PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 71, No. 1
July 2008
pp. 15–22

Bianchi Type-I cosmological mesonic stiff fluid models in Lyra's geometry

S D KATORE[1], S V THAKARE[2,*] and K S ADHAO[3]
[1]PGTD of Mathematics, Rajsthan Aryan Mahavidyalaya, Washim 444 505, India
[2]Department of Mathematics, Babasaheb Naik College of Engineering, Pusad 445 215, India
[3]PGTD of Mathematics, Sant Gadge Baba Amravati University, Amravati 444 602, India
*Corresponding author
E-mail: katoresd@rediffmail.com; s_v_thakare@rediffmail.com; ati_ksadhav@yahoo.co.in

MS received 13 October 2007; revised 25 February 2008; accepted 4 April 2008

Abstract. Bianchi Type-I cosmological models in Lyra's geometry are obtained when the source of gravitational field is a perfect fluid coupled with massless mesonic scalar field. Some physical and kinematical properties of the models are also discussed.

Keywords. Cosmology; Bianchi-I; Lyra's geometry; stiff fluid; massless mesonic scalar feld.

PACS Nos 98.80.K; 04.20.Ex

1. Introduction

Based on the cosmological principle, Einstein introduced the cosmological constant into his field equations in order to obtain a static model of the Universe since without the cosmological term his field equations admit only nonstatic cosmological models for nonzero energy density. Lyra [1] proposed a modification of Riemannian geometry by introducing a gauge function into the structureless manifold that bears close resemblance to Weyl's geometry [2]. Sen [3,4] found that the static model with finite density in Lyra's modified Riemannian geometry is similar to the static Einstein model. Halford [5] developed a cosmological theory within the framework of Lyra's geometry which gives rise to nonstatic perfect fluid models. The energy momentum tensor T^{ij} is not conserved in Lyra's geometry.

Further, it was shown by Halford [6] that the scalar–tensor treatment based on Lyra's geometry predicts some effects, within the observational limit, as in Einstein theory.

Eminent researchers such as Karade and Borikar [7], Reddy and Innaiah [8], Reddy and Venkateswarlu [9,10], Reddy *et al* [11], Bhamra [12], Bheesham [13], Hoyle [14], Singh and Singh [15], Singh and Desikan [16], Rahmann *et al* [17,18], Mahanta and Mukherjee [19], have studied several aspects of cosmological models based on Lyra's geometry.

In the present investigation, we have constructed Bianchi Type-I cosmological model in Lyra's geometry with time-dependent displacement vector field in the presence of perfect fluid coupled with massless mesonic scalar field. This paper is organised as follows: We have reproduced the field equations for LRS Bianchi Type-I metric in Lyra's geometry in §2 and mentioned the consequences of the field equations in §3. We have solved the field equations for three different cases and obtained exact solutions in §4. We have discussed some physical and kinematical properties of the models in §5.

2. The field equations

We consider the LRS metric for the spatially homogeneous Bianchi-Type I cosmological model

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2(\mathrm{d}x^2 + \mathrm{d}y^2) - B^2\mathrm{d}z^2, \tag{1}$$

where A and B are the functions of cosmic time t only.

The field equations in normal gauge for Lyra's geometry as obtained by Sen [4] are

$$R_{ij} - \frac{1}{2}g_{ij}R = -(T_{ij} + M_{ij}) - \frac{3}{2}\phi_i\phi_j + \frac{3}{4}g_{ij}\phi_\alpha\phi^\alpha, \tag{2}$$

where

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}, \tag{3}$$

and

$$M_{ij} = V_i V_j - \frac{1}{2}g_{ij}V_k V^k \tag{4}$$

are respectively the energy–momentum tensors corresponding to perfect fluid and massless mesonic scalar field satisfying the equation of state

$$T^{ij}_{;j} = 0, \tag{5}$$

and the Klein–Gordan wave equation

$$g^{ij}V_{;ij} = 0 \tag{6}$$

respectively.

Here ρ, p and u_i are respectively the energy density, equilibrium pressure and four-velocity vector of the cosmic fluid distribution. V is the massless scalar field and ϕ_i is the time-dependent Lyra's displacement vector, i.e.

$$\phi_i = (0\ , 0, 0, \lambda(t)).$$

The field equation (2) together with (5) and (6) for the space-time metric (1) leads to the following set of equations:

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}+\frac{3\lambda^2}{4}=-\left(p+\frac{V_4^2}{2}\right), \tag{7.1}$$

$$2\frac{A_{44}}{A}+\left(\frac{A_4}{A}\right)^2+\frac{3\lambda^2}{4}=-\left(p+\frac{V_4^2}{2}\right), \tag{7.2}$$

$$\left(\frac{A_4}{A}\right)^2+2\frac{A_4B_4}{AB}-\frac{3\lambda^2}{4}=\left(\rho+\frac{V_4^2}{2}\right), \tag{7.3}$$

$$(p+\rho)\left(2\frac{A_4}{A}+\frac{B_4}{B}\right)+\rho_4=0, \tag{7.4}$$

$$V_{44}+\left(2\frac{A_4}{A}+\frac{B_4}{B}\right)V_4=0. \tag{7.5}$$

Hereafterward the suffix 4 after a field variable represents ordinary differentiation with respect to time.

3. Consequences of the field equations

From eqs (7.1) and (7.2), we get

$$\frac{B_{44}}{B}+\frac{A_4B_4}{AB}=\frac{A_{44}}{A}+\left(\frac{A_4}{A}\right)^2. \tag{8}$$

We assume A to be some arbitrary function of B, say

$$A=\psi\left\{B\left(t\right)\right\} \tag{9}$$

so that eq. (8) becomes

$$\left(\frac{\psi'}{\psi}-\frac{1}{B}\right)B_{44}+\left[\frac{\psi'}{\psi}+\left(\frac{\psi'}{\psi}\right)^2-\frac{\psi'}{\psi B}\right]B_4^2=0 \tag{10}$$

which results in the following possibilities:
(i)

$$\frac{\psi'}{\psi}-\frac{1}{B}=0 \quad \text{and} \quad \frac{\psi''}{\psi}+\left(\frac{\psi'}{\psi}\right)^2-\frac{\psi'}{\psi B}=0. \qquad (11.1\text{a}), (11.1\text{b})$$

(ii)

$$B_{44} = 0 \quad \text{and} \quad \frac{\psi''}{\psi} + \left(\frac{\psi'}{\psi}\right)^2 - \frac{\psi'}{\psi B} = 0. \qquad (11.2a), (11.2b)$$

(iii)

$$B_4 = 0. \qquad (11.3)$$

Here prime over ψ denotes ordinary differentiation with respect to B.

4. Solutions of field equations

We have only five highly nonlinear field equations (7.1)–(7.5) containing six unknowns, viz. A, B, p, ρ, V and λ. In order to obtain its exact solution, we assume one more physically reasonable condition amongst the variables. We consider here the effective stiff fluid distribution (Banerjee *et al* [21], Mohanty and Sahu [20]).

$$p = \rho. \qquad (12)$$

Case i

Using eq. (11.1a) in eq. (11.1b) and then on integration we get

$$\psi = a_1 B + a_2, \qquad (13)$$

where a_1 and a_2 are the constants of integration. Using (13) in (11.1a) we obtain $a_2 = 0$ so that eq. (13) becomes

$$\psi = a_1 B. \qquad (14)$$

In view of (9) and (14) we have

$$A = a_1 B. \qquad (15)$$

Now using eq. (15) in eqs (7.1)–(7.3) we get

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 + \frac{3\lambda^2}{4} = -\left(p + \frac{V_4^2}{2}\right), \qquad (16)$$

$$3\left(\frac{B_4}{B}\right)^2 - \frac{3\lambda^2}{4} = \left(\rho + \frac{V_4^2}{2}\right). \qquad (17)$$

In order to obtain an explicit form of physical parameters, we consider here stiff fluid distribution given by (12). Therefore, eqs (16) and (17) yield

$$B = (a_3 t + a_4)^{1/3}, \qquad (18)$$

where a_3 ($\neq 0$) and a_4 are constants of integration.

In view of eqs (12), (15) and (18), eq. (7.4) gives

$$V = \frac{a_5}{a_1^2 a_3} \log\left(a_3 t + a_4\right) + a_6, \tag{19}$$

where a_5 ($\neq 0$) is the constant of integration and a_6 is the arbitrary constant.

In view of eqs (12), (15) and (18), eq. (7.4) gives

$$p = \rho = \frac{a_7}{\left(a_3 t + a_4\right)^2}, \tag{20}$$

where a_7 ($\neq 0$) is the constant of integration.

Using eqs (18), (19) and (20) in (7.2), we get

$$\lambda^2 = \frac{a_8}{\left(a_3 t + a_4\right)^2}, \tag{21}$$

where

$$a_8 = \frac{4}{9} a_3^2 - \frac{2}{3} \frac{a_5^2}{a_1^4} - \frac{4}{7} a_7 \quad \text{is the new arbitrary constant.}$$

In this case the metric (1) can be written in the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \left(a_3 t + a_4\right)^{2/3} \left\{a_1^2 \mathrm{d}x^2 + a_1^2 \mathrm{d}y^2 + \mathrm{d}z^2\right\} \tag{22}$$

which can be transformed, by a proper choice of coordinates and absorbing constants in the differentials, to the form

$$\mathrm{d}s^2 = \mathrm{d}T^2 - T^{2/3}(\mathrm{d}X^2 + \mathrm{d}Y^2 + \mathrm{d}Z^2). \tag{23}$$

Case ii

On integration, eq. (11.2a) yields

$$B = b_1 t + b_2, \tag{24}$$

where b_1 ($\neq 0$) and b_2 are the constants of integration.

Further, using (24) in (11.2b) and in view of (9), we get

$$A = \left(b_3 B^2 + b_4\right)^{1/2}, \tag{25}$$

where b_3 ($\neq 0$) and b_4 are constants of integration.

On integrating eq. (7.5), with the aid of eqs (25) and (24), we get

$$V = \frac{b_5}{b_1 b_4} \log \frac{(b_1 t + b_2)}{\sqrt{b_3 (b_1 t + b_2)^2 + b_4}} + b_6, \tag{26}$$

where b_5 and b_6 are constants of integration.

In view of eqs (12), (24) and (25), eq. (7.4) yields

$$p = \rho = \frac{b_7}{[b_1 t + b_2]^2 \left[b_3 (b_1 t + b_2)^2 + b_4\right]^2}. \tag{27}$$

Further, in view of eqs (24)–(27), eq. (7.2) yields

$$\lambda^2 = \frac{4B^2 b_1 b_3 (B - 2b_1)(b_3 B^2 + b_4) - 4b_7 - 2b_5^2}{3(b_3 B^2 + b_4)^2 B^2}, \tag{28}$$

where $B = (b_1 t + b_2)$.

In this case the metric (1) can be written as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \{b_3 (b_1 t + b_2)^2 + b_4\}\{\mathrm{d}x^2 + \mathrm{d}y^2\} - (b_1 t + b_2)^2 \mathrm{d}z^2$$

which can be transformed through a proper choice of coordinates to the form

$$\mathrm{d}s^2 = \mathrm{d}T^2 - (b_3 T^2 + b_4)(\mathrm{d}X^2 + \mathrm{d}Y^2) - T^2 \mathrm{d}Z^2. \tag{29}$$

Case iii

On integrating eq. (11.3) we get

$$B = c_1. \tag{30}$$

In view of eqs (12) and (30), eqs (7.1)–(7.5) yield

$$A = (c_2 t + c_3)^{1/2}, \tag{31}$$

$$p = \rho = \frac{c_4}{(c_2 t + c_3)^{1/2}}, \tag{32}$$

$$V = \frac{c_5}{c_2} \log(c_2 t + c_3) + c_6, \tag{33}$$

$$\lambda^2 = \frac{c_7}{(c_2 t + c_3)^2}, \tag{34}$$

where c_1, c_2, c_3, c_4, c_5 and c_6 are constants of integration and $c_7 = \frac{8}{3}(2c_2^2 - 2c_4 - c_5^2)$ is the new arbitrary constant.

In this case the line element (1) takes the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - (c_2 t + c_3)(\mathrm{d}x^2 + \mathrm{d}y^2) - c_1^2 \mathrm{d}z^2$$

which can be transformed, by a proper choice of coordinates and absorbing constants in the differentials, to the form

$$\mathrm{d}s^2 = \mathrm{d}T^2 - T^2(\mathrm{d}X^2 + \mathrm{d}Y^2) - \mathrm{d}Z^2. \tag{35}$$

5. Physical and kinematical properties of the solution

In this section we study the physical and geometrical properties of the models obtained in the preceding sections.

Case i

The massless mesonic scalar field V, energy density ρ, equilibrium pressure p and Lyra's displacement vector field λ for the model (23) are given by

$$p = \rho = \frac{a_7}{T^2}, \tag{36a, b}$$

$$\lambda^2 = \frac{a_8}{T^2}, \tag{36c}$$

$$V = a_9 \log T + a_{10}, \tag{36d}$$

where α_9 and α_{10} are the new arbitrary constants.

Case ii

The massless mesonic scalar field V, energy density ρ, equilibrium pressure p and Lyra's displacement vector λ for the model (29) are given by

$$V = b_8 \log \frac{T}{\sqrt{b_3 T^2 + b_4}} + b_6, \tag{37a}$$

$$p = \rho = \frac{b_7}{(b_3 T^3 + b_4 T)^2}, \tag{37b, c}$$

$$\lambda^2 = \frac{4T^2(T - 2b_1)(b_3 T^2 + b_4) b_1 b_3 - 4b_7 - 2b_5^2}{3(b_3 T^3 + b_4 T)^2}. \tag{37d}$$

Case iii

The massless mesonic scalar field V, energy density ρ, equilibrium pressure p and Lyra's displacement vector field λ for the model (35) are given by

$$V = c_8 \log T + c_6, \tag{38a}$$

$$p = \rho = \frac{c_4}{T^2}, \tag{38b, c}$$

$$\lambda^2 = \frac{c_7}{T^2}. \tag{38d}$$

6. Conclusion

In this paper we have obtained exact solutions of Sen equations in Lyra's geometry for time-dependent displacement vector field in the presence of nonmassive mesonic stiff fluid distribution. From eqs (36)–(38) we observed that the energy density, pressure and displacement vector λ decrease with the increase in the age of Universe. This is quite different from the situation in general relativity by Banerjee *et al* [21]. Here $p\ (=\rho) \to 0$ as $T \to \infty$ and $p\ (=\rho) \to \infty$ as $T \to 0$. This shows the presence of Big Bang singularity at initial epoch. It is interesting to note that, the models (23), (29) and (35) obtained so far are free from singularity $t = 0$. Moreover, we find that the energy density and pressure of the fluid decrease with the increase in the age of the Universe. This is quite different from the situation in general relativity by Banerjee *et al* [21].

Acknowledgment

Authors are grateful to Dr T M Karade (India) for constant encouragement and fruitful discussion and are very much thankful to the referee for his valuable comments for the improvement of this paper.

References

[1] G Lyra, *Math. Z.* **54**, 52 (1951)
[2] H Weyl, *Sber Preuss. Acad. Wiss.* 465 (1918)
[3] D K Sen and K A Dunn, *J. Math. Phys.* **12**, 578 (1971)
[4] D K Sen, *Z. Phys.* **149**, 311 (1957)
[5] W D Halford, *Aust. J. Phys.* **23**, 863 (1970)
[6] W D Halford, *J. Math. Phys.* **13**, 1399 (1972)
[7] T M Karade and S M Borikar, *Gen. Relativ. Gravit.* **9**, 431 (1978)
[8] D R K Reddy and P Innaiah, *Astrophys. Space Sci.* **123**, 49 (1978)
[9] D R K Reddy and R Venkateswarlu, *Astrophys. Space Sci.* **136**, 191 (1987)
[10] D R K Reddy and R Venkateswarlu, *Astrophys. Space Sci.* **149**, 287 (1988)
[11] D R K Reddy, R Venkateswarlu and C C R Murthy, *J. Indian Math. Soc.* **68(1–4)**, 145 (2001)
[12] K S Bhamra, *Aust. J. Phys.* **27**, 541 (1974)
[13] A Bheesham, *Aust. J. Phys.* **41**, 831 (1988)
[14] F Hoyle, *Mon. Not. R. Astron. Soc.* **108**, 252 (1948)
[15] T Singh and G P Singh, *J. Math. Phys.* **32**, Z456 (1991)
[16] G P Singh and Kalyani Desikan, *Pramana – J. Phys.* **49(2)**, 205 (1997)
[17] F Rahmann, N Begum and S Das, *Astrophys. Space Sci.* **XXX**, 1 (2004)
[18] F Rahmann, S Chakraborty, S Das, M Hossain and J Bera, *Pramana – J. Phys.* **60(3)**, 453 (2003)
[19] P Mahanta and Abhijit Mukherjee, *Ind. J. Pure Appl. Math.* **32(2)**, 199 (2001)
[20] G Mohanty and S K Sahu, *Astrophys. Space Sci.* **291**, 75 (2004)
[21] A Banerjee, S B Duttachoudhary and A K Sanyal, *J. Math. Phys.* **26(11)**, 3010 (1985)