PRAMANA — journal of physics © Indian Academy of Sciences Vol. 71, No. 3 September 2008 pp. 521–528

The investigation of the $2\nu\beta\beta$ decay by Pyatov method within quasiparticle random phase approximation formalism

S UNLU[1,*], T BABACAN[2], N CAKMAK[1] and C SELAM[1]
[1]Department of Physics, Anadolu University, Eskisehir, Turkey
[2]Department of Physics, Celal Bayar University, Manisa, Turkey
[*]Corresponding author. E-mail: serdarunlu@anadolu.edu.tr

MS received 7 May 2008; accepted 24 June 2008

Abstract. The violated commutation condition between the total shell model Hamiltonian and Gamow–Teller operator (GT) has been restored by Pyatov method (PM). The considered nuclear model Hamiltonian in PM includes the separable GT residual interaction in ph and pp channels and is differentiated from the traditional schematic model by h_0(restoration term). The influence of the h_0 effective interaction on the $2\nu\beta\beta$ decay of ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128,130}Te and ^{136}Xe is investigated. All the calculations have been done within the framework of standard QRPA. The results obtained by PM are compared with those of other approaches and experimental data. The influence of the restoration term on the stability of the $2\nu\beta\beta$ decay nuclear matrix elements is analysed.

Keywords. Pyatov method; Gamow–Teller; quasiparticle random phase approximation; $2\nu\beta\beta$.

PACS Nos 21.60.Cs; 23.40.-S

1. Introduction

Nuclear double beta decay process is a rare transition in which the nuclear charge changes by two units as the mass number remains the same. This process can occur only when ordinary single beta decay process is strongly suppressed due to a large change of spin or due to the states forbidden energetically. There are several modes for $\beta\beta$ decay. One of them is neutrinoless double beta decay which violates lepton number conservation and is thus forbidden in the standard model. Another decay mode is two neutrino double beta decay in which the lepton number is conserved.

Nuclear double beta decay phenomenon has recently been an interesting research topic for both nuclear and particle physicists. The reason why this interest has been increasing intensely is the consideration of the effective interaction in the particle–particle (pp) channel [1–13]. This effective interaction was included to get a better agreement between the calculated values of $2\nu\beta\beta$ decay nuclear matrix elements using quasiparticle random phase approximation (QRPA) method and the corresponding experimental data. However, the inclusion of the pp effective interaction leads to two different problems: the first problem is the sensitivity of nuclear matrix element (NME) to the pp effective interaction constant and the second is that NME suddenly goes to zero at a certain value of the constant. There are some attempts [8–13] to overcome the above-mentioned difficulties. These attempts are usually related to the use of higher versions of QRPA method, but the calculation results show that the use of the higher versions of QRPA is not successful enough in overcoming these difficulties. Fully renormalized QRPA (FR-QRPA) is one of these attempts towards the solution of the above problems. Two neutrino double beta decay within the framework of FR-QRPA formalism has been studied by Pacearescu *et al* [13]. They found that FR-QRPA method shifted the collapse to the larger values of the pp strength as compared to the QRPA and self-consistent QRPA (SCQRPA) methods, and thus the stability is increased in the FR-QRPA case. They consider the stability as the shift of the pp strength to larger values. This consideration may not be enough because the stability of the calculations is related to the deviation degree of the collapsed value of the pp strength ($\chi_{\rm pp}^{\rm collapsed}$) from the corresponding experimental value ($\chi_{\rm pp}^{\rm exp}$). The sensitivity of the $2\nu\beta\beta$ decay amplitude to the violated $SU(4)$ symmetry was studied in ref. [14] and the calculation results show that the sensitivity of the $2\nu\beta\beta$ decay amplitude to the ($\chi_{\rm pp}$) constant is related to the violation of $SU(4)$ symmetry.

In this study, the broken commutation condition between the shell model Hamiltonian operator and the GT operator in the quasiparticle space has been restored using Pyatov method, and then the dependence of the $2\nu\beta\beta$ decay NME on the ($\chi_{\rm ph}$) and ($\chi_{\rm pp}$) parameters has been studied. In other words, the aim of the present paper is to examine the influence of the mentioned restoration on $2\nu\beta\beta$ decay processes. The paper is organized as follows: In §2, the details of the restoration are given. In §3, the influence of this restoration on the characteristic quantities such as total $\beta^{\pm}$ transition strengths ($S^{\pm}$) and the $2\nu\beta\beta$ decay nuclear matrix element ($M^{2\nu}$) for ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128,130}Te and ^{136}Xe is investigated by comparing with the corresponding traditional schematic model (SM) calculation results. Moreover, the collapse problem of the QRPA solution and the stability degree of the calculated $M^{2\nu}$ values have also been studied. At the end of §3, the $M^{2\nu}$ and $t_{1/2}^{2\nu}$ values calculated by PM have been compared with the corresponding experimental data and the other theoretical calculations.

2. Theoretical formalism

As known, the commutation relation between GT operator

$$G^{\rho}_{1\mu} = \sum_k \sigma_{1\mu}(k)\tau_\rho(k); (\rho = \pm) \tag{1}$$

and shell model Hamiltonian operator is given as follows:

$$[H_{sp}, G^{\rho}_{1\mu}] = [V_1 + V_{\rm c} + V_{\rm ls}, G^{\rho}_{1\mu}]. \tag{2}$$

The same commutation relation in the quasiparticle representation will be different from that given in eq. (2) while the pairing interaction between nucleons is considered as given in ref. [15].

$$[H_{\rm sqp}, G^{\rho}_{1\mu}] \neq [V_1 + V_{\rm c} + V_{\rm ls}, G^{\rho}_{1\mu}]. \tag{3}$$

In this section, we will try to restore the commutation condition broken due to the pairing interaction using Pyatov method [16]. According to the method, the effective interaction term, which is added to the single quasiparticle Hamiltonian, is chosen in the following form:

$$h_0 = \sum_{\rho=\pm} \frac{1}{2\gamma_\rho} \times \sum_{\mu=0,\pm1} [H_{\rm sqp} - V_{\rm c} - V_{\rm ls} - V_1, G^{\rho}_{1\mu}]^{\dagger}[H_{\rm sqp} - V_{\rm c} - V_{\rm ls} - V_1, G^{\rho}_{1\mu}], \tag{4}$$

where V_1 is the isovector term in the central part of the shell model mean field potential, $V_{\rm c}$ is the Coulomb potential, and $V_{\rm ls}$ is the spin-orbit potential. γ_ρ is found from the condition,

$$[H_{\rm sqp} + h_0, G^{\rho}_{1\mu}] = [V_1 + V_{\rm c} + V_{\rm ls}, G^{\rho}_{1\mu}]$$

and given by the following expression:

$$\gamma_\rho = \frac{\rho}{2}\langle 0|[[H_{\rm sqp} - V_{\rm c} - V_{\rm ls} - V_1, G^{\rho}_{1\mu}], G^{\rho}_{1\mu}]|0\rangle.$$

Thus, the total Hamiltonian of the system according to PM is given as follows:

$$H = H_{\rm sqp} + h_0 + h_{\rm ph} + h_{\rm pp}. \tag{5}$$

The Hamiltonian of the system in SM is given in the following form:

$$H = H_{\rm sqp} + h_{\rm ph} + h_{\rm pp}. \tag{6}$$

In these equations, $h_{\rm ph}$ and $h_{\rm pp}$ are the GT effective interactions in the particle–hole and particle–particle channels, respectively. The eigenvalues and eigenfunctions of both Hamiltonian in eq. (5) and eq. (6) have been solved within the framework of QRPA method (for details, see ref. [17])

The total strengths $S^{(\pm)}$ in the $\beta^{(\pm)}$ channels and Ikeda sum rule (ISR) are calculated from the following equations:

$$S^{(\rho)} = \sum_{\mu,m} |\langle 1^+_m, \mu|G^{\rho}_{1\mu}|0^{+\rangle}_{\rm g.s.}|^2, \tag{7}$$

$$\mathrm{ISR} = S^{(-)} - S^{(+)} \cong 3(N-Z). \tag{8}$$

The nuclear matrix element of the $2\nu\beta\beta$ decay and the half-lives $t^{2\nu}_{1/2}$ are found from the equations:

$$M^{2\nu} = \sum_{\mu,m} \frac{\langle 0_{\rm f}^{+}({\rm g.s.})|G_{1\mu}^{-}|1_m^{+},\mu\rangle\langle 1_m^{+},\mu|G_{1\mu}^{-}|0_{\rm i}^{+}({\rm g.s.})\rangle}{\omega_m + W/2}, \tag{9}$$

$$[t_{1/2}^{2\nu}]^{-1} = f_{2\nu} M^{2\nu}, \tag{10}$$

where i(f) denotes the initial(final) ground state, W is the Q value of the $\beta\beta$ decay, each reduced matrix element is the GT matrix element between the mth intermediate 1^{+} state and the initial or final ground state, ω_m is the excitation energies of 1^{+} states calculated from the g.s. of the intermediate nucleus, and $f_{2\nu}$ is a lepton phase-space integral.

3. Results and discussions

In this section we present the $2\nu\beta\beta$ decay results obtained by Pyatov method for ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128,130}Te and ^{136}Xe, in comparison with the traditional QRPA calculations. The single particle energies are obtained using a Coulomb-corrected Woods–Saxon potential with Chepurnov [15] parametrization. The proton and neutron pairing gaps are determined as $C_{\rm n} = C_{\rm p} = 12/{\rm A}^{1/2}$ MeV. The BCS equations for the chemical potentials are solved for proton and neutron subsystems. The basis used in calculation contains all neutron–proton transitions which change the radial quantum number n by $\Delta n = 0, 1, 2, 3$. The quasiparticle Ikeda sum rule is fulfilled with ≈1–1.5% accuracy.

It is seen from eqs (5) and (6) that the difference between SM and PM Hamiltonians is the effective interaction term (h_0) found from the corresponding restoration in PM. In order to study the effect of h_0 on the quantities which characterize the single and $2\nu\beta\beta$ decay, the $\beta^{\pm}$ transition strengths, the Gamow–Teller resonance energies in intermediate nuclei, and the $2\nu\beta\beta$ nuclear matrix elements have been calculated by solving the QRPA equation of motion for Hamiltonians in eqs (5)

Table 1. The $\beta^{\pm}$ transition strengths ($S^{\pm}$), the $2\nu\beta\beta$ nuclear matrix elements (in units of $m_e^{-1} \times 10^{-2}$) and the Gamow–Teller resonance (GTR) energies in intermediate nuclei calculated by SM and PM.

	S^{-}		S^{+}		$M^{2\nu}$		$\omega_{\rm GTR}$ (MeV)	
Transitions	SM	PM	SM	PM	SM	PM	SM	PM
^{48}Ca → ^{48}Ti	24.676	23.676	1.193	0.203	6.13	2.26	16.35	7.03
^{76}Ge → ^{76}Se	36.681	35.654	1.194	0.161	10.94	2.03	15.83	9.85
^{82}Se → ^{82}Ke	42.397	41.558	1.003	0.168	7.90	1.37	17.29	11.75
^{96}Zr → ^{96}Mo	48.292	47.218	1.465	0.395	11.51	4.52	18.69	13.00
^{100}Mo → ^{100}Ru	48.538	47.226	1.766	0.480	16.96	7.25	17.84	12.73
^{116}Cd → ^{116}Sn	60.124	59.297	0.974	0.202	11.46	3.14	18.60	13.18
^{128}Te → ^{128}Xe	72.132	71.226	1.006	0.207	11.10	3.56	15.75	13.21
^{130}Te → ^{130}Xe	77.932	77.177	0.931	0.197	7.77	1.31	17.30	14.05
^{136}Xe → ^{136}Ba	83.504	82.804	0.932	0.259	5.41	1.02	19.08	14.83

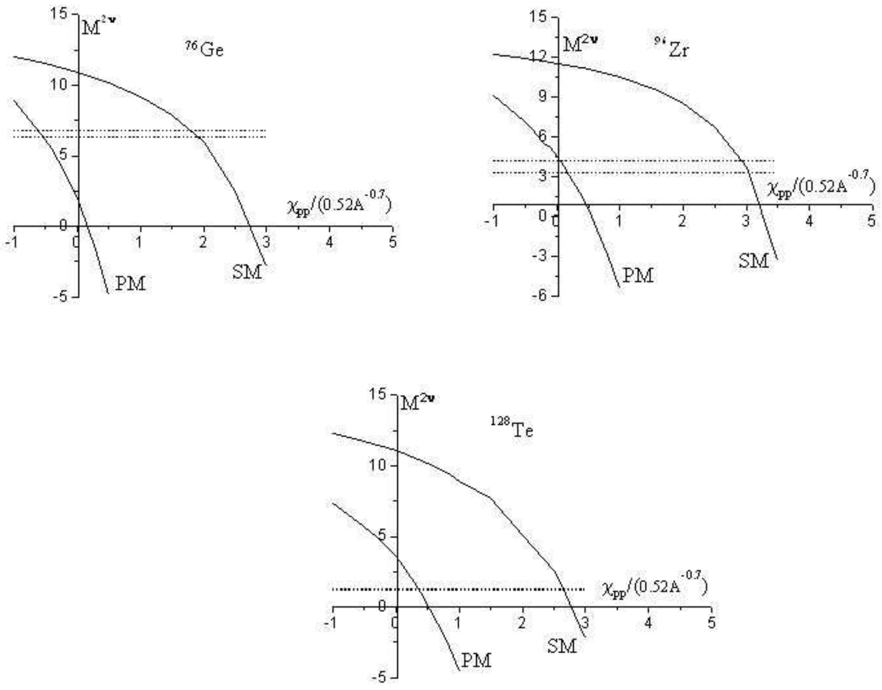

Figure 1. The dependence of the $M^{2\nu}$ on the $\chi_{\rm pp}$ constant for $\chi_{\rm ph}$ = 5.2 $A^{-0.7}$.

and (6) with the same $\chi_{\rm ph} = 1$ and $\chi_{\rm pp} = 0$ values. (In all tables and figures, the $\chi_{\rm pp}$ and $\chi_{\rm ph}$ values are in units of 0.52 $A^{-0.7}$ and 5.2 $A^{-0.7}$, respectively.) The calculation results for all the nuclei mentioned above are presented in table 1. As seen from the table, the effective interaction h_0 causes the decrease in $S^{\pm}$ strengths. The amount of the decrease in $S^{\pm}$ strengths is the same since the Ikeda sum rule is fulfilled with the same accuracy in both methods. Calculation results show that the influence of h_0 term on S^+ strengths is more considerable than that on S^- strengths. The decrease in the $\beta^{\pm}$ transition strengths in PM has also lead to the decrease in $2\nu\beta\beta$ decay nuclear matrix elements. This is clearly seen from the columns 6 and 7 of table 1. The GTR energy values ($\omega_{\rm GTR}$) in intermediate nuclei are given in the columns 8 and 9. As seen from the table, the calculated $\omega_{\rm GTR}$ values in PM shift to lower energies (by 2–10 MeV) in comparison with SM calculations. This shift in the nuclei under consideration shows that h_0 has an attractive character.

In the present paper, in order to understand the degree of sensitivity of the $M^{2\nu}$ values to the $\chi_{\rm pp}$ constant in PM, the dependence of $M^{2\nu}$ on the $\chi_{\rm pp}$ constant has been calculated at a certain value of $\chi_{\rm ph}$ for all nuclei under consideration. The calculation results for some nuclei (^{76}Ge, ^{96}Zr and ^{128}Te) have been compared with the corresponding SM calculations in figure 1. The horizontal line indicates

Table 2. The $\chi_{pp}^{collapsed} - \chi_{pp}^{exp}$, χ_{ph}^{exp} and χ_{pp}^{exp} values for nuclei under consideration.

	$\chi_{pp}^{collapsed} - \chi_{pp}^{exp}$		χ_{ph}^{exp}		χ_{pp}^{exp}	
Transitions	SM	PM	SM	PM	SM	PM
$^{48}Ca \rightarrow {}^{48}Ti$	0.20	0.23	0.5	1.30	2.65	−0.10
$^{76}Ge \rightarrow {}^{76}Se$	0.40	0.95	0.5	1.20	2.50	−0.80
$^{82}Se \rightarrow {}^{82}Ke$	0.45	0.60	0.5	1.00	2.25	−0.50
$^{96}Zr \rightarrow {}^{96}Mo$	0.125	0.45	0.5	1.00	3.20	0.10
$^{100}Mo \rightarrow {}^{100}Ru$	0.25	1.20	0.5	1.00	3.05	−0.50
$^{116}Cd \rightarrow {}^{116}Sn$	0.40	1.025	0.5	1.00	2.45	−0.70
$^{128}Te \rightarrow {}^{128}Xe$	0.10	0.175	0.75	1.00	2.70	0.375
$^{130}Te \rightarrow {}^{130}Xe$	0.25	0.30	0.5	0.85	2.00	−0.50
$^{136}Xe \rightarrow {}^{136}Ba$	–	–	0.5	1.00	2.75	−0.10

Table 3. A comparison of the calculated $M^{2\nu}$ (in units of $m_e^{-1} \times 10^{-2}$) and $t_{1/2}^{2\nu}$ values by PM with the corresponding experimental values and the other theoretical calculations.

Transitions	$(M_{exp}^{2\nu})^1$	$(M_{cal2}^{2\nu})^1$	$(M_{cal3}^{2\nu})^1$	$M_{PM}^{2\nu}$	$t_{1/2}^{2\nu}$ (exp)^1 (yr)	$t_{1/2}^{2\nu}$ (PM) (yr)
$^{48}Ca \rightarrow {}^{48}Ti$	$2.6^{+0.6}_{-1.0}$	7.0	5.5	2.59	$4.3^{+2.4}_{-1.1} \pm 1.4 \times 10^{19}$	4.335×10^{19}
$^{76}Ge \rightarrow {}^{76}Se$	$6.5^{+0.3}_{-0.2}$	7.4	8.3	6.36	$1.77^{+0.13}_{-0.11} \pm 1.4 \times 10^{21}$	1.849×10^{21}
$^{82}Se \rightarrow {}^{82}Ke$	$4.6^{+0.1}_{-0.5}$	4.60	5.2	4.29	$1.08^{+0.26}_{-0.06} \times 10^{20}$	1.242×10^{20}
$^{96}Zr \rightarrow {}^{96}Mo$	$3.7^{+0.5}_{-0.4}$	3.6	2.2	3.84	$3.9 \pm 0.9 \times 10^{19}$	3.619×10^{19}
$^{100}Mo \rightarrow {}^{100}Ru$	9.6 ± 1.0	19.7	5.9	9.83	$1.15^{+0.3}_{-0.2} \times 10^{19}$	1.097×10^{19}
$^{116}Cd \rightarrow {}^{116}Sn$	$6.9^{+0.8}_{-0.9}$	5.1	3.6	6.8	$2.6^{+0.9}_{-0.5} \pm 0.35 \times 10^{19}$	2.677×10^{19}
$^{128}Te \rightarrow {}^{128}Xe$	1.24 ± 0.03	4.6	0.56	1.23	$7.7 \pm 0.4 \times 10^{24}$	7.820×10^{24}
$^{130}Te \rightarrow {}^{130}Xe$	8.8 ± 0.2	2.8	1.6	4.03	$2.7 \pm 0.1 \times 10^{21}$	1.287×10^{22}
$^{136}Xe \rightarrow {}^{136}Ba$	< 2.4	4.0	3.8	1.37	$> 3.6 \times 10^{19}$	1.105×10^{21}

[1]The data have been taken from ref. [18]

the experimental values. As seen from figure 1, although the curves obtained by PM and SM show a similar behaviour, the curve in PM shifts to the lower χ_{pp} values, and the $M^{2\nu}$ approaches to the experimental value at smaller χ_{pp} values compared with SM. In some nuclei, for example ^{76}Ge, the fixed χ_{pp} may have a negative value. This shows that the h_{pp} effective interaction in PM has a repulsive character for some nuclei. (This conclusion will be discussed later.) In order to investigate the stability degree of the calculated $M^{2\nu}$ values in PM, the difference between the fixed value and the collapsed value (at which $M^{2\nu}$ is equal to zero) of the χ_{pp} constant has been calculated, and the calculation results have been given in table 2. It is clearly seen from the table that the $\chi_{pp}^{collapsed} - \chi_{pp}^{exp}$ difference in PM is larger than SM results. The largeness of this difference indicates that the $M^{2\nu}$ values calculated by PM are more stable. Furthermore, the effect of the h_0 effective interaction on the χ_{ph} and χ_{pp} values which are determined from the agreement of GTR energies and $M^{2\nu}$ values with the corresponding experimental data has been studied. The fixed values of the $\chi_{\rm ph}$ and $\chi_{\rm pp}$ constants for both methods have been given in table 2. It can be seen from this table that the $\chi_{\rm ph}$ values in PM are larger than those in SM. This means that the ph interaction in PM should be stronger than the effective interaction in SM. This is due to the fact that h_0 has an attractive character. On the other hand, the attractiveness of h_0 leads to a weaker $h_{\rm pp}$ interaction in PM in comparison with SM. In other words, the fixed $\chi_{\rm pp}$ values in PM should be much smaller than those in SM (see columns 6,7). Calculation results show that the $\chi_{\rm pp}$ values for some nuclei in PM have a minus sign. The reason for this is that the strength of the h_0 effective interaction in these nuclei is so large that the $h_{\rm pp}$ effective interaction becomes repulsive to get a good agreement of the $M^{2\nu}$ values with the corresponding experimental data.

Finally, in table 3 the calculated $M^{2\nu}$ and $t_{1/2}^{2\nu}$ values by PM have been compared with the corresponding experimental data and the other theoretical calculations. In the calculations, the $\chi_{\rm ph}$ and $\chi_{\rm pp}$ constants presented in table 2 have been used. It is clearly seen from the table that the calculated values in PM are closer to the experimental values in comparison with the other calculation results.

4. Conclusions

In summary, the violated commutation relation between the GT operator and the shell model Hamiltonian in the quasiparticle representation has been restored by Pyatov method. The considered nuclear model Hamiltonian in PM includes the separable GT residual interaction in ph and pp channels and is differentiated from the traditional schematic model by h_0. In order to determine the influence of the h_0 effective interaction on the calculation results, the single and $2\nu\beta\beta$ decay characteristic quantities have been calculated in both methods. All the calculations have been done within the framework of standard QRPA. The following conclusions have been drawn from our calculation results:

- The h_0 effective interaction coming from the restoration has an attractive character for the considered nuclei. This can be clearly seen from the fact that the centroid of the GT states found in PM is located in lower energies in comparison with SM.
- The h_0 effective interaction leads to decrease in the values of the $\beta^{\pm}$ transition strengths ($S^{\pm}$) and the $2\nu\beta\beta$ decay nuclear matrix elements ($M^{2\nu}$).
- Calculations show that the fixed $\chi_{\rm ph}$ values in PM are approximately two times larger and the fixed $\chi_{\rm pp}$ values are 10–30 times smaller than those in SM. Moreover, the $h_{\rm pp}$ effective interaction indicates a repulsive character in the investigated nuclei except for ^{96}Zr and ^{128}Te.
- $\chi_{\rm pp}^{\rm collapsed} - \chi_{\rm pp}^{\rm exp}$ in PM is larger than SM results. The largeness of this difference indicates that the $M^{2\nu}$ values calculated by PM are more stable.
- The calculated $M^{2\nu}$ and $t_{1/2}^{2\nu}$ values by the corresponding $\chi_{\rm ph}$ and $\chi_{\rm pp}$ values in PM are closer to the experimental values compared with other calculations.

Acknowledgments

This study is supported by the Scientific and Technological Research Council of Turkey with the project number of 104T152. The authors would like to thank Prof. A Kuliev for his fruitful discussions and important contributions to this study.

References

[1] K Muto, E Bender and H V Klapdor, *Z. Phys.* **A334**, 177 (1988)
[2] O Civitarese, A Faessler and T Tomoda, *Phys. Lett.* **B194**, 11 (1987)
[3] S K Balayev, A A Kuliev and D I Salamov, *Izv. AN SSSR, Ser. Fiz.* **54**, 855 (1990)
[4] J Toivanen and J Suhonen, *Phys. Rev. Lett.* **75**, 410 (1995); *Phys. Rev.* **C55**, 2314 (1997)
[5] J Schwieger, F Simkovic, A Faessler and W A Kaminski, *Phys. Rev.* **C57**, 1738 (1998)
[6] K Muto, *Phys. Lett.* **B391**, 243 (1997)
[7] G Hirsch, P O Hess and O Civitarese, *Phys. Rev.* **C54**, 1976 (1996)
[8] A A Raduta and J Suhonen, *Phys. Rev.* **C53**, 176 (1996); *J. Phys.* **G22**, 123 (1996)
[9] F Simkovic *et al*, *Phys. Lett.* **393**, 267 (1997)
[10] A Bobyk, W A Kaminski and P Zareba, *Nucl. Phys.* **A669**, 223 (2000)
[11] J Suhonen and O Civitarese, *Phys. Rep.* **300**, 123 (1998)
[12] S Stoica and H V Klapdor-Kleingrothaus, *Nucl. Phys.* **A694**, 269 (2001)
[13] L Pacearescu, V Rodin, F Simkovic and A Faessler, *Phys. Rev.* **C68**, 064310 (2003)
[14] V A Rodin, M H Urin and A Faessler, *Nucl. Phys.* **A747**, 295 (2005)
[15] V G Soloviev, *Theory of complex nuclei* (Pergamon, New York, 1976)
B A Chepurnov, *Yad. Phys.* **7**, 1199 (1968)
[16] N I Pyatov and D I Salamov, *Nucleonica* **22**, 127 (1977)
[17] D I Salamov *et al*, *Proceedings of 5th Conference on Nuclear and Particle Physics (NUPPAC 05)* (Cairo, August 2006), p. 361
[18] H Ejiri, *Phys. Rep.* **338**, 265 (2000)