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Anisotropic Bianchi-I universe with phantom field and cosmological constant

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Abstract. We study an anisotropic Bianchi-I universe in the presence of a phantom field and a cosmological constant. Cosmological solutions are obtained when the kinetic energy of the phantom field is of the order of anisotropy and dominates over the potential energy of the field. The anisotropy of the universe decreases and the universe transits to an isotropic flat FRW universe accommodating the present acceleration. A class of new cosmological solutions is obtained for an anisotropic universe in case an initial anisotropy exists which is bigger than the value determined by the parameter of the kinetic part of the field. Later, an autonomous system of equations for an axially symmetric Bianchi-I universe with phantom field in an exponential potential is studied. We discuss the stability of the cosmological solutions.

Keywords. Anisotropic cosmology; phantom field; accelerating universe.

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1. Introduction

Recent astrophysical data obtained from high redshift surveys of Supernovae, COBE to WMAP predict that the present universe is passing through an accelerating phase of expansion [1,2]. An accelerating phase of the universe at a later epoch is permitted with an equation of state $p = \omega\rho$, where $\omega < -1$. Such an equation of state is not possible with usual matters of the standard model. Thus a modification of the matter sector of the Einstein's equation with new fields, perhaps a new physics, is to be explored. It may be realistic to explore cosmological models with exotic fields, e.g. phantom field. Its appearance is not yet clear but it has many similarities with quantum field theory [3]. The interesting feature of the field is that it has unusual kinetic term in the Lagrangian suitable for describing dark energy, which originates in many theories, namely supergravity [4], higher derivative gravity theories [5], braneworld phantom energy [6], etc. Although such theories are known to be unstable with respect to quantum effects, it may be important to explore cosmologies with such fields because of its importance in the early and late era. One interesting aspect of the field is that the universe might end up with a singularity due to the appearance of divergence in scale factor $a(t)$, Hubble parameter and in its first derivative [7,8]. It is shown that the singularities with phantom matter are different from those of standard matter cosmology. It is also found that the relation between the phantom models and standard matter models are like the duality symmetry of string cosmology [9]. Faraoni [10] studied a spatially flat homogeneous and isotropic universe dominated by a phantom field by the phase space analysis and found that the late time attractors exist for a general phantom potential. Recently, Kujat *et al* [11] have studied phantom dark energy models with negative kinetic term and derived the conditions on the potential so that the result is consistent with current cosmological observations and yields a variety of different possible future fates of the universe. Nojiri and Odintsov [12] proposed that early inflation and late time acceleration of the universe may be unified in a single theory based on a phantom field. Although a fundamental quantum phantom is difficult to stabilize [13,14], it is shown that the solution is stable with respect to a small fluctuation of initial data when $m_p^2 \leq \frac{1}{2}$ and small fluctuations of the form of the potential [15]. The phantom field has some strange properties which may be interesting to explore cosmological model even though such theory is known to be unstable with respect to quantum effects. The energy density of phantom field increases with increasing scale factor and the phantom energy density becomes infinite at a finite t, known as 'big rip' condition [16]. However, the 'big rip' problem may be avoided in some models [17,18] which meets the current observations fairly well. The finite lifetime of a universe provides an explanation for the apparent coincidence between the current values of the observed matter density and the dark energy density [19]. When the phantom energy becomes strong enough, gravitational instability no longer works and the universe becomes homogeneous. Sami and Toporensky [20] examined the nature of future evolution of the universe with potential energy-dominated regime of the phantom field considering both massive and self-interacting phantom potential in addition to matter. It is found that the nature of the future evolution is dependent on the steepness of the field. The phantom cosmology has been analysed adopting phase space analysis technique and found that accelerated universe is an attractor with exponential potential [21]. Singh *et al* [22] studied the general features of the dynamics of the phantom field. Using inverse coshyperbolic function for the phantom potential it has been demonstrated that it admits $\omega < -1$. In the model it is noted that the de Sitter universe turns out to be the late time attractor. However, the time derivative of the field is considered to be zero initially. One of the features of the phantom field over scalar field is that it violates the strong energy condition naturally due to its negative kinetic energy. Historically, the pure negative kinetic energy term was first introduced by Hoyle [23] in order to reconcile the homogeneous density (based on perfect cosmological principle) by the creation of new matter in voids as a consequence of the expansion of the universe. Later it was reformulated by Hoyle and Narlikar [24] in the context of the steady state theory of the universe which is popularly known as 'creation' or C-field. The present objective of introducing similar field in modern cosmology is to look for an explanation of the present acceleration. In this paper we consider a phantom field with its kinetic energy of the order of the anisotropy in an anisotropic Bianchi-I universe. We consider also a cosmological constant in the theory which may appear due to phantom transition and assume that although in the early universe phantom field is negligible, it becomes important as the anisotropy gradually decreases in the early era. We explore cosmological solutions when (i) the anisotropy of the universe is more than the kinetic term and (ii) anisotropy of the universe is less than the kinetic energy of the field. It may be pointed out here that in the absence of both anisotropy and cosmological constant, a kinetic energy-dominated regime of the phantom field becomes unrealistic in cosmology, which however becomes important in its presence. We explore particularly this aspect of the field here.

The paper is organized as follows: the field equations are written down in §2. Section 3 is divided into two parts: in the first part we obtain cosmological solutions for kinetically-dominated phantom field with cosmological constant and in the second part we study the critical points corresponding to the set of autonomous equations of an axially symmetric Bianchi-I universe. Finally, in §4 we give a brief discussion.

2. Field equation

The Einstein's field equation is given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu}, \tag{1}$$

where $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, $g_{\mu\nu}$ is the metric and $T_{\mu\nu}$ is the energy–momentum tensor. We consider an anisotropic Bianchi-I metric which is given by

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \sum_{i=1}^{3} R_i^2(t)(\mathrm{d}x^i)^2, \tag{2}$$

where R_1, R_2, R_3 represent the directional scale factors for the universe. The energy–momentum tensor

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}, \tag{3}$$

where $u^\mu u_\mu = -1$, u^μ is the unit fluid velocity of matter, ρ is the energy density and p is the pressure. For a combination of different kinds of fluid model we have $\rho = \sum_{i=1}^{n} \rho_i$ and $p = \sum_{i=1}^{n} p_i$. The conservation equation is given by

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \Theta(\rho + p) = 0, \tag{4}$$

where Θ is the volume expansion rate. The directional Hubble parameters are defined by

$$H_i = \frac{\dot{R}_i}{R_i} \tag{5}$$

and the mean scale factor of the universe is $a(t) = (R_1 R_2 R_3)^{1/3}$. The expansion rate is now given by

$$\Theta = 3H = 3\frac{\dot{a}}{a} = \sum_{i=1}^{3} H_i \tag{6}$$

and the average anisotropy expansion is given by

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2. \tag{7}$$

For the Bianchi metric given by (2) we get

$$\sigma^{\mu\nu}\sigma_{\mu\nu} = \sum_{i=1}^{3}(H_i - H)^2 = \frac{6K^2}{a^6}, \quad \dot{K} = 0, \tag{8}$$

where $\sigma_{\mu\nu}$ represents the shear and the anisotropy parameter becomes

$$A = \frac{K^2}{H^2 V^2}, \tag{9}$$

where $V = R_1 R_2 R_3 = a^3(t)$. The field eq. (1) with eqs (7)–(9), can be written as

$$H^2 = \frac{8\pi G}{3}\rho + \frac{K^2}{a^6}. \tag{10}$$

It may be pointed out here that if one sets $K = 0$, the equations reduce to that obtained for a flat FRW universe. Thus when the universe is sufficiently large it behaves like a flat universe. One can write the energy density of the universe as $\rho = \rho_0 + \rho_1 + \rho_2$, where ρ_0 represents vacuum energy determined by cosmological constant, ρ_1 represents the field energy and $\rho_2 = \rho_{\rm rad}/a^4$ represents the radiation energy. We now consider a homogeneous field $\phi = \phi(t)$, and the energy density and pressure of the field can be written as

$$\rho = \frac{\epsilon}{2}\dot{\phi}^2 + V(\phi), \quad p = \frac{\epsilon}{2}\dot{\phi}^2 - V(\phi). \tag{11}$$

Here $\epsilon = 1$ corresponds to scalar field and $\epsilon = -1$ corresponds to phantom field. The evolution equation for ϕ is given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{1}{\epsilon}\frac{{\rm d}V(\phi)}{{\rm d}\phi} = 0. \tag{12}$$

For scalar field we put $\epsilon = 1$. The condition for inflation in this case is realized when $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$. For phantom field, we put $\epsilon = -1$, and the EOS parameter becomes

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 + 2V(\phi)}{\dot{\phi}^2 - 2V(\phi)}. \tag{13}$$

It is evident that the conditions $\omega_\phi < -1$ and $\rho_\phi > 0$ could be realized when $0 < \dot{\phi}^2 < 2V(\phi)$ with the phantom field also. The condition $\omega_\phi < -1$ is satisfied for phantom field only. In the case of a FRW universe with phantom field only the essential condition to satisfy is $0 < \dot{\phi}^2 < V(\phi)$, i.e., it requires potential energy to be large compared to kinetic energy [25]. However, in the presence of anisotropy it is evident that the above condition may be relaxed and one can begin with phantom field with its kinetic energy domination, i.e., $\dot{\phi}^2 > V(\phi)$ provided shear is more than the lower limit $\sigma^2 > \dot{\phi}^2 - V(\phi)$. However, it is important to look for cosmological behaviour for the regime $\dot{\phi}^2 > 2V(\phi)$ and its subsequent evolution. It may be pointed out here that a realistic solution is possible when the kinetic-dominated phantom field is considered in the presence of a cosmological constant. In this paper, considering an anisotropic Bianchi-I universe, cosmological solutions are explored which later may transit to an isotropic universe.

3. Cosmological solutions

3.1 *Kinetic energy-dominated field*

Let us consider a scalar/phantom field in Bianchi-I universe with a kinetic energy-dominated epoch in the presence of cosmological constant (Λ). Equation (12) becomes

$$\ddot{\phi} + 3H\dot{\phi} = 0. \tag{14}$$

On integrating the above equation we get

$$\dot{\phi} = \pm \frac{C}{a^3}, \tag{15}$$

where C is an integration constant. We assume an epoch when the kinetic energy ($\sim a^{-6}$) of the field dominates over both the radiation energy density ($\sim a^{-4}$) and matter density ($\sim a^{-3}$). The corresponding Einstein field equation (10) becomes

$$\left(\frac{\dot{a}}{a}\right)^2 = \beta^2 \left(1 + \frac{\alpha}{a^6}\right), \tag{16}$$

where we use the symbols $\alpha = \frac{K^2}{\beta^2} + \frac{\epsilon C^2}{2\Lambda}$ and $\beta = \sqrt{\frac{8\pi G\Lambda}{3}}$. The average scale factor of the universe is obtained on integrating eq. (16). In the next section we discuss different cosmological models:

- For a scalar field we set $\epsilon = 1$ in eq. (12) and on integration (16) one obtains

$$a(t) = \alpha^{1/6} \sinh^{1/3}(3\beta t), \tag{17}$$

where $\alpha = \frac{K^2}{\beta^2} + \frac{C^2}{2\Lambda} > 0$. The corresponding scalar field evolves as

$$\phi = \phi_0 \pm \frac{C}{3\beta\sqrt{\alpha}} \ln \tanh\left(\frac{3\beta}{2} t\right), \tag{18}$$

where ϕ_0 is a constant. The solution describes a universe from singularity which however transits to an inflationary phase at a later epoch [25]. It has a Big Bang singularity at $t = 0$, with asymptotic behaviour

$$a(t) \sim t^{1/3} \tag{19}$$

and the scalar field evolution near $t \to 0$ goes as

$$\phi(t) = \phi_0 - \frac{C}{3\beta\sqrt{\alpha}} \ln\left(\frac{3\beta}{2}t\right). \tag{20}$$

• For a phantom field, we set $\epsilon = -1$. In this case we discuss different regions determined by the anisotropy parameter.

(i) For $K^2 > (4\pi GC^2/3)$, eq. (16) admits a hyperbolic solution given by (17). But in this case $\alpha_{\text{phantom}} < \alpha_{\text{scalar}}$ which leads to a smaller universe in the case of phantom field-filled universe in the early epoch, than a universe filled with scalar field.

Another solution is permitted in the absence of cosmological constant ($\Lambda = 0$). The scale factor follows a power law expansion,

$$a(t) \sim t^{1/3}, \tag{21}$$

and the corresponding scalar field evolves slowly as

$$\phi = \phi_0 \pm \frac{C}{3\alpha} \ln t. \tag{22}$$

The anisotropy decreases as

$$A = \frac{K^2}{t} \tag{23}$$

leading to an isotropic universe. Thus even if initial anisotropy is large compared to the kinetic energy of the field, the anisotropic universe transits to an isotropic universe. In this case the universe is decelerating.

(ii) For $K^2 < (4\pi GC^2/3)$, eq. (16) admits a non-singular solution which is

$$a(t) = \tilde{\alpha}^{1/6} \cosh^{1/3}(3\beta t), \tag{24}$$

where $\tilde{\alpha} = -\alpha > 0$. The universe originates from a non-singular state. The corresponding evolution of the phantom field is given by

$$\phi = \phi_0 \pm \frac{2C}{3\beta\sqrt{\tilde{\alpha}^3}} \tan^{-1}\left[\tanh\left(\frac{3\beta}{2}t\right)\right]. \tag{25}$$

The anisotropy in this epoch decreases as

$$A = \frac{4K^2}{\tilde{\alpha}\beta^2 \sinh^2(3\beta t)}. \tag{26}$$

The universe quickly transits to an inflationary phase with a potential energy-dominated phase as the kinetic part decreases rapidly. The asymptotic behaviour of the scalar field at early epoch is

$$\phi_{\pm} = \phi_0 \pm \frac{C}{sqrt\tilde{\alpha}}\, t. \tag{27}$$

The above solution is obtained either with an increasing or a decreasing mode of the field. The solution is new and interesting as it admits an accelerating late universe.

(iii) For $K^2 = (4\pi GC^2/3)$, the kinetic energy density of the phantom field gets cancelled with the anisotropic contribution in the field equation. In this case we note that

• when $\Lambda \neq 0$, one obtains de Sitter expansion with kinetic-dominated phantom field determined by the cosmological constant given by

$$a(t) = a_0 \mathrm{e}^{\beta t}, \tag{28}$$

where $\beta = \sqrt{8\pi GV(\phi)/3}$. The solution permits a zero kinetic energy ($\dot{\phi} = 0$) phantom released at a distance from the origin which was considered in [22] to describe cosmological evolution with an inverse coshyperbolic potential with potential energy-dominated epoch.

• When $\Lambda = 0$, one obtains an interesting solution considering the coexistence of two fluids, namely, radiation and phantom field. The evolution of the universe is given by

$$a(t) = \left(\frac{32\pi G\rho_{\mathrm{rad}}}{3}\right)^{1/4} \sqrt{t} \tag{29}$$

and the corresponding field evolves as

$$\phi = \phi_0 \pm \frac{\phi_1}{\sqrt{t}} \tag{30}$$

which however attains a constant value at a later epoch. In the next section we consider phantom field in an exponential potential.

3.2 *Autonomous system with phantom in an exponential potential*

The field eq. (1) for anisotropic Bianchi-I metric can be written as a set of first-order non-linear partial differential equations by treating the shear as a massless scalar field [27]. The corresponding field equations with phantom field are

$$\dot{H} = -4\pi G(2\sigma^2 - \dot{\phi}^2), \tag{31}$$

$$\ddot{\phi} = -3H\dot{\phi} + \frac{\mathrm{d}\phi}{\mathrm{d}t}, \tag{32}$$

$$\dot{\sigma} = -3H\sigma, \tag{33}$$

together with the constraint equation

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\left(\sigma^2 - \frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \tag{34}$$

where dot denotes derivative with respect to cosmic time, shear $\sigma = \frac{1}{\sqrt{24\pi G}}\left(\frac{\dot{R}_1}{R_1} - \frac{\dot{R}_2}{R_2}\right)$ (taking $R_3 = R_2$ axially symmetric Bianchi-I metric). We consider here a phantom field in an exponential potential of the form $V(\phi) = V_0 \mathrm{e}^{-\sqrt{8\pi G}\lambda\phi}$, where V_0 and λ are free parameters. It is evident from eq. (31) that an exact exponential expansion $\dot{H} = 0$ is admissible in Bianchi-I universe with phantom field. Moreover, the field equations may admit $\dot{H} > 0$, a different result when the source consists of a self-interacting scalar field [27,28]. Now making use of the following dimensionless variables

$$x = \sqrt{\frac{4\pi G\dot{\phi}^2}{3H^2}}, \quad y = \sqrt{\frac{8\pi G V}{3H^2}}, \tag{35}$$

we obtain a set of plane autonomous system using eqs (31)–(34),

$$x' = -3x + 3x(1 - y^2) - \sqrt{\frac{3}{2}}\lambda y^2, \tag{36}$$

$$y' = -\sqrt{\frac{3}{2}}\lambda xy + 3y(1 - y^2), \tag{37}$$

where a prime represents derivative with respect to logarithm of the scale factor ($N = \ln\, a$). The constraint eq. (34) now becomes

$$1 = \frac{8\pi G\sigma^2}{3H^2} - x^2 + y^2. \tag{38}$$

The phantom density parameter Ω_ϕ and the effective phantom equation of state γ_ϕ are

$$\Omega_\phi = -x^2 + y^2, \quad \Omega_\phi\gamma_\phi = -2x^2. \tag{39}$$

It is evident that $\gamma_\phi < 0$, if $x^2 < y^2$.

Critical points and stability

We note the following

- $(0, 0)$ is a critical point. It is saddle if $\Omega_\phi = 0$ with $\omega_\phi =$ undefined.
- $\left(-\frac{\lambda}{\sqrt{6}}, \sqrt{1 + \frac{\lambda^2}{6}}\right)$ is a critical point. It is stable for $\Omega_\phi = 1$ with $\omega_\phi = -\lambda^2/3$.

It is a saddle point if λ is a complex number satisfying the inequality $\lambda^2 < -6$.

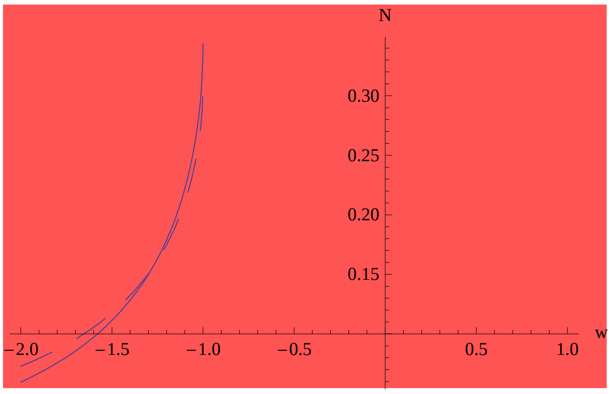

Figure 1. Variation of $\omega(w)$ is shown with N (for $\lambda = \sqrt{2}$ with continuous line and $\lambda = \sqrt{3}$ with broken line).

4. Discussions

We obtain cosmological models with phantom field in an anisotropic Bianchi-I universe with or without a cosmological constant. We explore kinetic-dominated phantom field. In the case of a vanishing cosmological constant, two fluid models (phantom field and radiation) are required for a physically relevant solution. We found that a singular solution obtained by Gron [26] (for $\epsilon = 1$) is revisited with phantom field for a restricted domain of initial anisotropy. But in the case of scalar field such a solution is permitted for any value of initial anisotropy. In the case of phantom field we obtain a new and interesting solution which accommodates a late accelerating universe provided the kinetic energy of the phantom field exceeds a lower limit determined by the anisotropy of an anisotropic universe. A pure de Sitter universe is obtained here when the kinetic energy of the phantom field is of the order of the anisotropy, i.e., at $K^2 = 4\pi GC^2/3$. However, in the absence of cosmological constant and for a kinetically-dominated phantom field the evolution of the universe at $K^2 = 4\pi GC^2/3$, is in accordance with the matter content in the universe. In another case we consider radiation with phantom field. In that case it leads to a late evolution satisfying the conditions necessary for inflationary universe with inverse coshyperbolic potential as was taken up by Singh *et al* [22], which may be realized even in an anisotropic background with non-zero anisotropy of the universe. We consider phantom field in an exponential potential in an axially symmetric Bianchi-I universe and obtain the set of autonomous equation. It is found that there exists one stable critical point which is interesting. We note that exponential potential admits $a(t) \sim t^{1/3}$ in the presence of shear. However, one obtains power-law inflation $a(t) \sim t^D$ with $D > 1$ for $V_0 > 1/4\pi G\lambda^2$ when $\sigma \to 0$. During this regime the field evolves slowly as $\phi = \sqrt{(1/2\pi G\lambda^2)}\ln t$. As the anisotropic universe transits to an isotropic universe, the evolution of the universe may be determined by the phantom field. To understand this we plot ω against N in figure 1 for $\lambda = \sqrt{2}$ (shown by continous line) and $\lambda = \sqrt{3}$ (shown by broken line) for $y = 1$. It is evident that the value of ω settles to a de Sitter value, admitting late acceleration of the universe. In [22,29] viable cosmological models with phantom field in various potentials have been explored in isotropic FRW model. It may be interesting to work with those potentials in anisotropic universe which will be discussed elsewhere.

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