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Regge behaviour of distribution functions and evolution of gluon distribution function in next-to-leading order at low-x

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Abstract. Evolution of gluon distribution function from Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equation in next-to-leading order (NLO) at low-x is presented assuming the Regge behaviour of quark and gluon at this limit. We compare our results of gluon distribution function with MRST2004, GRV98LO and GRV98NLO parametrizations and show the compatibility of Regge behaviour of quark and gluon distribution functions with perturbative quantum chromodynamics (PQCD) at low-x.

Keywords. Regge behaviour; DGLAP evolution equations; low-x; structure functions; gluon distribution function.

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1. Introduction

In our earlier communication [1], we derived the solution of DGLAP evolution equation for gluon distribution function in leading order (LO) at low-x considering Regge behaviour of distribution functions. Here, in continuation of the earlier work we solved DGLAP evolution equation for gluon distribution function at low-x in next-to-leading order (NLO) and the t and x-evolutions of gluon distribution function thus obtained have been compared with global MRST2004 and GRV98 parametrizations. In PQCD, since the higher-order terms in the leading logarithmic series $(\alpha_s(Q^2)\ln Q^2)^n$ are important, simply adding the leading order contributions of the quark and gluon is not enough to get the parton distributions. But at large $Q^2(Q^2 \gg \Lambda^2)$, which is the requirement of DGLAP equation, running coupling constant $\alpha_s(Q^2)$ is small and contributions of higher-order terms decrease very fast. At our current scale of Q^2 though $Q^2 \gg \Lambda^2$, $\alpha_s(Q^2)$ is not that much small. So higher-order terms, at least up to NLO, are also important [2–4]. Here, §1, 2, 3 and 4 are the introduction, theory, results and discussion, and conclusions respectively.

2. Theory

The DGLAP evolution equation for gluon distribution function up to NLO has the standard form [5,6]

$$\begin{aligned} Q^2 \frac{\partial}{\partial Q^2} G(x, Q^2) = & \frac{\alpha_s(Q^2)}{2\pi} \\ & \times \int_x^1 \left[P_{gg}^1(\omega) G(x/\omega, Q^2) + P_{gq}^1(\omega) F_2^s(x/\omega, Q^2) \right] \mathrm{d}\omega \\ & + \left[\frac{\alpha_s(Q^2)}{2\pi} \right]^2 \int_x^1 [P_{gg}^2(\omega) G(x/\omega, Q^2) \\ & + P_{gq}^2(\omega) F_2^s(x/\omega, Q^2)] \mathrm{d}\omega, \end{aligned} \quad (1)$$

where

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 Q^2} \left[1 - \frac{\beta_1 \ln Q^2}{\beta_0^2 Q^2} \right],$$

N_f being the number of flavours and Λ is the QCD cut-off parameter depends on the renormalization scheme, β_0 and β_1 are the expansion coefficients of the β-function and they are given by $\beta_0 = \frac{33-2N_\mathrm{f}}{3}$, $\beta_1 = \frac{306-38N_\mathrm{f}}{3}$. The splitting functions have their forms as given in [5,6]. At low-x, for $x \to 0$, only gluon splitting function matters [6,7]. So keeping the full form of other splitting functions, we make some approximation of the splitting function $P_{gg}^2(\omega)$ retaining only its leading term as $x \to 0$ [6], i.e., we take $P_{gg}^2(\omega) \cong \frac{52}{3}\frac{1}{\omega}$. After changing the variable Q^2 by t, where $t = \ln(Q^2/\Lambda^2)$ and putting the respective kernels we get

$$\begin{aligned} \frac{\partial G(x,t)}{\partial t} = & \frac{\alpha_s(t)}{2\pi} \left\{ 6 \left(\frac{11}{12} - \frac{N_\mathrm{f}}{18} + \ln(1-x) \right) G(x,t) + 6 I_g \right\} \\ & + \left[\frac{\alpha_s(t)}{2\pi} \right]^2 \int_x^1 \left[\frac{52}{3} \frac{1}{\omega} G(x/\omega, t) + A(\omega) F_2^s(x/\omega, t) \right] \mathrm{d}\omega, \end{aligned} \quad (2)$$

where

$$\begin{aligned} I_g = & \int_x^1 \mathrm{d}\omega \left[\frac{\omega G(x/\omega, t) - G(x,t)}{1-\omega} + \left(\omega(1-\omega) + \frac{1-\omega}{\omega} \right) G(x/\omega, t) \right. \\ & \left. + \frac{2}{9} \left(\frac{1 + (1-\omega)^2}{\omega} \right) F_2^s(x/\omega, t) \right], \end{aligned}$$

$$A(\omega) = C_F^2 A_1(\omega) + C_F C_G A_2(\omega) + C_F T_R N_\mathrm{f} A_3(\omega),$$

$$A_1(\omega) = -\frac{5}{2} - \frac{7}{2}\omega + \left(2 + \frac{7}{2}\omega\right)\ln\,\omega + \left(-1 + \frac{1}{2}\omega\right)\ln^2\,\omega$$
$$-2\omega\ln(1-\omega) + (-3\ln(1-\omega) - \ln^2(1-\omega))\frac{1+(1-\omega)^2}{\omega},$$
$$A_2(\omega) = \frac{28}{9} + \frac{65}{18}\omega + \frac{44}{9}\omega^2 + \left(-12 - 5\omega - \frac{8}{3}\omega^2\right)\ln\,\omega$$
$$+(4+\omega)\ln^2\omega + 2\omega\ln(1-\omega)$$
$$+\left(-2\ln\,\omega\,\ln(1-\omega) + \frac{1}{2}\ln^2\omega + \frac{11}{3}\ln(1-\omega)\right.$$
$$\left.+\ln^2(1-\omega) - \frac{1}{6}\pi^2 + \frac{1}{2}\right)\frac{1+(1-\omega)^2}{\omega}$$
$$-\frac{1+(1+\omega)^2}{\omega}\int_{\omega/1+\omega}^{1/1+\omega}\frac{\mathrm{d}z}{z}\ln\,\frac{1-z}{z}$$

and

$$A_3(\omega) = -\frac{4}{3}\omega - \left(\frac{20}{9} + \frac{4}{3}\ln(1-\omega)\right)\frac{1+(1-\omega)^2}{\omega}.$$

C_A, C_G, C_F and T_R are constants associated with the colour $SU(3)$ group and $C_A = C_G = N_C = 3, C_F = (N_C^2 - 1)/2N_C$ and $T_R = 1/2$. N_C is the number of colours.

Now let us consider the Regge behaviour of gluon distribution function [1,8–10] as

$$G(x,t) = T_1(t)x^{-\lambda}, \tag{3}$$

where $T_1(t)$ is a function of t only and λ is the Regge intercept. Since the DGLAP evolution equations of gluon and singlet structure functions in leading and next-to-leading order are in the same forms of derivative with respect to t, we consider the ansatz [1] for simplicity,

$$G(x,t) = K(x)F_2^s(x,t), \tag{4}$$

where $K(x)$ is a parameter to be determined from phenomenological analysis and we assume $K(x) = K$, ax^b or ce^{dx} where K, a, b, c and d are constants. Now

$$F_2^s(x/\omega,t) = \frac{1}{K(x/\omega)}G(x/\omega,t) = \frac{\omega^\lambda}{K(x/\omega)}G(x,t). \tag{5}$$

Putting eqs (3) and (5) in eq. (2), we get

$$\frac{\partial G(x,t)}{\partial t} - G(x,t)P(x,t) = 0, \tag{6}$$

where

$$P(x,t) = \frac{\alpha_s(t)}{2\pi}f_1(x) + \left(\frac{\alpha_s(t)}{2\pi}\right)^2 f_2(x),$$

$$f_1(x) = 6\left(\frac{11}{12} - \frac{N_\mathrm{f}}{18} + \ln(1-x)\right) + 6\int_x^1 \mathrm{d}\omega\left[\frac{(\omega^{\lambda+1}-1)}{1-\omega} + \left(\omega(1-\omega) + \frac{1-\omega}{\omega}\right)\omega^\lambda + \frac{2}{9}\left(\frac{1+(1-\omega)^2}{\omega}\right)\frac{\omega^\lambda}{K(x/\omega)}\right]$$

and

$$f_2(x) = \int_x^1 \left[\frac{52}{3}\omega^{\lambda-1} + A(\omega)\frac{\omega^\lambda}{K(x/\omega)}\right]\mathrm{d}\omega.$$

For possible solutions in NLO, we have taken the expression for $(\alpha_s(t)/2\pi)$ up to LO correction and we have to put an extra assumption $(\alpha_s(t)/2\pi)^2 = T_0(\alpha_s(t)/2\pi) = T_0T(t)$ [11,12], where T_0 is a numerical parameter and $T(t) = (\alpha_s(t)/2\pi)$. But T_0 is not arbitrary. We choose T_0 such that difference between $T^2(t)$ and $T_0T(t)$ is minimum in the region of our discussion (figure 1a). Equation (6) reduces to

$$\frac{\partial G(x,t)}{\partial t} - \frac{G(x,t)}{t}P(x) = 0 \tag{7}$$

with

$$P(x) = \frac{2}{\beta_0}f_1(x) + T_0\frac{2}{\beta_0}f_2(x).$$

Integrating eq. (7) we get

$$G(x,t) = Ct^{P(x)}, \tag{8}$$

where C is a constant of integration.

Applying initial conditions at $x = x_0, G(x,t) = G(x_0,t)$, and at $t = t_0, G(x,t) = G(x,t_0)$, we found the t and x-evolution equations for the gluon distribution function in NLO respectively as

$$G(x,t) = G(x,t_0)(t/t_0)^{P(x)} \tag{9}$$

and

$$G(x,t) = G(x_0,t)t^{\{P(x)-P(x_0)\}}. \tag{10}$$

Now ignoring the quark contribution to the gluon distribution function we get from the evolution eq. (2)

$$\frac{\partial G(x,t)}{\partial t} = \frac{\alpha_s(t)}{2\pi}\left\{6\left(\frac{11}{12} - \frac{N_\mathrm{f}}{18} + \ln(1-x)\right)G(x,t) + 6I'_g\right\} + \left[\frac{\alpha_s(t)}{2\pi}\right]^2\int_x^1\left[\frac{52}{3}\frac{1}{\omega}G(x/\omega,Q^2)\right]\mathrm{d}\omega, \tag{11}$$

where

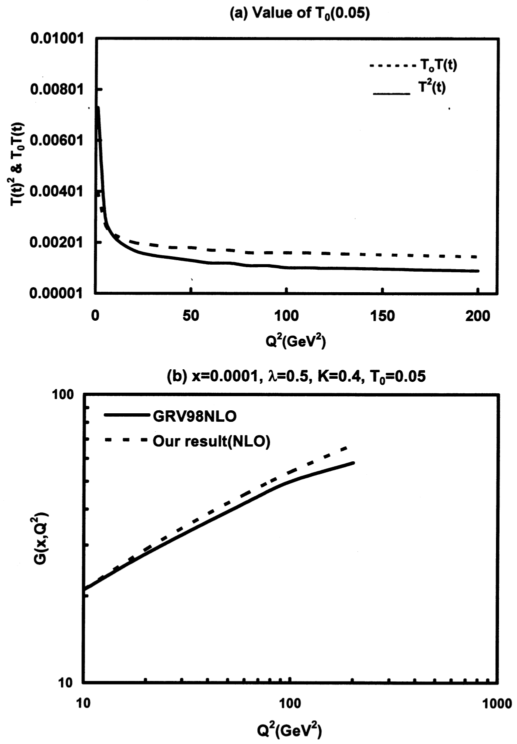

Figure 1. (**a**) The variation of $T(t)^2$ and $T_0T(t)$ with Q^2 for $T_0 = 0.05$. (**b**) The best-fit graphs of our result of t-evolution of gluon distribution function up to NLO for $\lambda = 0.5$, $T_0 = 0.05$ and $K(x) = K$ for the representative values of x presented with GRV98NLO parametrization $x = 10^{-4}$. Data points at lowest Q^2 values are taken as input to test the evolution equation (9).

$$I'_g = \int_x^1 \mathrm{d}\omega \left[\frac{\omega G(x/\omega,t) - G(x,t)}{1-\omega} + \left(\omega(1-\omega) + \frac{1-\omega}{\omega}\right) G(x/\omega,t)\right].$$

Pursuing the same procedure as above, we get the t and x-evolution equations for the gluon distribution function ignoring the quark contribution up to NLO respectively as

$$G(x,t) = G(x,t_0)(t/t_0)^{B(x)} \tag{12}$$

and

$$G(x,t) = G(x_0,t)t^{\{B(x)-B(x_0)\}}. \tag{13}$$

Here

$$B(x) = \frac{2}{\beta_0} f_3(x) + T_0 \frac{2}{\beta_0} f_4(x),$$

$$f_3(x) = 6\left(\frac{11}{12} - \frac{N_\mathrm{f}}{18} + \ln(1-x)\right)$$
$$+6\int_x^1 \mathrm{d}\omega \left[\frac{(\omega^{\lambda+1}-1)}{1-\omega} + \left(\omega(1-\omega) + \frac{1-\omega}{\omega}\right)\omega^\lambda\right]$$

and

$$f_4(x) = \int_x^1 \left[\frac{52}{3}\omega^{\lambda-1}\right] \mathrm{d}\omega.$$

3. Results and discussions

A new description of t and x-evolutions of gluon distribution function is presented through eqs (9) and (10), where we solved DGLAP evolution equation for gluon distribution function up to NLO considering Regge behaviour of distribution functions. In eqs (12) and (13), we obtained the description of gluon distribution function ignoring the quark contribution in the DGLAP evolution equation for gluon distribution function up to NLO. The contribution of quark to gluon distribution functions theoretically should decrease for $x \to 0, Q^2 \to \infty$. So we are interested to see the contribution of quark to gluon distribution functions in our region of discussion. We compare our result of t-evolution of gluon distribution function up to NLO with GRV98NLO [13] global parametrization at $Q^2 = 100$ GeV^2 and the result of x-evolution with MRST2004 [14], GRV98LO [13] and GRV98NLO [13] global parametrizations at several medium to high-Q^2 range. Along with the NLO results we also presented our LO results [1]. We compare our results from eqs (9) and (10) for $K(x) = K$, ax^b and ce^{dx}, where K, a, b, c and d are constants. In our work, we found that the values of the gluon distribution function remain almost same for $b < 0.00001$ and $d < 0.00001$. We have chosen $b = d = 0.00001$ for our calculation and the best-fit graphs are observed by changing the values of K, a and c. As the value of λ is close to 0.5 in quite a broad range of low-x [1,8,14], we have taken $\lambda = 0.5$ in our calculation.

In figure 1a we plot $T(t)^2$ and $T_0T(t)$, where $T(t) = \alpha_s(t)/2\pi$ against Q^2 in the Q^2 range $0 \le Q^2 \le 200$ GeV^2 as required by the data used by us. Here we observe that for $T_0 = 0.05$, errors become minimum in the Q^2 range $10 \le Q^2 \le 200$ GeV^2. From the graph it is clear that the difference between the values of $T(t)^2$ and $T_0T(t)$ in this range is negligible. In figure 1b we compare our result of t-evolution of gluon distribution function up to NLO with GRV98NLO gluon distribution parametrization for $K(x) = K$ at $x = 10^{-4}$. Also while compared with GRV98NLO gluon distribution parametrization for $K(x) = ax^b$ and ce^{dx} at $x = 10^{-4}$ we get the same graph. The best-fit results are for values $K = a = c = 0.4$. The figures show good agreement of our result with GRV98NLO parametrization at low-x. In figures 2a–2d we compare our result of x-evolution of gluon distribution function up to NLO each for $K(x) = K$ with GRV98NLO global parametrizations at $Q^2 = 20$, 40, 60, 100 GeV^2 respectively and best-fit results correspond to $K = -0.34$ for $Q^2 = 20$ GeV^2 and $K = -0.27$ for $Q^2 = 40$, 60 and 100 GeV^2. We have also compared our result for $K(x) = ax^b$ and ce^{dx} with GRV98NLO global parametrizations at $Q^2 = 20$, 40,

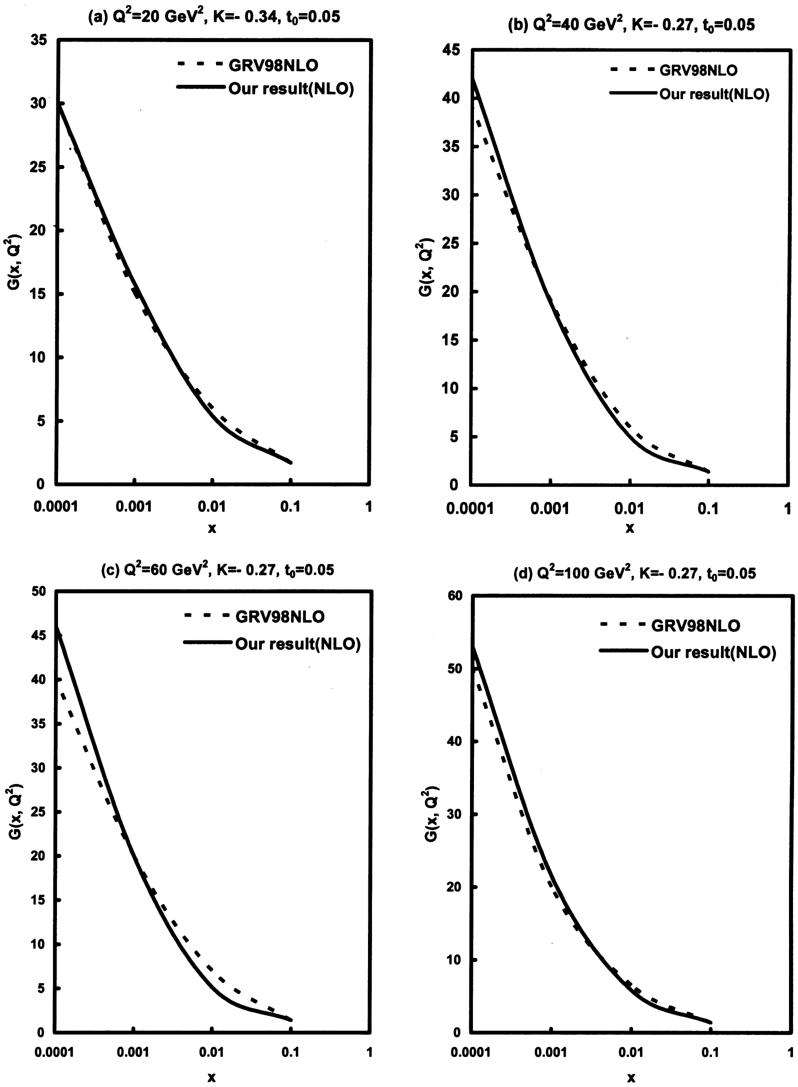

Figure 2. (**a**)–(**d**) are the best-fit x-evolution graphs of our result with GRV98NLO parametrization for $\lambda = 0.5$, $T_0 = 0.05$, $K(x) = K$ and $Q^2 = 20$, 40, 60 and 100 GeV^2 respectively. Data points at $x = 0.1$ are taken as input to test the evolution equation (10).

60, 100 GeV^2 respectively and found the same graphs as for $K(x) = K$. The best-fit results correspond to $a = c = -0.34$ for $Q^2 = 20$ GeV^2 and $a = c = -0.27$ for $Q^2 = 40$, 60 and 100 GeV^2. In some recent papers [15], Choudhury and Sahariah presented a form of gluon distribution function at low-x obtained from a unique solution with one single initial condition through the application of the method

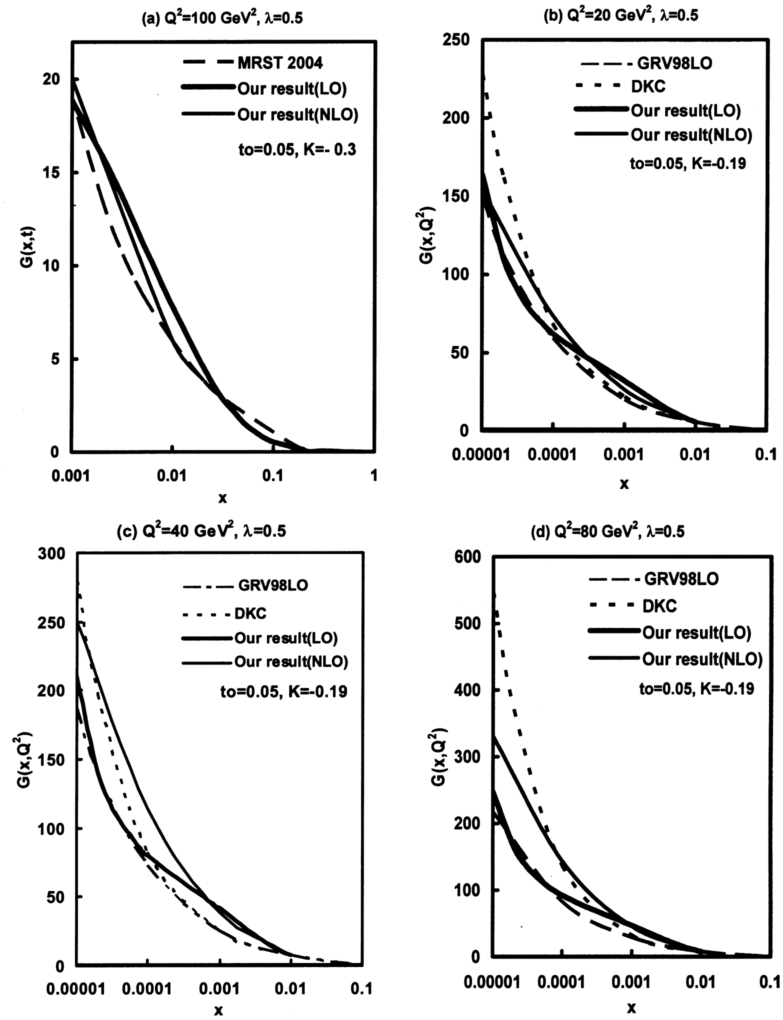

Figure 3. (**a**) The best-fit x-evolution graph of our result with MRST2004 parametrization for $Q^2 = 100$ GeV^2. Data points at $x = 0.02$ is taken as input to test the evolution equation (10). (**b**)–(**d**) are the best-fit x-evolution graphs of our result with GRV98LO parametrization for $Q^2 = 20$, 40 and 80 GeV^2 respectively. Data points at $x = 0.01$ are taken as input to test the evolution equation (10). Here graphs are observed for $\lambda = 0.5$, $T_0 = 0.05$ and $K(x) = K$. Along with the NLO result we presented our LO results also.

of characteristics [16]. They have overcome the limitations of non-uniqueness of some of the earlier approaches [17]. So, it is theoretically and phenomenologically favoured over the earlier approximations. We have presented these results with GRV98NLO parametrizations and our results, and found that with decreasing x we get a better fit of our result to GRV98LO parametrization in comparison with those

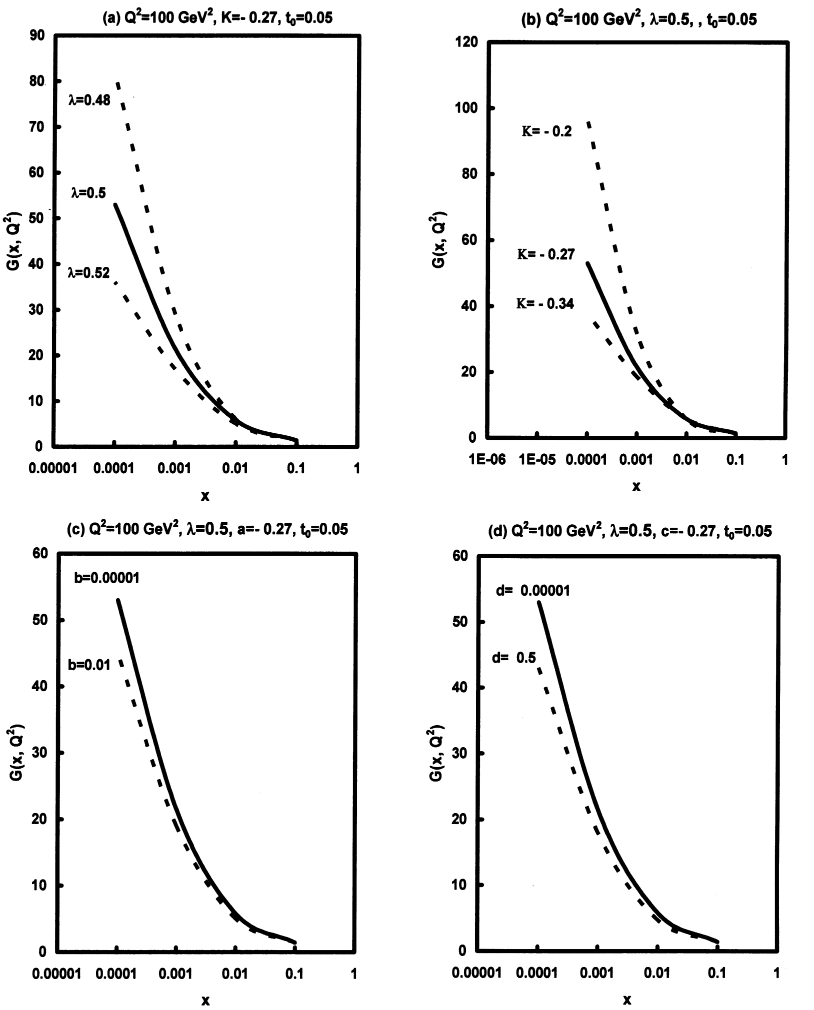

Figure 4. (**a**)–(**d**) The sensitivity of the parameters λ, K, b and d respectively at $Q^2 = 100$ GeV^2 with the best-fit graph of our results with GRV98NLO parametrization.

results. In figures 3a–3d we have compared our result of x-evolution of gluon distribution function up to NLO with MRST2004 and GRV98LO global parametrizations for $K(x) = K$ and with these results we have presented our LO results [1]. The best-fit result corresponds to $K = -0.3$ for $Q^2 = 100$ GeV^2 with MRST2004 global parametrization and $K = -0.19$ for $Q^2 = 40$, 60 and 80 GeV^2 with GRV98LO global parametrization. We have also compared our result for $K(x) = ax^b$ and ce^{dx} with MRST2004 and GRV98LO global parametrizations at $Q^2 = 40$, 60, 80 GeV^2 respectively and found the same graphs as for $K(x) = K$. The best-fit result corresponds to $a = c = -0.3$ for $Q^2 = 100$ GeV^2 with MRST2004 global parametrizations and $a = c = -0.19$ for $Q^2 = 40$, 60 and 80 GeV^2 with GRV98LO global parametrizations. From both the results it is seen that the NLO results follow the parametrization graphs more closely than the LO results. Figures 4a–4d show the sensitivity of the parameters λ, K, b and d respectively. The range of a and b are found the same as that of K. Taking the best-fit figures to the x-evolution of gluon distribution function up to NLO with GRV98NLO parametrization at $Q^2 = 100$ GeV^2, we have given the ranges of the parameters as $0.48 \leq \lambda \leq 0.52$, $-0.2 \leq K = a = c \leq -0.34$, $0.00001 \leq b \leq 0.01$ and $0.00001 \leq d \leq 0.5$.

4. Conclusions

In this paper we present an approximate analytical solution of the next-to-leading order DGLAP equation for the gluon structure function at low-x as a continuation of our earlier work at leading order [1]. We have considered the Regge behaviour of singlet structure function and gluon distribution function to solve DGLAP evolution equations. Here we find the t and x-evolutions of gluon distribution function up to NLO and compared them with GRV98NLO global parametrization. We also compared our results for both LO and NLO with MRST2004 and GRV98LO global parametrizations. From the graphs it can be concluded that our results for NLO are in good agreement with MRST2004, GRV98NLO and GRV98LO global parametrizations especially at low-x and high-Q^2 region. The x-evolution graphs obtained by solving DGLAP evolution equation up to LO and NLO can be compared and from the best-fitted graphs it is clear that the gluon distribution function up to NLO shows significantly better fitting to the parametrizations than up to LO. So the higher-order term up to NLO has appreciable contribution in the region of our discussion to the parton distribution function. The Regge behaviour of quark and gluon distribution functions is thus compatible with PQCD at that region. We were also interested to see the amount of contribution of quark to the gluon distribution function at different x and Q^2. It has been observed that in our x–Q^2 region of discussion, quark contributes appreciably to gluon distribution function even though we have taken the gluon distribution function up to NLO. So we cannot ignore the contribution of quark in our region of discussion. The results with and without quarks are not compared here since the results without quarks are far from the data as well as parametrization graphs. So, we have seen that considering Regge behaviour of distribution functions, DGLAP equations become quite simple to solve.

References

[1] U Jamil and J K Sarma, *Pramana – J. Phys.* **69**, 167 (2007), arXiv:hep-ph/0508066
[2] M Glück, E Reya and C Schuck, arXiv:hep-ph/0604116 (2006)
[3] A D Martin, W J Stirling, R S Thorne and G Watt, arXiv:hep-ph/07060459 (2007)
[4] Siegfried Bethke, arXiv:hep-ex/0407021 (2004)
[5] W Furmanski and R Petronzio, *Phys. Lett.* **B97**, 437 (1980); Ref. TH-2933-CERN (KEK scanned version); Ref. TH-2815-CERN (KEK scanned version)
[6] D K Choudhury and P K Sahariah, *Pramana – J. Phys.* **65**, 193 (2005)

[7] H Jung, QCD and Collider Physics, Lecture WS 2005/2006, University of Hamburg, http://www.desy.de/~jung/qcd_collider_physics_2005/
[8] J K Sarma and G K Medhi, *Eur. Phys. J.* **C16**, 481 (2000)
[9] G Soyez, arXiv:hep-ph/0401177 (2004)
G Soyez, *Phys. Rev.* **D69**, 096005 (2004); arXiv:hep-ph/0306113 (2003)
[10] P Bicudo, arXiv:hep-ph/0703114 (2007)
[11] A Deshamukhya and D K Choudhury, *Proc. 2nd Regional Conf. Phys. Research in North-East* (Guwahati, India, October 2001) p. 34
[12] R Rajkhowa and J K Sarma, *Indian J. Phys.* **79(1)**, 55 (2005); *Indian J. Phys.* **78(9)**, 979 (2004)
[13] M Glück, E Reya and A Vogt, *Z. Phys.* **C67**, 433 (1995); *Eur. Phys. J.* **C5**, 461 (1998)
[14] A D Martin, M G Ryskin and G Watt, arXiv:hep-ph/0406225 (2004)
[15] D K Choudhury and P K Sahariah, *Pramana – J. Phys.* **58**, 599 (2002); *ibid.* **60**, 563 (2003)
[16] S J Farlow, *Partial differential equations for scientists and engineers* (John Wiley, New York, 1982)
[17] J K Sarma, D K Choudhury and G K Medhi, *Phys. Lett.* **B403**, 139 (1997)