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Detection of directional energy damping in vibrating systems

B R AMRUTH[1] and SOURI BANERJEE[2,*]
[1]Department of Physics, Birla Institute of Technology and Science, Pilani 333 031, India
[2]Department of Physics, Birla Institute of Technology and Science, Pilani,
Hyderabad Campus, Shameepet Mandal 500 078, India
[*]Corresponding author. E-mail: souri.banerjee@gmail.com; souri@bits-hyderabad.ac.in

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Abstract. The transmission efficiency, frequency and amplitude alteration have been measured by a simple technique of coupled oscillators with a frequency gradient and in a system of non-Newtonian fluid in the form of corn-flour slime. The system of coupled oscillators was found to exhibit preferential energy transfer towards the low frequency end with the reverse propagation severely damped. Energy transfer in all directions was damped in the non-Newtonian fluid in comparison with water. Also the damping in non-Newtonian fluids works only after a lower limit for input amplitude. While most of the previous studies focussed on dissipation of energy within shock-absorbing systems, we demonstrate the contribution of re-distribution of energy reaching the output end to achieve shock absorbing.

Keywords. Coupled oscillators; non-Newtonian fluid; shock absorber; density gradient.

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1. Introduction

Frequency distribution in the propagating media has been analysed extensively because of its application in constructing shock-proof devices [1–3] and media with composite frequencies are found to be good shock absorbers [4,5]. They exhibit an upper limit for the amplitude, that could be effectively damped in the corresponding frequency range [6]. High efficiency shock absorbers also show a lower limit, which makes them operate in a narrow range of frequencies and input energies. Even materials like non-Newtonian fluids [7] have been studied as smart materials in modifying vibrations owing to their differential behaviour under the application of vibrations. However, there are a couple of issues relating to shock absorbers that still need to be addressed: (1) The sophisticated nature of the fabrication of such a system which has generally been intricate and expensive [6]. (2) Most of the previous study on the behaviour of non-Newtonian fluids indicated [7] that a density gradient is developed as a vibration is fed. However, as the natural frequency of oscillation depends inversely on the square root of density, it is quite plausible that there would exist a frequency gradient in the system which was not obvious from earlier studies [6]. Keeping the above issues in mind, we report a novel, cost-effective and simple technique to build a shock-absorbing system maintaining high efficiency of shock absorption using a chain of oscillators with linearly varying frequencies, as discussed later. We also test non-Newtonian fluids for their shock-absorbing potential by choosing corn-flour slime for our tests. Our system is found to behave as an effective shock absorber in a narrow range of energy and frequencies. It is to be noted that similar oscillator systems have been studied by other researchers [8] with respect to their ability to modulate the frequency and amplitude of oscillations. We extend such systems by introducing a non-linear frequency gradient in the system while keeping forced oscillations to a minimum.

We make use of two phenomena that occur in mechanical systems where a preferential transfer of energy is introduced. The first is the build-up of energy per oscillation in a system of coupled oscillators where the phase change rate decreases along the direction of propagation. Such systems also show an upper limit for driving amplitude after which shock absorption reduces [9]. The second phenomenon that we use in our system is the stretching of a wave due to wavefront acceleration in a medium with increasing velocity along the direction of propagation [7,10]. This wavefront acceleration stretches a propagating impulse as it enters the region of higher velocity of propagation resulting in broken wave pulses.

2. Experimental design

The experimental design for the detection of directional energy damping in vibrating systems consisted of two parts. In the first part, we design a system of oscillators such that individual oscillators retain their frequencies even in the presence of a driving force fed at a slightly different frequency. In the second part, we design a set-up to detect the shock-absorbing ability of corn-flour slime.

The experimental set-up for the oscillator system consisted of a 'test system' with a series of springs of increasing frequencies attached to the sides of a sealed column of water, which acts as an efficient coupling agent. A typical four-spring system is shown in figure 1a. The springs were made of soft-steel with spring constant 100 N/m. The system was built using a set of hollow tubes of 2 cm length that were filled with water before covering them with rubber membranes. Such a closed column of water was used in between the springs since it effectively transmits the input impulses without any internal damping according to Pascal's law [11]. The rubber membranes serve the purpose of allowing the two ends of the tube to vibrate without forcing them to be coherent with each other, which would be the case if a rigid connector was used. This also breaks linear transfer of vibrations between adjacent oscillators introducing non-linearity which plays a crucial role in controlling forced vibrations. To feed input vibrations of the required amplitude, a mechanical spring was attached to the input end as a tapping tool and the output end was connected to an oscilloscope (Tektronix 305 5 MHz) through a milli-Volt microphone. A 'control system' was similarly set up to serve as reference, with successive springs having identical frequencies. For the measurement of the

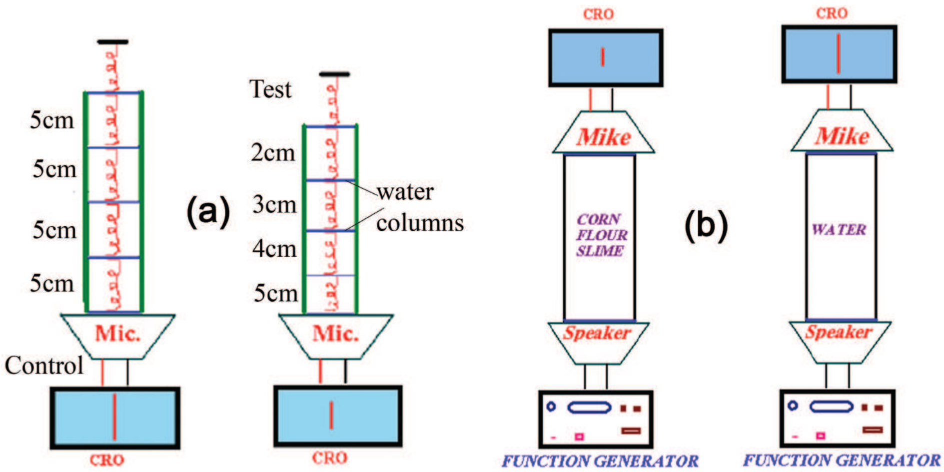

Figure 1. Schematic diagram of the experiments to detect (**a**) directional energy damping in an oscillator system with a frequency gradient and (**b**) shock-absorbing properties in corn-flour slime.

time period of oscillation a digital stop-clock was used. As the rubber membranes and water columns allow individual oscillators to retain their frequencies even in the presence of a driving force, fed at a slightly different frequency, it is possible to observe a gradient in phase change rate across the system, which can change the energy distribution of the propagating vibration.

The experiment to detect the shock-absorbing ability of corn-flour slime, as shown in figure 1b, consisted of a hollow tube of length 5 cm, covered by rubber membrane on one end and connected to an oscilloscope (5 MHz Tektronix 305) through a milli-Volt microphone on the other end. The tube was filled with corn-flour slime and placed on a speaker membrane connected to a function generator (Aplab-Model 2219U) to feed input vibrations. As discussed later, such a set-up would allow us to measure the amount of damping taking place within the system for different ranges of input frequencies and energies, thereby allowing us to detect shock-absorbing properties, if any.

3. Results and discussions

Our first objective was to analyse the nature of output vibrations as we change the input frequency and amplitude fed by a function generator. By increasing the input amplitude of the mechanical tap from 0.5 cm to 5 cm in steps of 0.5 cm in the oscillator system of figure 1a, variation of maximum output amplitude was recorded using the oscilloscope. The maximum output amplitude for the ‘test’ and ‘control’ systems were plotted against input amplitude to measure the amount of modulation taking place in the ‘test system’, as shown in figure 2a. Since the output amplitude in the ‘test system’ was consistently found to be lower than the ‘control’, it clearly suggests that the ‘test system’ shows a lower transmission than

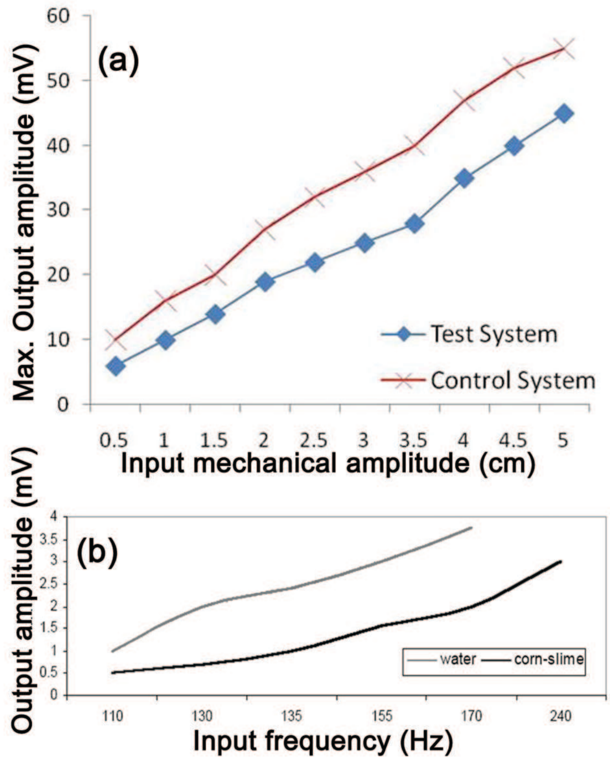

Figure 2. Maximum output amplitude of transmitted vibrations against input amplitude for (**a**) test and control systems for oscillator system and (**b**) water and corn-flour slime systems. It is obvious from the figure that both the test system of oscillators as well as the corn-flour system exhibited a lower output amplitude.

the 'control system', which does not have any frequency gradient in it. The result obtained was similar to that obtained in systems with auxiliary vibrators which have been found to exhibit low transmission efficiencies [12,13]. In these systems, the authors have used auxiliary components to distribute and dissipate energy thereby reducing energy transmission.

In order to measure the frequency alterations corresponding to amplitude fall, the oscillator systems were subjected to input vibrations from the mechanical tap and frequency changes were measured as the vibrations propagate through the oscillator system. To average out any experimental errors, time period for 10 oscillations was recorded for each of the spring oscillator in separate trials. The variation of mean time period for successive oscillators as a function of oscillator position, measured by taking the spring nearest to the input end as the first one, is plotted in figure 3. The theoretical values for their time period, calculated by considering them as independent oscillators are plotted in the same figure. It can be seen that the measured time period for successive oscillators vary in tune with the theoretical values of time period which they would have if they maintained their natural frequency.

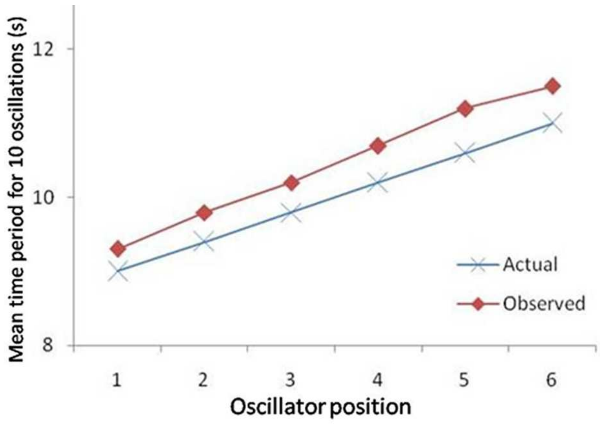

Figure 3. Plot of variation of time period of successive oscillators in the test system of oscillators as a function of oscillator position. The figure shows that observed frequencies vary in tune with the theoretical values suggesting that the oscillators retained their natural frequency.

This showed that individual oscillators retained their natural frequencies as long as the driving amplitude was below a threshold which was consistent with the theoretical studies by several researchers [14,15]. That the individual oscillators retained their natural frequencies may be argued as follows: The water column in between the springs in the oscillator system which acts as a coupling agent, also serves the purpose of allowing for a greater degree of independence in the oscillation of successive oscillators because of the rubber membranes as explained earlier. This also ensures that forced oscillations do not dominate between successive oscillators and hence allows the oscillators to retain their natural frequencies.

The next set of experiments were carried out to measure the extent of energy modulation in the oscillator system (figure 1a). In such oscillator systems, the intensity of vibration is given by $I = (E/AT) = 2\pi^2 f^2 a^2 \rho v$ where E is the energy carried through an area A in a time T and f is the frequency, a is the amplitude of vibration, ρ is the density of the medium and v is the distance travelled by the vibration in the given time. For one impulse which is measured by the maximum amplitude at the output end, $v = c/f$ and $T = 1/f$. Therefore, the energy of the output impulse $E = 2\pi^2 a^2 \rho c$ where amplitude is the only variable. Hence measurement of output amplitude can be directly related to the degree of energy modulation in our system. This allows us to get a measure of the degree of modulation in the 'test system' of oscillators by analysing how the ratio of output amplitude in the 'test system' to the output amplitude in the 'control system' varies as the input amplitude of the mechanical tap is increased from 0.5 cm to 5 cm in steps of 0.5 cm as before. The plot of degree of modulation so calculated against input mechanical amplitude in the 'test system' is shown in figure 4. It is noticed that the degree of shock absorbing displayed by the 'test system' gets stabilized and then shows a consistent dip after around 3.5 cm input mechanical amplitude as the driving amplitude increased. Since forced oscillations take over outside a narrow range of

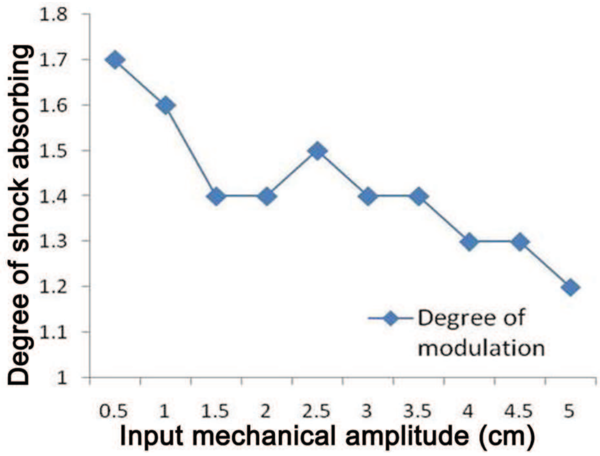

Figure 4. The variation of degree of modulation as the input mechanical amplitude of the 'test' system is increased. The degree of modulation was calculated as the ratio of output amplitudes in the test and control systems of oscillators. It can be seen that the degree of shock absorbing decreases as the amplitude increases.

energies, this dip was found to be consistent with theoretical predictions which use non-linear evolution equations to model forced oscillations [14,15].

We then tested the 'test system' for transmission efficiency by calculating the total energy of oscillations at the input and output ends for each trial. Each oscillation of the spring oscillator was assumed to carry energy proportional to the square of the amplitude and the total energy of oscillations in 10 s was calculated for the springs at the input and output ends. Energy transmission efficiency in the oscillator system was then calculated as the percentage of input energy that was transferred to the output end. The energy transmission efficiency in the 'test system' was found to vary between 50% and 65% for high frequency to low frequency transmission keeping the reverse transmission from low frequency to high frequency end negligible (<3%) for the same input amplitude at the starting end. It can be noted that Nunes [16] reported transmission efficiencies of 60% in composite metallic systems for both directions. In contrast, our system shows a similar transmission efficiency for high to low frequency transmission while the reverse transmission is severely damped adding to the shock-absorbing efficiency for low to high frequency transmission.

Though preferential energy transfer was apparent in the 'test system' which consisted of oscillators of unequal frequencies, it is important to look for the physics behind such observations. The most likely reason for preferential energy transfer could be the deviation of the relative phase difference between the adjacent oscillators from π, which is the value for phase difference between two coupled oscillators of identical frequencies. As a result, a part of the energy of every interaction goes in damping the vibration of the couple. For energy transmission towards low frequency end, phase change is negative since successive oscillators oscillate slower than their predecessors. Because of this, every time the high frequency oscillator thrusts a bit more energy to the low frequency oscillator than it would if there was no phase difference, thereby losing energy towards the low frequency end. For reverse transmission, phase change is positive. Therefore, the low frequency oscillator will lose less energy than it would if there was no phase change and thereby damping the transmission towards high frequency end. Also, since the energy per oscillation is more towards the low frequency end, the damping effect of the low frequency oscillator dominates that of the high frequency oscillator and hence the system exhibits a preferential energy transfer towards the low frequency end which is consistent with simulation results on particle acceleration damping [17]. Energy transfer in the case of stick-slip pendulums also exhibited similar behaviour as studied elsewhere [15].

Now to test for similar shock-absorbing properties in the corn-flour system of figure 1b, it was put to similar tests by changing the frequency and amplitude of the impulses fed from the frequency generator and recording the output amplitudes. The variation of maximum output amplitude with increasing input frequencies for corn-flour slime and water columns are shown in figure 2b. It clearly suggests a lower transmission for corn-flour slime in comparison with the water column. It should be noted that the transmission curves for water and corn-flour shown in figure 2b converge towards the low frequency end. This makes the system damp the input vibrations significantly only after a limiting frequency and amplitude is exceeded. In fact, reports showed that there existed a similar lower limit for energy for the formation of worm-like micelles in corn-flour slimes which alter their behaviour and properties [18]. However, the frequency of output vibrations measured in the oscilloscope was found to be identical with the frequency fed at the input end by the function generator. Thus no alteration in frequency was found for the corn-flour slime system between the input and output ends indicating that the observed shock-absorbing behaviour of non-Newtonian fluids might not be due to frequency gradient as one might expect.

In corn-flour slime, the flour particles get puffed up when a sudden pressure is applied to them, allowing extra water to enter their cavity. This not only creates a scarcity of water in the surrounding medium and makes it difficult for the particles to move, but also increases the size and mass of these particles resulting in a density gradient in the media [7]. The density decreases as we move away from the input end since the intensity of impulses reaching the point also decreases. It is to be noted that this density gradient does not translate into frequency gradient, as one would intuitively expect. But the density gradient creates a velocity gradient in the media since velocity inversely depends on the square root of density, $v \propto (1/\sqrt{\rho})$. The decreasing density gradient away from the input end creates an increasing velocity gradient due to which the wavefront accelerates as it propagates, resulting in the breaking of the wave [10]. Also some amount of energy is utilized in puffing up the particles and pushing the water into their cavity, and this energy is expressed as heat. The combined effect manifests as reduced energy transmission by the media. Studies elsewhere [19] indicate that a very identical system involving water–glycerin mixture, consistently gave higher energy dissipation than expected from thermal dissipation. Further, the dissipation curve also closely resembles the results we have obtained in figure 2b. This suggests that damping due to the frequency gradient that manifests in corn-flour slime might hold true for all such non-Newtonian fluids.

However it is to be noted that, while previous studies focus on energy dissipation, our experiments detect an effective damping purely because of the way the energy is presented at the output end. This is identical to one large blow being presented as several smaller blows at the output end, within the same period of time.

However, if the applied pressure on the corn-flour slime is lower than that required to push enough water from the surrounding into the corn-flour particles, no density gradient would be established eliminating shock absorption due to wavefront acceleration which requires the velocity in the medium to increase as the wave propagates forward. This implies that a lower limit for input energy must exist for effective shock absorbing to be seen in non-Newtonian fluids. The observed lower limit for input energy in our experiments supports the above argument. Such a lower limit for input energy is also found to exist for nonlinear vibration modes of elastic panels where the elastic properties of the panels come to picture only after a limiting input energy is exceeded [20].

4. Conclusion

The transmission efficiency, frequency and amplitude alteration have been measured in a system of coupled oscillators with a frequency gradient and in a system of non-Newtonian fluid. The experiments show directional energy damping in such systems, which are able to retain a frequency gradient. They also show that non-Newtonian fluids can behave as shock absorbers with a lower limit on energy scale after which they display shock-absorbing properties. In this context it would be interesting to find out the heat produced in the system as a function of input energy, which would give us a realistic estimate about the extent of shock absorbing coming from dissipative effects in the medium and also look for other non-Newtonian fluids like polyvinyl alcohol slimes, which are more stable, as a potential alternative. It is equally crucial and relevant to assess the reliability of the system adopted and to understand how these shock-absorbing properties scale with energy and frequency. Alternative approach would be to build oscillator systems that operate in successive frequency bands. All these investigations are already underway which would throw more light into the shock-absorbing properties of such an oscillator system with a frequency gradient.

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References

[1] W A Courtney and S O Oyadiji, *J. Mater. Process. Technol.* **119**, 379 (2001)
[2] A H El-Sinawi, *J. Sound Vib.* **271**, 323 (2004)

[3] M Terrones *et al*, *Appl. Phys. Lett.* **75**, 3932 (1999)
[4] A G Demeshkin, M E Kozeko, V M Kornev and V D Kurguzov, *J. Appl. Mech. Tech. Phys.* **42**, 169 (2001)
[5] S S Morye, P J Hine, R A Duckett, D J Carr and I M Ward, *Composites Sci. Technol.* **60**, 2631 (2000)
[6] W J Cantwell and J Morton, *Composites* **22**, 347 (1991)
[7] R P Chhabra, *Bubbles, drops, and particles in non-Newtonian fluids* (CRC Press, Boca Raton, FL, 2006)
[8] S Simonian, personal communication
[9] A Carpio and L L Bonilla, *Phys. Rev.* **E67**, 056621 (2003)
[10] M O Starzewski and J Trebickib, *Mech. Mat.* **38**, 840 (2006)
[11] D Halliday, R Resnik and J Walker, *Fundamentals of physics* (Wiley, New York, 2001) 6th ed.
[12] P Lorrain, *J. Appl. Phys.* **46**, 62 (1975)
[13] F Georgiades, A F Vakakis, M McFarland and L A Bergman, *Int. J. Bifurcat. Chaos* **15**, 1989 (2005)
[14] S Fifer, *J. Appl. Phys.* **22**, 1421 (1951)
[15] B O Enflo, C M Hedberg and O V Rudenko, *J. Acoustical Soc. Am.* **117**, 601 (2005)
[16] J Nunes, US Naval Res. Lab., Shock Vib. Bull, 20 (1976)
[17] T Hikihara, Y Kondo and Y Ueda, *IEICE Transactions on Fundamentals of Electronics* **E82-A**, 1701 (1999)
[18] E Miller and J P Rothstein, *J. Non-Newtonian Fluid Mech.* **143**, 22 (2007)
[19] W Tao Shi, C L Goodridge and D P Lathrop, *Phys. Rev.* **E56**, 4 (1997)
[20] A L Tukmakov, *J. Appl. Mech. Tech. Phys.* **41**, 171 (2000)