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Bianchi type-V cosmological models with perfect fluid and heat flow in Saez–Ballester theory

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Abstract. In this paper we discuss the variation law for Hubble's parameter with average scale factor in a spatially homogeneous and anisotropic Bianchi type-V space-time model, which yields constant value of the deceleration parameter. We derive two laws of variation of the average scale factor with cosmic time, one is of power-law type and the other is of exponential form. Exact solutions of Einstein field equations with perfect fluid and heat conduction are obtained for Bianchi type-V space-time in these two types of cosmologies. In the cosmology with the power-law, the solutions correspond to a cosmological model which starts expanding from the singular state with positive deceleration parameter. In the case of exponential cosmology, we present an accelerating non-singular model of the Universe. We find that the constant value of deceleration parameter is reasonable for the present day Universe and gives an appropriate description of evolution of Universe. We have also discussed different types of physical and kinematical behaviour of the models in these two types of cosmologies.

Keywords. Cosmology; Hubble's parameter; deceleration parameter; heat conduction.

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1. Introduction

In the last few decades there has been much interest in alternative theories of gravitation, especially the scalar–tensor theories proposed by Brans and Dicke [1], Nordvedt [2], Barber [3], Saez and Ballester [4], Lau and Prokhovnik [5] etc. The latest inflationary models [6], extended inflation [7], hyper-extended inflation and extended chaotic inflation [8] are based on these scalar–tensor theories of gravitation. In Saez and Ballester theory [4], the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields in which an anti-gravity regime appears despite the dimensionless behaviour of the scalar field. This theory suggests a possible way to solve the missing matter problem in non-flat FRW models.

The cosmological models which are spatially homogeneous and anisotropic play significant roles in the description of the Universe at its early stages of evolution. Bianchi I–IX spaces are very useful tools in constructing spatially homogeneous cosmological models. The importance of Bianchi type-V models is due to the fact that the space of constant negative curvature is contained in it as a special case. Several authors have studied cosmological models based on Saez and Ballester [4] theory of gravitation. Singh and Agrawal [9,10] have studied Bianchi type I–IX models in the presence of perfect fluids. Reddy and Rao [11] and Reddy *et al* [12] have obtained solutions of Bianchi type-I models with perfect fluid in this theory.

It has been a subject of considerable interest to study the Universe near the Big-Bang singularity, which would be in a highly dense state having exotic and unusual behaviour. The distribution of matter in the Universe is essentially inhomogeneous and anisotropic. As the matter is not expected to attain thermal equilibrium in the early stages of the evolution of the Universe, evidently there would be heat flow in the Universe. The effect of heat flow in the evolution of the Universe has been investigated by several authors such as Deng [13], Mukherjee [14], Novello and Reboucas [15], Reboucas and Lima [16], Bradley and Sviestins [17]. Banerjee and Sanyal [18] have considered Bianchi type-V cosmologies with viscosity and heat flow. It has also been shown that it is possible for dissipative Bianchi type-V Universe models not to be in thermal equilibrium in their early stages. Coley [19] has investigated Bianchi type-V spatially homogeneous imperfect fluid cosmological models which contain both viscosity and heat flow. Coley and Hoogen [20] have also generalized the work of Coley and Dunn [21] who assumed a locally rotationally symmetric Bianchi type-V metric for an imperfect fluid source with both viscosity and heat conduction. Recently, Singh [22] has presented some new Bianchi type-V cosmological models in the presence of perfect fluid with heat flow.

The Einstein's field equation are a coupled system of highly non-linear differential equations and there are no standard methods for solving them. In order to obtain physically realistic solutions, one has to make assumptions generally at the cost of physics of the problem or for mathematical convenience. Solutions of field equations can be generated by applying the law of variation for Hubble's parameter proposed by Berman [23], which yields a constant value of deceleration parameter. The law of variation for Hubble's parameter gives a new approach for solving field equations that is quite general and suitable for the description of present day Universe. In this paper we obtain two categories of exact solutions of field equations with perfect fluid and heat flow in Saez and Ballester theory [4] for a Bianchi type-V space time, by using two distinct forms of the average scale factor, derived from the variation law of Hubble's parameter. We also discuss the physical and kinematical behaviour of the different parameters such as expansion scalar, anisotropy parameter and shear scalar in these two singular and non-singular cosmological models with constant deceleration parameter.

2. The model and basic equations

We consider the line element for an anisotropic Bianchi type-V metric, given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}[B^2(t)dy^2 + C^2(t)dz^2], \tag{1}$$

where A, B and C are metric functions and m is a constant.

The average scale-factor a, the spatial volume V and the generalized Hubble's parameter H for the space-time (1) are defined by

$$a = (ABC)^{1/3}, \tag{2}$$

$$V = a^3 = ABC, \tag{3}$$

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{4}$$

where $H_1 = \dot{A}/A$, $H_2 = \dot{B}/B$ and $H_3 = \dot{C}/C$ are the directional Hubble's parameters in the directions of x, y and z respectively. The dot denotes differentiation with respect to t.

From eqs (2)–(4), we have the important relation

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \tag{5}$$

We introduce the kinematical quantities such as the expansion (θ), the shear scalar (σ^2) and the anisotropy parameter (Am), defined as follows:

$$\theta = u^{\mu}_{;\mu}, \tag{6}$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu}, \tag{7}$$

$$Am = \frac{1}{3}\sum_{\mu=1}^{3}\left(\frac{H_\mu - H}{H}\right)^2, \tag{8}$$

where $u^\mu = (0, 0, 0, 1)$ is the matter 4-velocity vector and

$$\sigma_{\mu\nu} = \frac{1}{2}(u_{\mu;\alpha}P^\alpha_\nu + u_{\nu;\alpha}P^\alpha_\mu) - \frac{1}{3}\theta P_{\mu\nu}. \tag{9}$$

Here the projection tensor $P_{\mu\nu}$ has the form

$$P_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu. \tag{10}$$

These dynamical scalars, in Bianchi type-V, have the forms

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{11}$$

$$2\sigma^2 = \left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{3}. \tag{12}$$

An important quantity q, the deceleration parameter in cosmology, is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{13}$$

3. Field equations and their quadrature solutions

The field equations in the scalar–tensor theory, proposed by Saez and Ballester [4], are given by

$$G_{\mu\nu} - \omega\phi^r \left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2} g_{\mu\nu}\phi_{,k}\phi^{,k}\right) = -T_{\mu\nu}, \tag{14}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ and $8\pi G = c = 1$. The scalar field ϕ satisfies the equation

$$2\phi^r \phi^{;\mu}_{;\mu} + r\phi^{r-1}\phi_{,k}\phi^{,k} = 0. \tag{15}$$

Here r is an arbitrary constant and ω is a dimensionless coupling constant. Comma and semi-colon respectively denote ordinary and covariant derivative with respect to cosmic time t. $T_{\mu\nu}$ is the energy–momentum tensor of the matter.

The energy–momentum tensor is the source of the gravitational field through which the effect of the perfect fluid with heat flow in the evolution of the Universe is performed. The energy–momentum tensor of a perfect fluid with heat flow has the form given by [22]

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} + h_\mu u_\nu + h_\nu u_\mu, \tag{16}$$

where ρ is the energy density, p is the thermodynamic pressure, u_μ is the four-velocity of the fluid and h_μ is the heat flow vector satisfying

$$h^\mu u_\mu = 0. \tag{17}$$

We assume that the heat flow is in x direction only so that $h_\mu = (h_1, 0, 0, 0)$, h_1 being a function of time. Considering the form of the energy–momentum tensor (16), the Einstein's field equations (14), for the Bianchi type-V space-time (1) in Saez–Ballester theory, are given explicitly as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{18}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{19}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{m^2}{A^2} = -p + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{20}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3m^2}{A^2} = \rho - \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{21}$$

$$m\left(2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = h_1, \tag{22}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{r}{2\phi}\dot{\phi}^2 = 0. \tag{23}$$

From the energy conservation equation $T^{\nu}_{\mu;\nu} = 0$, we obtain

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{2m}{A^2}q_1. \tag{24}$$

Equations (18)–(21) can be written in terms of H, q, σ^2 and ϕ as

$$\rho = 3H^2 - \sigma^2 - \frac{3m^2}{A^2} + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \tag{25}$$

$$p = H^2(2q-1) - \sigma^2 + \frac{m^2}{A^2} + \frac{1}{2}\omega\phi^r\dot{\phi}^2. \tag{26}$$

Now, we follow the approaches of Saha and Rikhvitsky [24] and Singh and Chaubey [25] to solve the field equations (18)–(21). Subtracting eq. (18) from (19), eq. (18) from (20) and eq. (19) from (20), we get the following relations respectively:

$$\frac{A}{B} = d_1 \exp\left(k_1 \int \frac{\mathrm{d}t}{a^3}\right), \tag{27}$$

$$\frac{A}{C} = d_2 \exp\left(k_2 \int \frac{\mathrm{d}t}{a^3}\right), \tag{28}$$

$$\frac{B}{C} = d_3 \exp\left(k_3 \int \frac{\mathrm{d}t}{a^3}\right), \tag{29}$$

where d_1, d_2, d_3 and k_1, k_2, k_3 are constants of integration. From eqs. (27)–(29), the metric functions can be obtained explicitly as

$$A(t) = l_1 a \exp\left(\frac{X_1}{3}\int \frac{\mathrm{d}t}{a^3}\right), \tag{30}$$

$$B(t) = l_2 a \exp\left(\frac{X_2}{3}\int \frac{\mathrm{d}t}{a^3}\right), \tag{31}$$

$$C(t) = l_3 a \exp\left(\frac{X_3}{3}\int \frac{\mathrm{d}t}{a^3}\right), \tag{32}$$

where

$$l_1 = \sqrt[3]{d_1 d_2}, \quad l_2 = \sqrt[3]{{d_1}^{-1} d_3}, \quad l_3 = \sqrt[3]{(d_2 d_3)^{-1}},$$
$$X_1 = k_1 + k_2, \quad X_2 = k_3 - k_1, \quad X_3 = -(k_2 + k_3),$$

where the constants X_1, X_2, X_3 and l_1, l_2, l_3 satisfy the relations

$$X_1 + X_2 + X_3 = 0, \quad l_1 l_2 l_3 = 1. \tag{33}$$

The quadrature expression for the dimensionless scalar field function ϕ, from eq. (23), is found as

$$\phi = \left[\frac{\phi_0(r+2)}{2} \int \frac{\mathrm{d}t}{a^3}\right]^{2/(r+2)}, \tag{34}$$

where ϕ_0 is a constant.

It is clear from eqs (30)–(34) that once we get the value of the average scale factor a, we can easily calculate the metric functions A, B, C and the scalar function ϕ.

4. Variation law of Hubble's parameter

In order to find the exact solutions of the metric functions in terms of cosmic time t, we assume that the Hubble parameter H is related to the average scale factor a by the relation

$$H = la^{-n}, \tag{35}$$

where $l > 0$ and $n(\geq 0)$ are constants. Such type of relation has already been considered by Berman [23] and Berman and Gomide [26] for solving field equations in FRW models. Such relation gives a constant value of deceleration parameter. It may be noted that though the current observations of SNe Ia and CMB favour accelerating models ($q < 0$), they do not altogether rule out the decelerating ones which are also consistent with these observations. Later on, several authors (see ref. [27] and references therein) have considered FRW cosmological models with constant deceleration parameter. Singh and Kumar [27,28] and Kumar and Singh [29,30] have extended these types of works to anisotropic Bianchi types-I and II cosmological models in general relativity and some scalar–tensor theories. In this paper, our intention is to solve the field equations of Bianchi V perfect fluid with heat flow in the framework of Saez–Ballester theory of gravitation using relation (35), since this relation refers to Bianchi V space-time in any physical context.

From eqs (5) and (35), we get

$$\dot{a} = la^{-n+1}, \tag{36}$$

$$\ddot{a} = -l^2(n-1)a^{-2n+1}. \tag{37}$$

From eqs (13), (36) and (37), we obtain

$$q = n - 1. \tag{38}$$

Equation (38) indicates that the deceleration parameter q is constant. Thus the special law of variation of Hubble's parameter (35) yields a constant value of q.

The sign of q indicates whether the model inflates or not. The positive sign of q corresponds to standard decelerating model whereas the negative sign indicates inflation. Integrating eq. (36), we obtain the laws of variation for the average scale factor a as

$$a = (nlt + c_1)^{1/n}, \quad n \neq 0, \tag{39}$$

$$a = c_2 \exp(lt), \quad n = 0, \tag{40}$$

where c_1 and c_2 are integration constants. Equation (39) implies that the condition for expanding Universe is $n = 1 + q > 0$.

In the next section, we solve the quadrature equations (30)–(34) using the values of the average scale factor obtained in eqs (39) and (40) in two different physically viable cosmologies for $n \neq 0$ and $n = 0$ respectively, which are of physical interests in describing the decelerated and accelerated phases of the Universe.

5. Exact solutions

Case 1: *Power-law solution* (*when* $n \neq 0$)

Using the value of the average scale factor in eq. (39) into eqs (30)–(32), we obtain

$$A(t) = l_1 (nlt + c_1)^{1/n} \exp\left[\frac{X_1}{3l(n-3)}(nlt + c_1)^{(n-3)/n}\right], \tag{41}$$

$$B(t) = l_2 (nlt + c_1)^{1/n} \exp\left[\frac{X_2}{3l(n-3)}(nlt + c_1)^{(n-3)/n}\right], \tag{42}$$

$$C(t) = l_3 (nlt + c_1)^{1/n} \exp\left[\frac{X_3}{3l(n-3)}(nlt + c_1)^{(n-3)/n}\right]. \tag{43}$$

The solution for the scalar function ϕ, from eq. (34), is obtained as

$$\phi = \left[\frac{\phi_0 (r+2)}{2l(n-3)}\right]^{2/(r+2)} \cdot (nlt + c_1)^{2(n-3)/n(r+2)}. \tag{44}$$

By using the values of the metric functions in eqs (41)–(43) into eq. (22), the expression for the heat flow function h_1 is given by

$$h_1 = m X_1 (nlt + c_1)^{-3/n}. \tag{45}$$

The above solutions are valid for $n \neq 3$. The dynamical scalars θ and σ^2 are given as

$$\theta = 3l(nlt + c_1)^{-1} \tag{46}$$

$$\sigma^2 = \frac{(X_1^2 + X_2^2 + X_3^2)}{18}(nlt + c_1)^{-6/n}. \tag{47}$$

The directional Hubble's parameters have the following expressions:

$$H_1 = l(nlt + c_1)^{-1} + \frac{X_1}{3}(nlt + c_1)^{-3/n}, \tag{48}$$

$$H_2 = l(nlt + c_1)^{-1} + \frac{X_2}{3}(nlt + c_1)^{-3/n}, \tag{49}$$

$$H_3 = l(nlt + c_1)^{-1} + \frac{X_3}{3}(nlt + c_1)^{-3/n}. \tag{50}$$

Also the average generalized Hubble's parameter is given by

$$H = l(nlt + c_1)^{-1}. \tag{51}$$

The anisotropy parameter Am is given by

$$Am = \frac{(X_1^2 + X_2^2 + X_3^2)}{27l^2}(nlt + c_1)^{2(n-3)/n}. \tag{52}$$

The spatial volume is given as

$$V = (nlt + c_1)^{3/n} \exp(2mx). \tag{53}$$

From eqs (25) and (26), the energy density and pressure are given by

$$\begin{aligned}\rho = {} & 3l^2(nlt + c_1)^{-2} + \left[\frac{1}{2}\omega\phi_0^2 - \frac{(X_1^2 + X_2^2 + X_3^2)}{18}\right](nlt + c_1)^{-6/n} \\ & - \frac{3m^2}{l_1^2}(nlt + c_1)^{-2/n} \exp\left[\frac{-2X_1}{3l(n-3)}(nlt + c_1)^{(n-3)/n}\right],\end{aligned} \tag{54}$$

$$\begin{aligned}p = {} & (2n-3)l^2(nlt + c_1)^{-2} + \left[\frac{1}{2}\omega\phi_0^2 - \frac{(X_1^2 + X_2^2 + X_3^2)}{18}\right] \\ & \times (nlt + c_1)^{-6/n} + \frac{m^2}{l_1^2}(nlt + c_1)^{-2/n} \\ & \times \exp\left[\frac{-2X_1}{3l(n-3)}(nlt + c_1)^{(n-3)/n}\right],\end{aligned} \tag{55}$$

provided $n \neq 3$.

From eqs (54) and (55), we find that

$$\frac{\mathrm{d}p}{\mathrm{d}\rho} = \frac{M}{N}, \tag{56}$$

where

$$
\begin{aligned}
M = & -2nl^3(2n-3)(nlt+c_1)^{-3} \\
& + \left[\frac{(X_1^2+X_2^2+X_3^2)l}{3} - 3\omega\phi_0^2 l\right](nlt+c_1)^{-(6/n)-1} \\
& - \frac{2m^2 l}{l_1^2}(nlt+c_1)^{-(2/n)-1}\exp\left[\frac{-2X_1}{3l(n-3)}(nlt+c_1)^{(n-3)/n}\right] \\
& - \frac{2m^2 X_1}{3l_1^2}(nlt+c_1)^{-5/n}\exp\left[\frac{-2X_1}{3l(n-3)}(nlt+c_1)^{(n-3)/n}\right], \qquad (57)
\end{aligned}
$$

$$
\begin{aligned}
N = & -6nl^3(nlt+c_1)^{-3} \\
& + \left[\frac{(X_1^2+X_2^2+X_3^2)l}{3} - 3\omega\phi_0^2 l\right](nlt+c_1)^{-(6/n)-1} \\
& + \frac{6m^2 l}{l_1^2}(nlt+c_1)^{-(2/n)-1}\exp\left[\frac{-2X_1}{3l(n-3)}(nlt+c_1)^{(n-3)/n}\right] \\
& + \frac{2m^2 X_1}{l_1^2}(nlt+c_1)^{-5/n}\exp\left[\frac{-2X_1}{3l(n-3)}(nlt+c_1)^{(n-3)/n}\right]. \qquad (58)
\end{aligned}
$$

For the physical reality, we must have $\mathrm{d}p/\mathrm{d}\rho \leq 1$ to determine the region for the physical validity of the solution. This leads to a lengthy and complicated inequation which is very difficult to solve to determine the region for the physical validity of the solution.

It is observed that the energy conservation equation (24) is identically satisfied. Initially at $t = t_1$, where $t_1 = -c_1/nl$, we observe that the spatial volume and all the three scale factors are zero. The expansion scalar, the anisotropy parameter for $n < 3$ and the heat function h_1 are all infinite at this epoch. This shows that the Universe starts evolving with zero volume and infinite heat flow at $t = t_1$ and expands with an infinite rate of expansion. Also the anisotropy of expansion is infinite for $n < 3$ and will be zero for $n > 3$, at this point. The energy density and pressure along with shear scalar are infinite at $t = t_1$, which clearly indicate the point singularity at this epoch. The directional Hubble's parameters and the generalized Hubble's parameter are both infinite at this singularity point. The scalar field function (ϕ) is infinity for $n < 3$ and zero for $n > 3$ at this initial singularity. The Universe follows the power-law expansion just after the Big-Bang singularity. With the increase in cosmic time t, the volume scalar and all three scale factors increase and will become infinite when $t \to \infty$. The expansion scalar and the anisotropy parameter for $n < 3$ decrease with time and will completely be zero for the large time. This shows that the expansion of the Universe slows down with the decrease in anisotropy. For the large time, the expansion will be completely finished and the model will become isotropic as $\lim_{t\to\infty}(\sigma^2/\theta) \to 0$. The energy density, pressure and shear scalar decrease with time and will be zero when the time reaches maximum which is the indication of the empty Universe for the late time of its evolution. There will not be any heat flow for $t \to \infty$. The heat flow diminishes as time increases. The directional Hubble's parameter, the average generalized Hubble's parameter along with shear scalar will be zero at this epoch. There will not be any scalar field for the large time provided $n < 3$. Therefore, this is an expanding model of the Universe with initial Big-Bang start approaching isotropy at late times.

Case 2: *Exponential solution* (*when* $n = 0$): In this case, we study an exponentially expanding non-singular cosmological model. The values of the metric functions (30)–(32), by using the value of average scale factor from eq. (40), are given as

$$A(t) = c_2 l_1 \exp\left[lt - \frac{X_1}{9lc_2^3}\exp(-3lt)\right], \tag{59}$$

$$B(t) = c_2 l_2 \exp\left[lt - \frac{X_2}{9lc_2^3}\exp(-3lt)\right], \tag{60}$$

$$C(t) = c_2 l_3 \exp\left[lt - \frac{X_3}{9lc_2^3}\exp(-3lt)\right]. \tag{61}$$

The scalar function is obtained as

$$\phi = \left[\frac{\phi_0(r+2)}{6lc_2^3}\right]^{2/(r+2)} \exp\left(-\frac{6lt}{r+2}\right). \tag{62}$$

The expression for the heat flow (h_1) is given by

$$h_1 = \frac{mX_1}{c_2^3}\exp(-3lt). \tag{63}$$

The dynamical scalars θ and σ^2 are given as

$$\theta = 3l, \tag{64}$$

$$\sigma^2 = \frac{(X_1^2 + X_2^2 + X_3^2)}{18c_2^6}\exp(-6lt). \tag{65}$$

The directional Hubble's parameters H_1, H_2 and H_3 are obtained as

$$H_1 = l + \frac{X_1}{3c_2^3}\exp(-3lt), \tag{66}$$

$$H_2 = l + \frac{X_2}{3c_2^3}\exp(-3lt), \tag{67}$$

$$H_3 = l + \frac{X_3}{3c_2^3}\exp(-3lt), \tag{68}$$

whereas the average generalized Hubble's parameter $H = l$.

The anisotropy parameter is calculated as

$$Am = \frac{(X_1^2 + X_2^2 + X_3^2)}{27l^2 c_2^6} \exp(-6lt). \tag{69}$$

The spatial volume and the deceleration parameter in this case, are obtained as

$$V = c_2^3 \exp(2mx + 3lt), \tag{70}$$

$$q = -1. \tag{71}$$

The values of the energy density and pressure are obtained as

$$\rho = 3l^2 + \left[\frac{1}{2}\omega\frac{\phi_0^2}{c_2^6} - \frac{(X_1^2 + X_2^2 + X_3^2)}{18c_2^6}\right] \exp(-6lt) - \frac{3m^2}{c_2^2 l_1^2} \exp\left[-2\left(lt - \frac{X_1}{9c_2^3 l}\exp(-3lt)\right)\right], \tag{72}$$

$$p = -3l^2 + \left[\frac{1}{2}\omega\frac{\phi_0^2}{c_2^6} - \frac{(X_1^2 + X_2^2 + X_3^2)}{18c_2^6}\right] \exp(-6lt) + \frac{m^2}{c_2^2 l_1^2} \exp\left[-2\left(lt - \frac{X_1}{9c_2^3 l}\exp(-3lt)\right)\right]. \tag{73}$$

From eqs (72) and (73), we have

$$\frac{\mathrm{d}p}{\mathrm{d}\rho} = \frac{P}{Q}, \tag{74}$$

where

$$P = \left[\frac{(X_1^2 + X_2^2 + X_3^2)l}{3c_2^6} - \frac{3\omega\phi_0^2 l}{c_2^6}\right] \exp(-6lt) - \frac{2m^2}{c_2^2 l_1^2} \times \left[l + \frac{X_1}{3c_2^3}\exp(-3lt)\right] \cdot \exp\left[-2\left(lt - \frac{X_1}{9c_2^3 l}\exp(-3lt)\right)\right], \tag{75}$$

$$Q = \left[\frac{(X_1^2 + X_2^2 + X_3^2)l}{3c_2^6} - \frac{3\omega\phi_0^2 l}{c_2^6}\right] \exp(-6lt) + \frac{6m^2}{c_2^2 l_1^2} \times \left[l + \frac{X_1}{3c_2^3} \cdot \exp(-3lt)\right] \cdot \exp\left[-2\left(lt - \frac{X_1}{9c_2^3 l}\exp(-3lt)\right)\right]. \tag{76}$$

It is easily seen that the physical reality condition $\mathrm{d}p/\mathrm{d}\rho$ is less than unity.

The conservation equation (24) is identically satisfied. It can easily be observed that the spatial volume, all three scale factors, the energy density, pressure, the scalar field function and all other physical and kinematical parameters along with the heat flow are constants at $t = 0$. This shows that the model is free from the initial singularity. The expansion scalar is constant throughout the evolution. This indicates that the Universe starts evolving with constant volume and expands with exponential rate. There is a constant uniform rate of expansion in this model. The negative value of q indicates inflation. As the cosmic time increases, the spatial volume and all the three metric functions increase and will become infinity for time $t \to \infty$. The heat flow dies out as time increases. The scalar function, shear scalar and the anisotropy parameter decrease with time and will finally be zero for the maximum time. This shows that the model will become isotropic for the late time evolution since the limiting value of σ^2/θ is 0 as $t \to \infty$. The energy density, pressure, the directional Hubble's parameters and the average generalized Hubble's parameter will become constant for large time. Thus, we will have the relation $p = -\rho$ for maximum cosmic time which shows that at late times the Universe is dominated by vacuum energy which drives the expansion of the Universe [31–34]. Therefore, this model indicates that the Universe starts evolving with constant spatial volume and expands exponentially with constant rate of expansion and will finally approach isotropy at late times.

6. Conclusion

We have presented two categories of Bianchi type-V cosmological solutions of the field equations with perfect fluid and heat flow in Saez and Ballester theory of gravitation using the power-law form and the exponential type of the average scale factor derived from the variation law of Hubble's parameter, which gives a constant value of the deceleration parameter. In the first category of models, the Universe begins to start expanding from the singular state. The rate of expansion slows down and vanishes as $t \to \infty$. The scale function ϕ decreases with time for $n < 3$. The model represents a shearing, non-rotating and expanding Universe which approaches isotropy for large values of t. In the second category of models, the Universe has no singular state. The Universe starts expanding with constant expansion rate from a constant volume where all physical quantities are well-behaved. We have observed that the model shows equation of state $p = -\rho$ for large time which is responsible for the acceleration of Universe. We have also discussed the physical and kinematical properties of the cosmological models of the Universe in these two types of cosmologies. The models presented in this paper could give an appropriate description of the evolution of the Universe. More realistic models may be analysed by using this technique, which may lead to interesting and different physical behaviour of the evolution of the Universe.

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