(t) = V_i = (X_i, X_{i+\tau}, X_{i+2\tau}, \ldots, X_{i+(m-1)\tau})$, $V_i \in R^m$ where m is the embedding dimension and τ is an appropriate time delay. The dynamical information in one-dimensional data has been converted to spatial information in the m-dimensional set. A system which has a υ-dimensional attractor in its phase space will have its Takens vectors lying on a υ-dimensional subset of the embedding space R^m [14]. To find the correlation dimension υ or D_2 we make use of the correlation integral $C(\varepsilon)$ introduced by Grassberger and Procaccia [13]

$$C(\varepsilon) = \frac{1}{N^2} \sum \theta(\varepsilon - |X_i - X_j|), \tag{2}$$

where θ is the Heaviside step function. $C(\varepsilon)$ will survive for all values of $|X_i - X_j| < \varepsilon$. The correlation dimension υ is defined by the limit

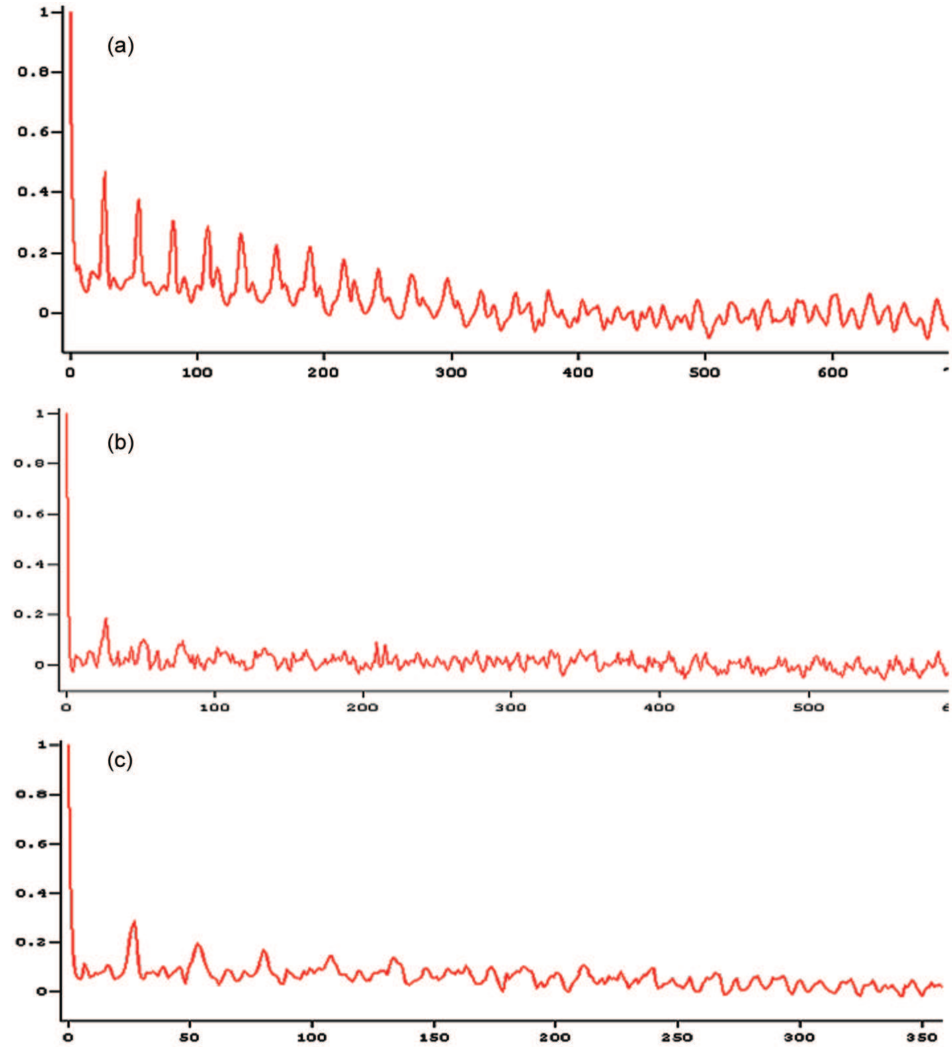

Figure 1. (**a**) The autocorrelation function as a function of the time lag for velocity profile for the period 1998–2006. The x-axis is in days and y-axis is the autocorrelation function. (**b**) The autocorrelation function as a function of the time lag for density profile for the period 1998–2006. The x-axis is in days and y-axis is the autocorrelation function. (**c**) The autocorrelation function as a function of the time lag for temperature profile for the period 1998–2006. The x-axis is in days and y-axis is the autocorrelation function.

$$v = \lim_{\varepsilon \to 0} \lim_{N \to \infty} \frac{\mathrm{d} \log C(\varepsilon, N)}{\mathrm{d} \log \varepsilon}. \tag{3}$$

Typically, one reconstructs the attractor in a suitable space and computes $C(\varepsilon)$ and its slopes as a function of ε. For correlation dimension (attractor dimension)

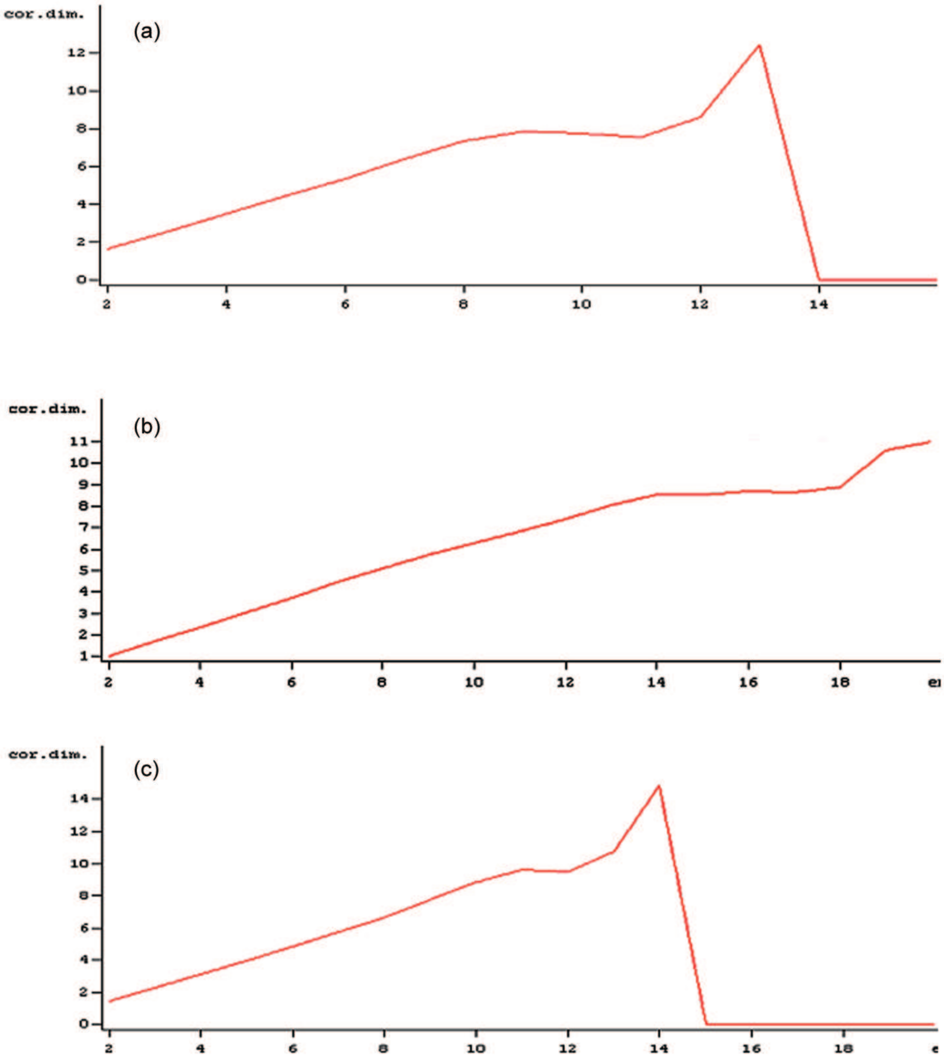

Figure 2. (**a**) Graph showing the variations of correlation dimension with embedding dimension for velocity profile. (**b**) Graph showing the variations of correlation dimension with embedding dimension for density profile. (**c**) Graph showing the variations of correlation dimension with embedding dimension for temperature profile.

corresponding to each parameter one would look for a range of ε values where υ saturates or where it is relatively constant. This υ will be the attractor dimension D_2. We have obtained D_2 as 7.84 from figure 2a for the original data for velocity whose embedding dimension is nine. For density profile D_2 is obtained as 8.54 as in figure 2b and for temperature it is obtained as 9.67 as in figure 2c which are all fractals depicting the chaotic nature of the problem.

Table 1. Characteristics of solar wind attractors for the solar wind data during the period Jan. 1998 to Oct. 2006 (consolidated data).

Characteristics	Velocity	Density	Temperature
Numbers of data points	3181	3181	3181
Sampling time Δt	1 day	1 day	1 day
Autocorrelation time	5 days	11 days	4 days
Correlation dimension D_2	7.84 bits/	8.54 bits/	9.67 bits/
Embedding dimension	9	14	11
Entropy K_2	0.37	0.55	0.47
LLE	0.349	0.493	0.403

2.3 *Kolmogorov entropy*

Using time series of equally spaced original data we have constructed a large number of vectors in the embedding phase space of dimension m. Then we have divided this space into a large number $M(\varepsilon)$ of equal hypercubes of size ε which cover the presumed attractor. If P_j is the probability measure that a point from a time series falls in a typical jth hypercube, using q-order function $I_q(\varepsilon) = \sum (P_j)^q$ where $j = 1, \ldots, M$ [5]. Renyi–Kolmogorov information entropy can be generally defined as

$$K_q = \lim_{\varepsilon \to 0} \lim_{M \to \infty} \frac{1}{1-q} \ln I_q(\varepsilon). \tag{4}$$

Mathematical theory of information in the context of communication was studied in 1940s by Shanon, who defines the information content of an event as the logarithm of the inverse of the probability of occurrence, $I = -\log P_i$. For more than one event the information entropy $I = -\sum P_c \log P_i$ which is known as Kolmogorov entropy. The conventional Boltzmann entropy of zero order ($q = 0$) gives no information about how the system evolves. But Renyi–Kolmogorov entropy of order two ($q = 2$) provides information about the evolution of the system. This correlation entropy provides important information about the dynamics of the system and is useful for nonlinear deterministic system [8].

As discussed in Redaelli and Macek [8] we have estimated order 2 correlation entropy K_2 directly from unfiltered data, using the following equation:

$$K_2 = \frac{1}{\Delta m} \left\{ \ln \frac{C_m(\varepsilon)}{C_m(\varepsilon) + \Delta m(\varepsilon)} \right\}. \tag{5}$$

The calculated entropy for velocity, density and temperature profiles are given in table 1. In all the three cases positive values of K_2 were obtained which reveal the chaotic nature.

2.4 *The largest Lyapunov exponent* (*LLE*)

In nonlinear time series analysis, it is of great interest to measure the Lyapunov characteristic exponents which if positive, by definition are the most striking evidence for chaos (Holger Kantz [12]). One can use an algorithm which directly exploits the definition of the Lyapunov exponent in the state space and therefore does not need the underlying equations of motion. By this, in practice, only the maximal exponent can be computed.

Let $x(t)$ be the time evolution of some initial condition $x(0)$ in an appropriate state space. Then the maximal Lyapunov exponent is found with probability one by Holger Kantz [12],

$$\lambda_{\max} = \lim_{t\to\infty} \lim_{\varepsilon\to 0} \frac{1}{t} \frac{\ln[|x(t) - x_\varepsilon(0)|]}{\varepsilon}, \qquad |x(t) - x_\varepsilon(0)| = \varepsilon \tag{6}$$

for almost all difference vectors $x(t) - x_\varepsilon(0)$. To compute $\lambda_{\max}$ numerically one can apply eq. (6) directly in real space by searching for pairs of neighbouring trajectories and see how they diverge.

In all the three cases positive values of LLEs were obtained which reveal the chaotic nature.

3. Data analysis

We have analysed the solar wind data from ACE spacecraft measured *in situ* in the heliosphere at 1 AU. We have selected the daily average values of solar wind velocity, density and temperature from January 1998 to October 2006. It is well-known that various disturbances are superimposed on the overall structure of the solar wind including mainly Alfven waves, which move away from Sun [7]. In this work we have used raw data instead of filtered data in order to give a faithful representation of the nonlinear behaviour of the solar wind. Figures 3a–c show the original data without filtering. This is a novel step compared to the previous related works on solar wind data. The input parameter here is the time delay selected. In this case we have calculated the autocorrelation time by plotting the autocorrelation function for choosing the proper time delay. After getting the autocorrelation time t_{a} we have selected a time delay which is slightly greater than t_{a}. Autocorrelation function has been plotted as in figures 1a–c.

We have repeated the time series analysis for three more samples. The sample data consist of *in situ* measurements of the hourly averaged data for the years 2000, 2002 and 2004 for 4 months. In this analysis we got the same response as the consolidated data had been exhibited that each parameter has its own characteristic attractor.

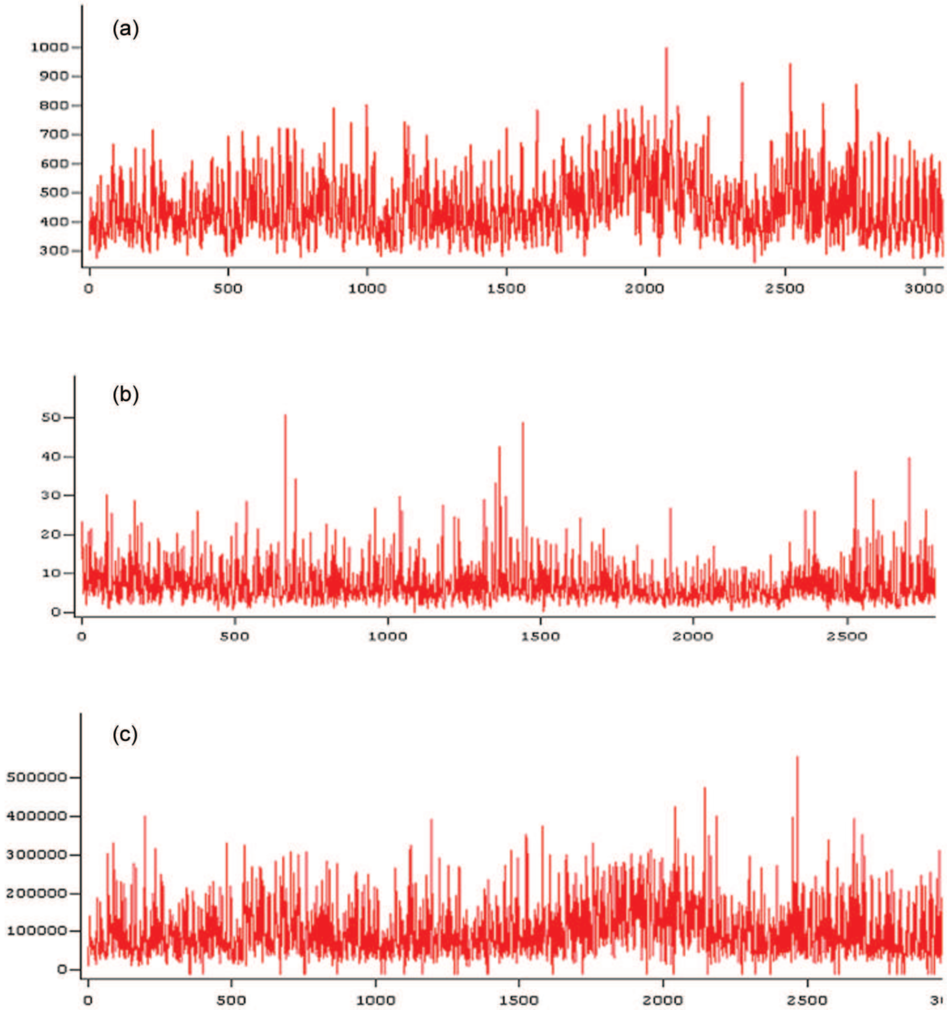

Figure 3. (**a**) The original daily average data profile for velocity for the period 1998–2006. The x-axis is in days and y-axis is in km/s. (**b**) The original daily average data profile for density for the period 1998–2006. The x-axis is in days and y-axis is in cm^{-3}. (**c**) The original daily average data profile for the temperature for the period 1998–2006. The x-axis is in days and y-axis is in degree Kelvin.

4. Results and discussions

We have analysed the ACE data using plasma parameters measured inside in the heliosphere at 1 AU. We have selected the daily averaged solar wind velocity from ACE data from January 1998 to October 2006. First we have calculated the natural logarithm of the correlation sum $C_{\mathrm{m}}(\varepsilon)$ vs. $\ln \varepsilon$ for various embedding dimensions.

Plot the slope of the curves for various embedding dimensions as in figures 2a–c. The slope for which saturation occurs is the correlation dimension D_2 of the attractor. We have calculated the attractor dimension for the velocity profile as 7.84 bits/ and the corresponding embedding dimension as 9. We have obtained a positive value for $\lambda_{\max}$ which is equal to +0.349 for the velocity data and Kolmogorov entropy for this case is 0.37. The chaotic behaviour is caused by the superposition of more than two modes of oscillation and is due to strong nonlinear coupling between them. According to Sasidharan *et al* [9] there exist two strange attractors with embedding space dimensions 7 and 18 for the pressure profile in coronal loops and the trajectories can land upon either of these attractors. Since the attractors are strange attractors, the trajectories could jump from one to the other and this can lead to chaos.

Macek and Redaelli [7] have used the filtered low speed solar wind data at 0.32 AU using nonlinear Schreiber filter and obtained the correlation dimension as 3.1 ± 0.2. But in our work we have obtained the correlation dimension as 7.84 bits/ for unfiltered data of solar wind velocity at 1 AU. Hence our results are bonafiding the fact that filtering can reduce the nonlinear behaviour of the data. Therefore, to get actual attractor dimension it is necessary to use the unfiltered data. In this study we took the daily average solar wind data by taking into account both the fast and slow winds which have more complex behaviour.

We have extended this idea of time series analysis to unfiltered density and temperature profiles to calculate the attractor dimensions. All the properties of the chaotic attractor has been calculated using density and temperature profiles. For the density profiles the attractor dimension is obtained as 8.54 bits/ as in figure 2b for the embedding dimension 14. The calculated LLE and Kolmogorov entropies are 0.4938 and 0.55. This shows that the density profile also is chaotic. From figure 2b it is obvious that there has been more than one attractor in the density profile. In the case of temperature profile we have got different properties for the chaotic attractor. The correlation dimension is 9.67 bits/ as in figure 2c for an embedding dimension 11. The LLE and Kolmogorov entropy are 0.403 and 0.47. These results show that it forms another chaotic attractor. So it has been interpreted that the nature of interactions would change depending upon the parameters. Each parameter has its own characteristic attractor.

The main achievement of our results is that, by using various techniques for chaotic studies, we found that the dynamical parameters are more significant than the static parameters. The scatter is more for the static parameters when compared to dynamic parameters. The correlation dimension is a static parameter while the Kolmogorov entropies and largest Lyapunov exponents are dynamic ones. Of these, Kolmogorov entropy exhibits more scatter from the mean as compared to Lyapunov exponents. Hence, even in the dynamic parameters one can make a classification based on the scatter from the mean. We have shown that LLEs are the most dynamic chaotic parameters in the problem. To the best of our knowledge this is a new and revealing result, and thus demand further work in this direction. We propose that this result cannot be expected from the time series analysis where a series is generated from a given chaotic equation since our case is a natural phenomenon. Nature holds back informations from a modelled equation.

5. Conclusions

We have analysed the unfiltered ACE spacecraft data of solar wind at 1 AU. The correlation dimension (attractor dimension) using time series analysis of the solar wind parameters, the velocity, density and temperature, has been estimated in our study. The technique of time series analysis of unfiltered solar wind data allows a more realistic estimation of the characteristics of complex dynamical solar wind system. The entropies and largest Lyapunov exponents obtained in this work are positive and finite, which provide the chaotic behaviour of the solar wind system which we have analysed. Depending on the solar wind parameters considered, the attractor characteristics are found to be different. A comparative study of the results reveal the possibility of a classification of various methods used for chaotic study and we found that LLEs are the most powerful and dynamic ones among them.

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