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Revisiting non-degenerate parametric down-conversion

JOSEPH AKEYO OMOLO[1]
Department of Physics, Maseno University, P.O. Private Bag, Maseno, Kenya
[1]Abdus Salam ICTP, Strada Costiera, 11-34014, Trieste, Italy
E-mail: ojakeyo@maseno.ac.ke; ojakeyo04@yahoo.co.uk

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Abstract. The quantum dynamics of a two-mode non-resonant parametric down-conversion process is studied by recasting the time evolution equations for the basic operators in an equivalent spin equation form with simpler exact solutions for a pump field with harmonic time dependence. Expectation values of suitable operators for studying important features such as squeezing and quantum revivals are presented in simple forms.

Keywords. Parametric down-conversion; squeezing; quantum revivals.

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1. Introduction

We consider a model of non-degenerate parametric down-conversion process composed of two coupled linear harmonic oscillators described by a two-mode Hamiltonian of the form

$$\hat{H} = \hbar\omega_a\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \hbar\omega_b\left(\hat{b}^\dagger\hat{b} + \frac{1}{2}\right) + i\hbar(g(t)\hat{a}\hat{b} - g^*(t)\hat{a}^\dagger\hat{b}^\dagger), \tag{1}$$

where $(\hat{a}, \hat{a}^\dagger)$ and $(\hat{b}, \hat{b}^\dagger)$ are the annihilation and creation operators of the oscillators, while the time-dependent coupling parameter $g(t)$ describes an arbitrary classical pump field.

The model described by the Hamiltonian in eq. (1) has been widely studied in quantum optics, particularly under resonance conditions [1–3] where the time evolution equations are easily solved. The more general non-resonant two-mode parametric down-conversion process was studied in detail by Rekdal and Skagerstam [4]. The Lie algebraic method was applied to solve the equation for the time evolution operator within the interaction picture. Besides providing the desired solutions in terms of fairly complicated expressions, the method also involves application of operator expansion theorems, making the calculations quite tedious.

In the present paper, we develop a simpler approach by considering that the basic operators of the coupled system are the annihilation and creation operators whose time evolution can be determined directly through Heisenberg's equation,

$$i\hbar\frac{\mathrm{d}\hat{Q}}{\mathrm{d}t}=[\hat{Q},\hat{H}(t)], \tag{2}$$

for an operator $\hat{Q}$. In this respect, we set $\hat{Q}=\hat{a};\,\hat{b}^{\dagger}$ and use $H(t)$ from eq. (1) in eq. (2) to obtain the time evolution equations,

$$i\hbar\frac{\mathrm{d}\hat{a}}{\mathrm{d}t}=\hbar(\omega_a\hat{a}-ig^{*}(t)\hat{b}^{\dagger}), \tag{3a}$$

$$i\hbar\frac{\mathrm{d}\hat{b}^{\dagger}}{\mathrm{d}t}=-\hbar(\omega_b\hat{b}^{\dagger}+ig(t)\hat{a}). \tag{3b}$$

We solve these equations through a matrix method, which provides an appropriate time evolution matrix.

Introducing a matrix,

$$A=\begin{pmatrix}\hat{a}\\ \hat{b}^{\dagger}\end{pmatrix}, \tag{4}$$

we express eqs (3a) and (3b) in the matrix form,

$$i\hbar\frac{\mathrm{d}A}{\mathrm{d}t}=\mathcal{H}(t)A, \tag{5a}$$

where the matrix

$$\mathcal{H}(t)=\hbar\begin{pmatrix}\omega_a & -ig^{*}(t)\\ -ig(t) & -\omega_b\end{pmatrix}, \tag{5b}$$

plays the role of a time evolution generator.

1.1 *Equivalent spin equation*

Introducing the Pauli matrices

$$\sigma_x=\begin{pmatrix}0&1\\1&0\end{pmatrix};\quad \sigma_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix};\quad \sigma_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}, \tag{6a}$$

with

$$\sigma_+=\sigma_x+i\sigma_y=\begin{pmatrix}0&2\\0&0\end{pmatrix};\quad \sigma_-=\sigma_x-i\sigma_y=\begin{pmatrix}0&0\\2&0\end{pmatrix};\quad I=\begin{pmatrix}1&0\\0&1\end{pmatrix} \tag{6b}$$

$$\sigma_+\sigma_- = 4\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = 2(I+\sigma_z); \quad \sigma_-\sigma_+ = 4\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = 2(I-\sigma_z), \tag{6c}$$

we express the time evolution generator $\mathcal{H}(t)$ in eq. (5b) in the interaction form

$$\begin{aligned}\mathcal{H}(t) &= \frac{1}{2}\hbar\Delta I + \frac{1}{2}\hbar E\sigma_z - \frac{i}{2}\hbar(g(t)\sigma_- + g^*(t)\sigma_+) \\ &= \frac{1}{2}\hbar\Delta I + \hbar\mathbf{B}(t)\cdot\vec{\sigma},\end{aligned} \tag{7a}$$

after defining

$$\Delta = \omega_a - \omega_b; \quad E = \omega_a + \omega_b \tag{7b}$$

and introducing a vector $\mathbf{B}(t)$ and a Pauli matrix vector $\vec{\sigma}$ defined by

$$\mathbf{B}(t) = \left(-ig_{\mathrm{R}}(t), -ig_{\mathrm{I}}(t), \frac{1}{2}E\right); \quad \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z), \tag{7c}$$

where $g_{\mathrm{R}}(t)$ and $g_{\mathrm{I}}(t)$ are the real and imaginary parts of $g(t)$ defined as usual by

$$g(t) = g_{\mathrm{R}}(t) + ig_{\mathrm{I}}(t); \quad g^*(t) = g_{\mathrm{R}}(t) - ig_{\mathrm{I}}(t). \tag{7d}$$

The time evolution generator $\mathcal{H}(t)$ is seen to be equivalent to the Hamiltonian of a spin-$\frac{1}{2}$ or a two-level system interacting with an external field [5,6] $\mathbf{B}(t)$. The interaction is characterized by the pump parameter $g(t)$. This means that eq. (5a) is a spin equation governing the dynamics of a spin-$\frac{1}{2}$ or two-level system equivalent to the non-degenerate parametric down-conversion process. Such spin-$\frac{1}{2}$ or two-level systems constitute qubits for implementation of quantum computation [7–9].

2. General solution

The desired general solution to eq. (5a) can be obtained by specifying the form of the pump parameter $g(t)$ and then applying the usual procedures for solving spin equations of similar form. Following ref. [4], we consider the pump parameter to have harmonic time dependence in the form

$$g(t) = g\mathrm{e}^{i\omega t}; \quad g^*(t) = g\mathrm{e}^{-i\omega t}; \quad g = |g(t)| = \text{constant}, \tag{8}$$

which we use in eq. (7a) to express the time evolution generator in the form

$$\mathcal{H}(t) = \frac{1}{2}\hbar\Delta I + \frac{1}{2}\hbar E\sigma_z - \frac{i}{2}\hbar g(\mathrm{e}^{i\omega t}\sigma_- + \mathrm{e}^{-i\omega t}\sigma_+). \tag{9}$$

We move to the rotating frame through the application of a unitary matrix

$$T(t) = \mathrm{e}^{\frac{i}{2}\omega t\sigma_z}; \quad T^\dagger(t) = T^{-1}(t) = \mathrm{e}^{-\frac{i}{2}\omega t\sigma_z}, \tag{10}$$

so that under the unitary transformations (equivalent to rotation through ωt about the z-axis),

$$\bar{A} = TA; \quad A = T^\dagger \bar{A}; \quad \bar{H} = T\mathcal{H}(t)T^\dagger - i\hbar T \frac{\mathrm{d}T^\dagger}{\mathrm{d}t}, \tag{11}$$

we express eq. (5a) in the form

$$i\hbar \frac{\mathrm{d}\bar{A}}{\mathrm{d}t} = \bar{H}\bar{A}. \tag{12}$$

Noting that

$$T = \mathrm{e}^{\frac{i}{2}\omega t \sigma_z} = \begin{pmatrix} \mathrm{e}^{\frac{i}{2}\omega t} & 0 \\ 0 & \mathrm{e}^{-\frac{i}{2}\omega t} \end{pmatrix}; \quad T^\dagger T = TT^\dagger = I, \tag{13}$$

we easily obtain

$$T\sigma_z T^\dagger = \sigma_z; \quad T\mathrm{e}^{\frac{i}{2}\omega t}\sigma_- T^\dagger = \sigma_-; \quad T\mathrm{e}^{-\frac{i}{2}\omega t}\sigma_+ T^\dagger = \sigma_+, \tag{14}$$

which we use in eq. (11), with $\mathcal{H}(t)$ as defined in eq. (9) to obtain

$$\bar{H} = \frac{1}{2}\hbar\Delta I + \frac{1}{2}\hbar(E - \omega)\sigma_z - i\hbar g \sigma_x. \tag{15}$$

With all parameters Δ, E, ω and g constant, the time evolution generator $\bar{H}$ in the rotating frame is time independent.

We put $\bar{H}$ from eq. (15) in eq. (12) and then introduce

$$\Omega = \omega - E = \omega - \omega_a - \omega_b; \quad \vec{\beta} = \left(-g, 0, \frac{i}{2}\Omega\right), \tag{16}$$

to write

$$\frac{\mathrm{d}\bar{A}}{\mathrm{d}t} = \left(-\frac{i}{2}\Delta I + \vec{\beta}\cdot\vec{\sigma}\right)\bar{A}, \tag{17}$$

which is easily integrated to obtain

$$\bar{A}(t) = \mathrm{e}^{-\frac{i}{2}\Delta t}\mathrm{e}^{\vec{\beta}\cdot\vec{\sigma}t}\bar{A}(0). \tag{18}$$

We use a standard result [10],

$$\mathrm{e}^{\vec{\beta}\cdot\vec{\sigma}t} = I\cosh\beta t + \hat{\beta}\cdot\vec{\sigma}\sinh\beta t, \tag{19a}$$

where

$$\beta = \sqrt{\beta_x^2 + \beta_y^2 + \beta_z^2}; \quad \hat{\beta} = \frac{\vec{\beta}}{\beta}; \quad \hat{\beta}\cdot\ \vec{\sigma} = \frac{1}{\beta}\begin{pmatrix} \frac{i}{2}\Omega & -g \\ -g & -\frac{i}{2}\Omega \end{pmatrix}. \tag{19b}$$

We use $\vec{\beta}$ from eq. (16) to obtain

$$\beta = \pm g\sqrt{1-k^2}; \quad k = \frac{\Omega}{2g}, \tag{19c}$$

which on substituting into eq. (19a) gives

$$\mathrm{e}^{\vec{\beta}\cdot\vec{\sigma}t} = \begin{pmatrix} \cosh\beta t + i\dfrac{\Omega}{2\beta}\sinh\beta t & -\dfrac{g}{\beta}\sinh\beta t \\ -\dfrac{g}{\beta}\sinh\beta t & \cosh\beta t - i\dfrac{\Omega}{2\beta}\sinh\beta t \end{pmatrix}. \tag{20}$$

Multiplying eq. (18) by $T^\dagger(t)$ from the left and using eq. (11) together with

$$T(0) = I; \quad A(0) = T^\dagger(0)\bar{A}(0) = \bar{A}(0), \tag{21}$$

we obtain

$$A(t) = U(t)A(0), \tag{22a}$$

after introducing a time evolution matrix $U(t)$ defined by

$$U(t) = \mathrm{e}^{-\frac{i}{2}\Delta t}T^\dagger(t)\mathrm{e}^{\vec{\beta}\cdot\vec{\sigma}t}; \quad U(0) = I. \tag{22b}$$

We apply eq. (13) for $T^\dagger(t)$ and then use eq. (20), together with the definitions,

$$\mu(t) = \cosh\beta t + i\frac{\Omega}{2\beta}\sinh\beta t; \quad \nu(t) = -\frac{g}{\beta}\sinh\beta t, \tag{23}$$

to express the time evolution matrix in eq. (22b) in the form

$$U(t) = \begin{pmatrix} \mu(t)\mathrm{e}^{-\frac{i}{2}(\Omega+2\omega_a)t} & \nu(t)\mathrm{e}^{-\frac{i}{2}(\Omega+2\omega_a)t} \\ \nu(t)\mathrm{e}^{\frac{i}{2}(\Omega+2\omega_b)t} & \mu^*(t)\mathrm{e}^{\frac{i}{2}(\Omega+2\omega_b)t} \end{pmatrix}, \tag{24}$$

where we have used

$$\omega + \Delta = \Omega + 2\omega_a; \quad \omega - \Delta = \Omega + 2\omega_b,$$

according to the definitions in eqs (7b) and (16).

Using eq. (24) in eq. (22a) with mode operators at initial time denoted by $\hat{a}(0) = \hat{a}$, $\hat{b}^\dagger(0) = \hat{b}^\dagger$, we obtain the general non-resonant solutions to eqs (3a) and (3b) in the form $(\mu = \mu(t), \nu = \nu(t))$,

$$\hat{a}(t) = \mathrm{e}^{-\frac{i}{2}(\Omega+2\omega_a)t}(\mu\hat{a} + \nu\hat{b}^\dagger); \quad \hat{b}^\dagger(t) = e^{\frac{i}{2}(\Omega+2\omega_b)t}(\mu^*\hat{b}^\dagger + \nu\hat{a}). \tag{25}$$

These solutions take much simpler form compared to the solutions obtained in ref. [4] through the Lie algebraic method. The results are valid for all values $k^2 = 0$, $k^2 < 1$, $k^2 = 1$, and $k^2 > 1$, in accordance with the definition of β in eq. (19c).

We now check the consistency of our solutions. We start by considering resonance as a special case. Under resonance, we have

$$\omega = \omega_a + \omega_b, \quad \Omega = 0; \quad k = 0; \quad \beta = g, \tag{26}$$

which we use in eqs (23) and (25) to obtain the resonant solutions

$$\hat{a}(t) = e^{-i\omega_a t}(\cosh(gt)\hat{a} - \sinh(gt)\hat{b}^\dagger);$$
$$\hat{b}^\dagger(t) = e^{i\omega_b t}(\cosh(gt)\hat{b}^\dagger - \sinh(gt)\hat{a}), \tag{27}$$

which agree exactly with the result obtained in ref. [4]. We observe that factors $e^{-i\omega_a t}$ and $e^{i\omega_b t}$ have been left out in writing down the final results in ref. [4], even though these factors occur in the expressions at intermediate stages of the calculations.

Next, we take appropriate (Hermitian) conjugations of eq. (25) to obtain

$$\hat{a}^\dagger(t) = e^{\frac{i}{2}(\Omega+2\omega_a)t}(\mu^*\hat{a}^\dagger + \nu\hat{b}); \quad \hat{b}(t) = e^{-\frac{i}{2}(\Omega+2\omega_b)t}(\mu\hat{b} + \nu\hat{a}^\dagger), \tag{28}$$

where $\nu^*(t) = \nu(t)$ in accordance with eq. (23). We then use eqs (25) and (28) to obtain

$$[\hat{a}(t), \hat{a}^\dagger(t)] = [\hat{b}(t), \hat{b}^\dagger(t)] = |\mu(t)|^2 - \nu^2(t), \tag{29a}$$

$$\hat{a}^\dagger(t)\hat{a}(t) - \hat{b}^\dagger(t)\hat{b}(t) = (|\mu(t)|^2 - \nu^2(t))(\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}). \tag{29b}$$

Using eq. (23), with $\beta^2 = g^2 - \frac{\Omega^2}{4}$ from eq. (19c), we obtain

$$|\mu(t)|^2 - \nu^2(t) = 1, \tag{30}$$

which in eqs (29a) and (29b) leads to the desired consistency results,

$$[\hat{a}(t), \hat{a}^\dagger(t)] = 1; \quad [\hat{b}(t), \hat{b}^\dagger(t)] = 1, \tag{31a}$$

$$\hat{a}^\dagger(t)\hat{a}(t) - \hat{b}^\dagger(t)\hat{b}(t) = \hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b} = \text{constant}. \tag{31b}$$

The first of these consistency conditions, eq. (31a), is the fundamental quantum commutation bracket which must be satisfied at all times, while the second condition in eq. (31b) governs the simultaneous production of signal and idler photons in the parametric down-conversion process.

2.1 *Photon statistics*

We now apply our results to perform calculations of expectation values characterizing photon statistics. The photon number operators for the two modes are given by

$$\hat{n}_a(t) = \hat{a}^\dagger(t)\hat{a}(t); \quad \hat{n}_b(t) = \hat{b}^\dagger(t)\hat{b}(t), \tag{32}$$

where $(\hat{a}(t), \hat{b}^\dagger(t))$ and $(\hat{a}^\dagger(t), \hat{b}(t))$ are given in eqs (25) and (28), respectively.

Working in the Fock state $|n\rangle_a|m\rangle_b = |nm\rangle$ for the two modes, we easily obtain the mean photon numbers, defined by

$$\bar{n}_a(t) = \langle mn|\hat{n}_a(t)|nm\rangle; \quad \bar{n}_b(t) = \langle mn|\hat{n}_b(t)|nm\rangle \tag{33}$$

to be

$$\bar{n}_a(t) = |\mu|^2 n + \nu^2(m+1); \quad \bar{n}_b(t) = |\mu|^2 m + \nu^2(n+1), \tag{34a}$$

giving

$$\bar{n}_a(t) + \bar{n}_b(t) = |\mu|^2(n+m) + \nu^2(n+m+2); \quad \bar{n}_a(t) - \bar{n}_b(t) = n - m, \tag{34b}$$

where we have used eq. (30) to obtain the second result in eq. (34b).

To calculate photon number fluctuations, we introduce notations

$$\overline{n_j^2(t)} = \langle mn|\hat{n}_j^2(t)|nm\rangle, \quad j = a, b,$$

with normal order forms,

$$\overline{n_a^2(t)} = \langle mn|\hat{a}^{\dagger 2}(t)\hat{a}^2(t)|nm\rangle + \bar{n}_a(t), \tag{35a}$$

$$\overline{n_b^2(t)} = \langle mn|\hat{b}^{\dagger 2}(t)\hat{b}^2(t)|nm\rangle + \bar{n}_b(t). \tag{35b}$$

Applying eqs (25) and (28) we obtain

$$\langle mn|\hat{a}^{\dagger 2}(t)\hat{a}^2(t)|nm\rangle = |\mu|^4 n(n-1) + 4|\mu|^2\nu^2 n(m+1) + \nu^4(m+1)(m+2) \tag{36a}$$

$$\langle mn|\hat{b}^{\dagger 2}(t)\hat{b}^2(t)|nm\rangle = |\mu|^4 m(m-1) + 4|\mu|^2\nu^2 m(n+1) + \nu^4(n+1)(n+2), \tag{36b}$$

which we put in eqs (35a) and (35b) to obtain

$$\overline{n_a^2(t)} = (\bar{n}_a(t))^2 + \bar{n}_a(t) - |\mu|^4 n + 2|\mu|^2\nu^2 n(m+1) + \nu^4(m+1), \tag{37a}$$

$$\overline{n_b^2(t)} = (\bar{n}_b(t))^2 + \bar{n}_b(t) - |\mu|^4 m + 2|\mu|^2\nu^2 m(n+1) + \nu^4(n+1). \tag{37b}$$

Using eq. (30) to substitute $|\mu|^2 = 1 + \nu^2$ in eqs (34a), (37a) and (37b), we express

$$\bar{n}_a(t) = n + \nu^2(n+m+1); \quad \bar{n}_b(t) = m + \nu^2(n+m+1), \tag{38a}$$

$$-|\mu|^4 n + 2|\mu|^2\nu^2 n(m+1) + \nu^4(m+1) = 2\nu^2 nm + \nu^4(2nm+n+m+1) - n, \tag{38b}$$

$$-|\mu|^4 m + 2|\mu|^2\nu^2 m(n+1) + \nu^4(n+1) = 2\nu^2 nm + \nu^4(2nm+n+m+1) - m. \tag{38c}$$

The photon number fluctuations

$$(\Delta n_j(t))^2 = \overline{n_j^2(t)} - (\bar{n}_j(t))^2, \; j = a, b$$

are then easily obtained from eqs (37a)–(38c) in the form

$$(\Delta n_a(t))^2 = (\Delta n_b(t))^2 = \nu^2(1+\nu^2)(2nm+n+m+1). \tag{39}$$

In the vacuum state where $n = m = 0$, we obtain

$$\bar{n}_a(t) = \bar{n}_b(t) = n_0(t) = \nu^2 \tag{40a}$$

$$(\Delta n_a(t))^2 = (\Delta n_b(t))^2 = (\Delta n_0(t))^2 = n_0(t)(1+n_0(t)), \tag{40b}$$

where we have adopted the notation

$$n_0(t) = \nu^2 = \frac{g^2}{\beta^2}\sinh^2(\beta t) = \frac{1}{1-k^2}\sinh^2(gt\sqrt{1-k^2}) \tag{40c}$$

defining (squeezed) vacuum photon number in ref. [4].

Following ref. [4], we define Mandel's quality factor in the form

$$Q_j(t) = \frac{(\Delta n_j(t))^2 - \bar{n}_j(t)}{\bar{n}_j(t)}, \quad j = a, b, \tag{41a}$$

which on using eqs (37a), (37b), (38b), (38c) and (40a), takes the forms

$$Q_a(t) = \frac{2n_0(t)nm + n_0^2(t)(2nm+n+m+1) - n}{n + n_0(t)(n+m+1)}, \tag{41b}$$

$$Q_b(t) = \frac{2n_0(t)nm + n_0^2(t)(2nm+n+m+1) - m}{m + n_0(t)(n+m+1)}. \tag{41c}$$

These results agree exactly with the results obtained in ref. [4].

2.2 *Characteristic features of the dynamics*

We notice that the results of expectation values are generally expressed in terms of $n_0(t) = \nu^2(t)$. According to eq. (40c), the value of the parameter k determines the characteristic features of the dynamics of the parametric down-conversion process. There are four distinct cases to consider.

2.2.1 *Case* $k^2 = 0$: The case $k^2 = 0$ characterizes parametric resonance, with $\omega = \omega_a + \omega_b$ so that $\Omega = 0$ according to eq. (16). Under this condition, we have

$$\beta = g; \quad \nu_0^2(t) = \sinh^2(gt). \tag{42}$$

The two-mode operators evolve in time according to eq. (27), which exhibits squeezing action described by two-photon coherent states or pure squeezed states.

2.2.2 *Case* $k^2 < 1$: The case $k^2 < 1$ characterizes squeezing action with de-tuning in the parametric down-conversion process. Under this condition, we have

$$\beta = g\sqrt{1-k^2}; \quad \nu^2_{<1}(t) = \frac{1}{1-k^2}\sinh^2(gt\sqrt{1-k^2}). \tag{43}$$

The squeezing property can be demonstrated through a calculation of the uncertainties in canonically conjugate quadrature components, showing that the uncertainty in one quadrature component grows exponentially, while the uncertainty in the other quadrature component decays exponentially.

2.2.3 *Case* $k^2 > 1$: The case $k^2 > 1$ characterizes oscillatory behaviour. Under this condition, we have

$$\beta = i\alpha, \quad \alpha = g\sqrt{k^2-1};$$
$$\cosh(\beta t) = \cos(\alpha t); \quad \sinh(\beta t) = i\sin(\alpha t), \tag{44a}$$

$$\nu^2_{>1}(t) = \frac{1}{k^2-1}\sin(gt\sqrt{k^2-1}). \tag{44b}$$

The periodic nature of the dynamics leads to quantum revival features in the parametric down-conversion process.

2.2.4 *Case* $k^2 = 1$: The case $k^2 = 1$ is the critical condition signalling the transition from squeezing properties characterized by $k^2 < 1 \geq 0$ to oscillatory behaviour with quantum revival features characterized by $k^2 > 1$ in the parametric down-conversion process. Under this condition, we have

$$\beta \to 0; \quad \nu_1(t) \approx -gt; \quad \mu_1(t) \approx 1 + igt, \tag{45a}$$

$$\nu_1^2(t) = g^2t^2. \tag{45b}$$

Details of the characteristic features specified above have been presented in ref. [4] and therefore need not be repeated here.

3. Conclusion

Working within the framework of Heisenberg's picture of quantum mechanics, we have developed a matrix method which transforms the time evolution equations for the annihilation and creation operators (or any canonically conjugate operators) in the non-degenerate parametric down-conversion process into an equivalent spin equation with much simpler solutions. The method avoids the complications of operator expansion theorems involved in the Lie algebraic and operator ordering techniques generally applied in solving the time evolution equations within the Schroedinger picture. Expectation values for studying photon statistics have been evaluated in much simpler forms.

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