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A relativistic quark–diquark model for the nucleon

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Abstract. We developed a constituent quark–diquark model for the nucleon and its resonances using a harmonic oscillator potential for the interaction. The effects due to relativistic kinetic energy correction are studied. Finally, charge form factor of the model is calculated and compared with experimental data.

Keywords. Baryon spectrum; relativistic effects; quark–diquark model.

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1. Introduction

In the framework of phenomenological constituent quarks models (CQM), many works have been initially realized [1,2] with a nonrelativistic approach giving satisfactory results for the static properties of the nucleon and of its excited states. However, relativistic versions of the model have been proposed later improving the theoretical predictions [3,4], specially for the study of the baryonic electromagnetic observables.

Also quark–diquark models have been proposed for the study of the baryonic spectroscopy [5] to avoid the problem of missing resonances. The main objective of the present work is to develop an effective quark–diquark model, introducing in a perturbative way the kinetic energy relativistic correction that was not considered previously. Furthermore, we use a harmonic oscillator (HO) potential that directly ensures confinement for the quark–diquark system.

We also test how the model mass spectrum is affected by this correction. The diquark is assumed to be a single particle [5], with quantum numbers corresponding to two coupled quarks. This approximation has given good results also reducing the three-body problem to a two-body one. For the quark–diquark interaction we have chosen a HO potential which presents some advantages of mathematical type and, moreover, gives confinement as it is strictly required in quark physics. In summary, we show that the use of a confining HO potential and the relativistic energy correction can give a good description of the baryonic spectrum.

Finally, we calculate in the nonrelativistic approximation the proton charge form factor by means of the wave function obtained by solving the eigenvalue equation of the model. The comparison of the results with the experimental data clearly shows that a fully relativistic generalization of the model is required for calculating the wave function of the system and, moreover, the matrix elements of the electromagnetic current. The manifestly covariant spectator model for spin-1/2 particles [6] must be generalized to the present case where the diquark is a bosonic state with $S = 0$ or 1.

2. The Hamiltonian and the wave functions

Taking advantage of some existing nonrelativistic models [5], we propose the following Hamiltonian with relativistic corrections for the present quark–diquark model:

$$\begin{aligned} H = {} & E_0 + \frac{p^2}{2m} + \frac{1}{2}kr^2 + (B + D\delta_{N,0})\delta_{s_{12},1} + C\delta_{N,0} \\ & +2A(-1)^{l+1}\mathrm{e}^{-\lambda^2 r^2}[\vec{s}_{12}\cdot\vec{s}_3 + \vec{t}_{12}\cdot\vec{t}_3 + 2\vec{s}_{12}\cdot\vec{s}_3\vec{t}_{12}\cdot\vec{t}_3] \\ & -\frac{\rho}{r} - \frac{p^4}{8\eta^3}. \end{aligned} \tag{1}$$

The first term represents the constant rest energy of the system, the second one is the nonrelativistic kinetic energy in terms of the standard reduced mass $1/m = 1/m_{\mathrm{q}} + 1/m_{\mathrm{d}}$, where $m_{\mathrm{q/d}}$ represents the quark/diquark mass. The third term is the HO (analytically solvable) confining potential. The next term is used for a phenomenological description of the $N - \Delta$ splitting that is usually represented by the hyperfine interaction. Then, we have the so-called exchange interaction term. With respect to [5], in this term we have replaced an exponential spatial dependence with a Gaussian one in order to obtain analytic matrix elements in our HO basis. The standard Coulomb-like short distance term $-\rho/r$ [5] is also considered in our quark–diquark model. Finally, the p^4 term represents the relativistic correction to the kinetic energy with $1/\eta^3 = 1/m_{\mathrm{q}}^3 + 1/m_{\mathrm{d}}^3$, as obtained by a standard reduction of the quark and diquark relativistic energies. The Hamiltonian H is defined in the rest frame of the system and $\vec{p}$ represents the relative quark–diquark momentum.

The analytic solutions for the first three terms of the previous Hamiltonian are

$$\Psi_{N,l,m}(\vec{r}) = R_{N,l}(r)Y_{l,m}(\theta,\varphi) \tag{2}$$

with the radial wave functions

$$R_{N,l}(r) = \left[\frac{2(\frac{N-l}{2})!}{\Gamma(\frac{N+l+3}{2})}\right]^{1/2} \alpha^{-3/2}(\xi)^l \mathrm{e}^{-\xi^2/2} L^{l+\frac{1}{2}}_{\frac{N-l}{2}}(\xi^2). \tag{3}$$

In the previous equation α is the dimensional parameter of the wave function, related to the oscillator energy by $\alpha^2 = m\omega$; ξ is the dimensionless variable that is defined as $\xi = \alpha r$. $L^{l+\frac{1}{2}}_{\frac{N-l}{2}}(\xi^2)$ are the well-known Laguerre associate polynomials [7]. The corresponding energy eigenvalues are

$$E_{N,l} = \left(N + \frac{3}{2}\right)\omega + E_0,$$
$$N = 0, 1, 2, 3..., \quad l = N,\ N-2,\ N-4, ...(l \geq 0). \tag{4}$$

The spin and isospin wave functions can be obtained by coupling the corresponding quark–diquark quantities.

$$\chi^{s_{12}}_{S,S_3} = [\chi^{(1)}_{s_{12}} \otimes \chi^{(2)}_{\frac{1}{2}}]_{S,S_3}, \quad G_{S,S_3;T,T_3} = \chi^{s_{12}}_{S,M_s}\phi^{t_{12}}_{T,T_3}, \tag{5}$$

where the diquark spin and isospin are $s_{12} = t_{12} = 0, 1$. Finally, we can build the total wave function in the following way:

$$\Psi = R_{N,l}[Y_l \otimes [\chi_{s_{12}} \otimes \chi_{\frac{1}{2}}]_{S,S_3}]_{J,M} \cdot [\phi_{T_{12}} \otimes \phi_{\frac{1}{2}}]_{T,T_3}, \tag{6}$$

where J is the quantum number which appears when the orbital angular momentum and the spin are coupled. The states of the quark–diquark model can be expressed by means of the ket

$$|\Psi\rangle = |N, l, s_{12}, \tfrac{1}{2}, S; J, M; T_{12}, \tfrac{1}{2}; T, T_3\rangle. \tag{7}$$

3. The perturbative calculation

The contributions of the other terms of the Hamiltonian, that are calculated perturbatively for the r- and p-dependent terms, finally give the following equation for energy spectrum of the system:

$$E = E_0 + \left(N + \frac{3}{2}\right)\omega + (B + D\delta_{N,0})\delta_{s_{12},1} + C\delta_{N,0}$$
$$+2A(-1)^{l+1}F_1(\alpha,\lambda)F_2(S,T,s_{12},t_{12}) - F_3(\beta,\eta) - F_4(\rho,\alpha). \tag{8}$$

The explicit expressions for the matrix elements F_1, F_2, F_3, F_4 are the following:

$$F_1 = \int_0^\infty R_{n,l}(r)\mathrm{e}^{-\lambda^2 r^2}R_{n,l}(r)r^2\mathrm{d}r, \tag{9}$$

$$F_2 = [\langle\vec{s}_{12}\cdot\vec{s}_3\rangle + \langle\vec{t}_{12}\cdot\vec{t}_3\rangle + 2\langle\vec{s}_{12}\cdot\vec{s}_3\rangle\langle\vec{t}_{12}\cdot\vec{t}_3\rangle], \tag{10}$$

$$F_3 = \int_0^\infty R_{n,l}(p)\frac{p^4}{8\eta}R_{n,l}(p)p^2\mathrm{d}p, \tag{11}$$

$$F_4 = \int_0^\infty R_{n,l}(r)\frac{\rho}{r}R_{n,l}(r)r^2\mathrm{d}r. \tag{12}$$

In particular, F_2 is calculated by means of the standard Lande equation, i.e. $\langle\vec{s}_{12}\cdot\vec{s}_3\rangle = \frac{1}{2}[S(S+1) - s_{12}(s_{12}+1) - 3/4]$ and analogously for the isospin term. Furthermore, F_3 is easily obtained by using the spatial HO wave functions in the momentum representation [8]. All the integrals of the previous equations are calculated analytically in the HO basis.

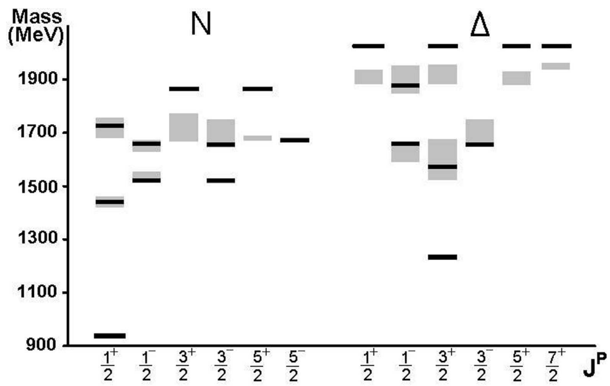

Figure 1. The calculated masses (black lines) and experimental masses (grey boxes).

Table 1. Fit of the relativistic correction $\Delta E_{\rm cal}$ as a function of the strength parameter. In the last line the experimental values are shown. All the entries are in MeV.

$\frac{\beta^{-4}}{16\eta^3}$	N(1440)	N(1535)	N(1680)
74	682.4	695.68	1108
75	667.8	690.68	1096
76	652.8	685.68	1084
78	622.83	675.68	1066
79	607.86	670.68	1021
80	592.84	665.68	1036
80.5	583.34	663.17	1030
$\Delta E_{\rm exp}$	492–532	582–617	712–812

4. The spectrum of the model

The energy spectrum is characterized by some free parameters. The following values have been chosen: $E_0 = 574$ MeV, $\omega = 840$ MeV, $B = 245$ MeV, $D = 207$ MeV, $C = 85$ MeV, $A = 125$ MeV, $\alpha = 1.875$ fm^{-1}, $\lambda = 1.64$ fm^{-1}, $(\beta^{-4}/16\eta^3) = 80.5$ MeV, $\rho = 320$ MeV $\cdot$ fm^{-1}. The result is shown in figure 1.

The model gives a good description in the lower part of the spectrum. However, for high energy states we find some discrepancies with respect to experimental data [9]. This fact can be due to the relativistic and field effects that have not been taken into account by the model.

In order to study how the relativistic corrections affect the spectrum, we analysed the effect of the strength parameter $\beta^{-4}/16\eta^3$. Specifically, we studied the calculated energy difference $\Delta E_{\rm cal}$ of the states N(1440), N(1535), N(1680) with respect to the nucleon. We started by using the value $(\beta^{-4}/16\eta^3) = 74$ MeV. Then we changed the value of the strength parameter until we obtained the best result. Such fit is shown in table 1.

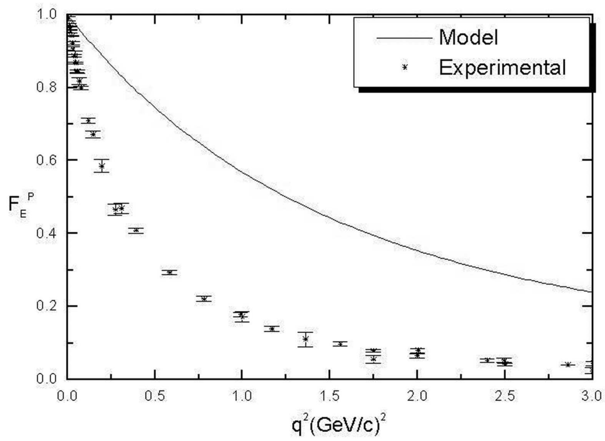

Figure 2. Proton charge form factor.

However, we stopped the fitting procedure at the strength value of 80.5 MeV, in order to keep the calculation in a correct perturbative regime. A different approach, such as a variational method, should be used to study larger relativistic effects.

5. Proton charge form factor

The form factor is defined nonrelativistically as the matrix element of the charge density transition operator. For the nucleon case, the electric form factor in the quark–diquark HO model takes the following form:

$$F_E(\vec{q}) = e_{\rm d}{\rm e}^{-(m_{\rm q}q/2\alpha M)^2} + e_{\rm q}{\rm e}^{-(m_{\rm d}q/2\alpha M)^2}, \tag{13}$$

where $e_{\rm q/d}$ is the quark/diquark charge. Also, we used the values of the parameters obtained for fitting the spectrum: $m_{\rm d} = 626.6$ MeV, $m_{\rm q} = 313.3$ MeV, $M = 940$ MeV, $\alpha = 370$ MeV. For the proton, the electric factor form is displayed in figure 2. We find that our quark–diquark HO form factor calculation decreases faster than the experimental data, in the same way as in the standard nonrelativistic quark models. A relativistic calculation of the four-current matrix element [6] is strictly needed to reproduce the experimental data. For consistency, also a fully relativistic wave function must be used.

We conclude that the results of this preliminary work, showing that the nucleon spectrum can be adequately reproduced by a quark–diquark HO model with relativistic corrections, motivate to develop a completely relativistic quark–diquark model for the nucleon.

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