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Deformed special relativity with an invariant minimum speed and its cosmological implications

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Abstract. The paper aims to introduce a new symmetry principle in the space-time geometry through the elimination of the classical idea of rest and by including a universal minimum speed limit in the subatomic world. Such a limit, unattainable by particles, represents the preferred reference frame associated with a universal background field that breaks Lorentz symmetry. Thus, the structure of space-time is extended due to the presence of a vacuum energy density, which leads to a negative pressure at cosmological scales. The tiny values of the cosmological constant and the vacuum energy density shall be successfully obtained, which are in good agreement with current observational results.

Keywords. Minimum speed; ultra-referential; background field; cosmological constant.

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1. Introduction

Driven by an urge to search for new fundamental symmetries in nature [1], the paper attempts to implement a uniform background field into the flat space-time. Such a background field connected to a uniform vacuum energy density represents a preferred reference frame, which leads us to postulate a universal minimum speed limit for particles with very large wavelengths (very low energies).

The idea that some symmetries of the fundamental theory of quantum gravity may have non-trivial consequences for cosmology and particle physics at very low energies is interesting and indeed quite reasonable. So, it seems that the idea of a universal minimum speed as one of the first attempts of Lorentz symmetry violation could have the origin from the fundamental theory of quantum gravity at very low energies (very large wavelengths).

Besides quantum gravity for the Planck minimum length l_{P} (very high energies), the new symmetry idea of a minimum velocity V could appear due to the indispensable presence of gravity at quantum level for particles with very large wavelengths (very low energies). So we expect that such a universal minimum velocity V also depends on fundamental constants such as, for instance, G (gravitation) and $\hbar$ (quantum physics). In this sense, there could be a relation between V and l_{P} since $l_{\mathrm{P}} \propto (G\hbar)^{1/2}$. The origin of V and a possible connection between V and l_{P} shall be deeply investigated in a future work.

The hypothesis of the lowest non-null limit of speed for low energies ($v \ll c$) in the space-time results in the following physical reasonings:

– In non-relativistic quantum mechanics, the plane-wave wave function ($A\mathrm{e}^{\pm ipx}$) which represents a free particle is an idealization that is impossible to conceive under physical reality. In the event of such an idealized plane-wave, it would be possible to find with certainty the reference frame that cancels its momentum ($p = 0$), so that the uncertainty on its position would be $\Delta x = \infty$. However, the presence of an unattainable minimum limit of speed emerges in order to forbid the ideal case of a plane-wave wave function ($p =$ constant or $\Delta p = 0$). This means that there is no perfect inertial motion ($v =$ constant) such as a plane-wave, except the privileged reference frame of a universal background field connected to an unattainable minimum speed limit V, where p would vanish. However, since such a minimum speed V (universal background frame) is unattainable for the particles with low energies (large length scales), their momentum can actually never vanish when one tries to be closer to such a preferred frame (V).

On the other hand, according to special relativity (SR), the momentum cannot be infinite since the maximum speed c is also unattainable for a massive particle, except for the photon ($v = c$) as it is a massless particle.

This reasoning allows us to think that the electromagnetic radiation (photon: ‘$c - c$’$= c$) as well as the massive particle (‘$v - v$’$> V(\neq 0)$ for $v < c$) are in equal footing in the sense that it is not possible to find a reference frame at rest ($v_{\mathrm{relative}} = 0$) for any speed transformation in a space-time with both maximum and minimum speed limits. Therefore, such a deformed special relativity will be termed as symmetrical special relativity (SSR). We will look for new speed transformations of SSR in the next section.

The dynamics of particles in the presence of a universal background reference frame connected to V is within the context of ideas of Sciama [2], Schrödinger [3] and Mach [4], where there should be an ‘absolute’ inertial reference frame in relation to which we have the inertia of all moving bodies. However, we must emphasize

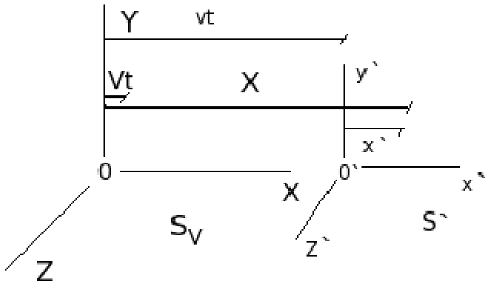

Figure 1. S' moves with a velocity v with respect to the background field of the covariant ultra-referential S_V. If $V \to 0$, S_V is eliminated (empty space) and thus the Galilean frame S takes place, recovering Lorentz transformations.

that the approach used here is not classical as Machian ideas, since the lowest (unattainable) speed limit V plays the role of a privileged (inertial) reference frame of background field instead of the 'inertial' frame of fixed stars.

It is very interesting to notice that the idea of universal background field was sought in vain by Einstein [5], suggested first by Lorentz. It was Einstein who coined the term ultra-referential as the fundamental aspect of reality to represent a universal background field [6]. Based on such a concept, let us call ultra-referential S_V to be the universal background field of a fundamental inertial reference frame connected to V.

2. Transformations of space-time and velocity in the presence of the ultra-referential S_V

SSR should contain three postulates, namely:

(1) The constancy of the speed of light c.

(2) The non-equivalence (asymmetry) of the reference frames in such a space-time, i.e., we cannot exchange the speed v (of S') for $-v$ (of S_V) by the inverse transformations, since we cannot find the rest for S' ('$v - v$' $> V$) (see figure 1). Such an asymmetry will be clarified later.

(3) The covariance of the ultra-referential (background frame) S_V connected to an unattainable minimum speed limit V (figure 1). This postulate is directly related to the second one. Such a connection will be clarified by studying the new velocity transformations to be obtained soon.

Let us assume the reference frame S' with a speed v in relation to the ultra-referential S_V according to figure 1.

Hence, to simplify, consider the motion only at one spatial dimension, namely $(1+1)D$ space-time with the background field S_V. So we write the following transformations:

$$\mathrm{d}x' = \Psi(\mathrm{d}X - \beta_* c\,\mathrm{d}t) = \Psi(\mathrm{d}X - v\,\mathrm{d}t + V\,\mathrm{d}t), \tag{1}$$

where $\beta_* = \beta\epsilon = \beta(1-\alpha)$, $\beta = v/c$ and $\alpha = V/v$, so that $\beta_* \to 0$ for $v \to V$ or $\alpha \to 1$.

$$\mathrm{d}t' = \Psi\left(\mathrm{d}t - \frac{\beta_*\,\mathrm{d}X}{c}\right) = \Psi\left(\mathrm{d}t - \frac{v\,\mathrm{d}X}{c^2} + \frac{V\,\mathrm{d}X}{c^2}\right), \tag{2}$$

being $\vec{v} = v_x\mathbf{x}$. We have $\Psi = \frac{\sqrt{1-\alpha^2}}{\sqrt{1-\beta^2}}$. If we make $V \to 0$ ($\alpha \to 0$), we recover Lorentz transformations, where the ultra-referential S_V is eliminated and simply replaced by the Galilean frame S at rest for the classical observer.

In order to get the transformations (1) and (2) above, let us consider the following more general transformations: $x' = \theta\gamma(X - \epsilon_1 vt)$ and $t' = \theta\gamma(t - \frac{\epsilon_2 vX}{c^2})$, where θ, ϵ_1 and ϵ_2 are factors (functions) to be determined. We hope all these factors depend on α, such that, for $\alpha \to 0$ ($V \to 0$), we recover Lorentz transformations as a particular case ($\theta = 1$, $\epsilon_1 = 1$ and $\epsilon_2 = 1$). By using those transformations to perform $[c^2t'^2 - x'^2]$, we find the identity: $[c^2t'^2 - x'^2] = \theta^2\gamma^2[c^2t^2 - 2\epsilon_1 vtX +$

$2\epsilon_2 vtX - \epsilon_1^2 v^2 t^2 + \frac{\epsilon_2^2 v^2 X^2}{c^2} - X^2]$. Since the metric tensor is diagonal, the crossed terms must vanish and so we assure that $\epsilon_1 = \epsilon_2 = \epsilon$. Due to this fact, the crossed terms $(2\epsilon vtX)$ are cancelled between themselves and finally we obtain $[c^2t'^2 - x'^2] = \theta^2\gamma^2(1 - \frac{\epsilon^2 v^2}{c^2})[c^2t^2 - X^2]$. For $\alpha \rightarrow 0$ ($\epsilon = 1$ and $\theta = 1$), we reinstate $[c^2t'^2 - x'^2] = [c^2t^2 - x^2]$ of SR. Now we write the following transformations: $x' = \theta\gamma(X - \epsilon vt) \equiv \theta\gamma(X - vt + \delta)$ and $t' = \theta\gamma(t - \frac{\epsilon vX}{c^2}) \equiv \theta\gamma(t - \frac{vX}{c^2} + \Delta)$, where we assume $\delta = \delta(V)$ and $\Delta = \Delta(V)$, so that $\delta = \Delta = 0$ for $V \rightarrow 0$, which implies $\epsilon = 1$. So from such transformations we extract: $-vt + \delta(V) \equiv -\epsilon vt$ and $-\frac{vX}{c^2} + \Delta(V) \equiv -\frac{\epsilon vX}{c^2}$, from where we obtain $\epsilon = (1 - \frac{\delta(V)}{vt}) = (1 - \frac{c^2\Delta(V)}{vX})$. As ϵ is a dimensionless factor, we immediately conclude that $\delta(V) = Vt$ and $\Delta(V) = \frac{VX}{c^2}$, so that we find $\epsilon = (1 - \frac{V}{v}) = (1 - \alpha)$. On the other hand, we can determine θ as follows: θ is a function of α ($\theta(\alpha)$), such that $\theta = 1$ for $\alpha = 0$, which also leads to $\epsilon = 1$ in order to recover Lorentz transformations. So, as ϵ depends on α, we conclude that θ can also be expressed in terms of ϵ, namely $\theta = \theta(\epsilon) = \theta[(1 - \alpha)]$, where $\epsilon = (1 - \alpha)$. Therefore, we can write $\theta = \theta[(1 - \alpha)] = [f(\alpha)(1 - \alpha)]^k$, where the exponent $k > 0$. The function $f(\alpha)$ and k will be estimated by satisfying the following conditions:

(i) As $\theta = 1$ for $\alpha = 0$ ($V = 0$), this implies $f(0) = 1$.

(ii) The function $\theta\gamma = \frac{[f(\alpha)(1-\alpha)]^k}{(1-\beta^2)^{1/2}} = \frac{[f(\alpha)(1-\alpha)]^k}{[(1+\beta)(1-\beta)]^{1/2}}$ should have a symmetrical behaviour, that is to say it goes to zero when closer to V ($\alpha \rightarrow 1$) and in the same way it goes to infinity when closer to c ($\beta \rightarrow 1$). In other words, this means that the numerator of the function $\theta\gamma$, which depends on α should have the same shape of its denominator, which depends on β. Due to such conditions, we naturally conclude that $k = 1/2$ and $f(\alpha) = (1 + \alpha)$, so that $\theta\gamma = \frac{[(1+\alpha)(1-\alpha)]^{1/2}}{[(1+\beta)(1-\beta)]^{1/2}} = \frac{(1-\alpha^2)^{1/2}}{(1-\beta^2)^{1/2}} = \frac{\sqrt{1-V^2/v^2}}{\sqrt{1-v^2/c^2}} = \Psi$, where $\theta = \sqrt{1-\alpha^2} = \sqrt{1-V^2/v^2}$.

The transformations shown in (1) and (2) are the direct transformations from S_V $[X^\mu = (X, ict)]$ to S' $[x'^\nu = (x', ict')]$, where we have $x'^\nu = \Omega^\nu_\mu X^\mu$ $(x' = \Omega X)$, so that we obtain the following matrix of transformation:

$$\Omega = \begin{pmatrix} \Psi & i\beta(1-\alpha)\Psi \\ -i\beta(1-\alpha)\Psi & \Psi \end{pmatrix}, \tag{3}$$

such that $\Omega \rightarrow L$ (Lorentz matrix of rotation) for $\alpha \rightarrow 0$ ($\Psi \rightarrow \gamma$). We should investigate whether the transformations (3) form a group and if so whether an invariant object like the Lorentz invariant length of a vector can be defined for this. However, these investigations can form the basis of further work.

We obtain $\det\Omega = \frac{(1-\alpha^2)}{(1-\beta^2)}[1 - \beta^2(1-\alpha)^2]$, where $0 < \det\Omega < 1$. Since V (S_V) is unattainable ($v > V$), this assures that $\alpha = V/v < 1$ and therefore the matrix Ω admits inverse ($\det\Omega \neq 0$ (> 0)). However, Ω is a non-orthogonal matrix ($\det\Omega \neq \pm 1$) and so it does not represent a rotation matrix ($\det\Omega \neq 1$) in such a space-time due to the presence of the privileged frame of background field S_V that breaks strongly the invariance of the norm of the 4-vector (limit $v \rightarrow V$ in (15) or (16)). Actually, such an effect ($\det\Omega \approx 0$ for $\alpha \approx 1$) emerges from a new relativistic physics of SSR for treating much lower energies at infra-red regime closer to S_V (very large wavelengths).

We notice that $\det\Omega$ is a function of the speed v with respect to S_V. In the approximation for $v \gg V$ ($\alpha \approx 0$), we obtain $\det\Omega \approx 1$ and so we practically reinstate the rotation behaviour of Lorentz matrix as a particular regime for higher energies. If we make $V \to 0$ ($\alpha \to 0$), we recover $\det\Omega = 1$.

The inverse transformations (from S' to S_V) are

$$\mathrm{d}X = \Psi'(\mathrm{d}x' + \beta_* c\,\mathrm{d}t') = \Psi'(\mathrm{d}x' + v\,\mathrm{d}t' - V\,\mathrm{d}t'), \tag{4}$$

$$\mathrm{d}t = \Psi'\left(\mathrm{d}t' + \frac{\beta_*\mathrm{d}x'}{c}\right) = \Psi'\left(\mathrm{d}t' + \frac{v\,\mathrm{d}x'}{c^2} - \frac{V\,\mathrm{d}x'}{c^2}\right). \tag{5}$$

In matrix form, we have the inverse transformation $X^\mu = \Omega^\mu_\nu x'^\nu$ ($X = \Omega^{-1}x'$), so that the inverse matrix

$$\Omega^{-1} = \begin{pmatrix} \Psi' & -i\beta(1-\alpha)\Psi' \\ i\beta(1-\alpha)\Psi' & \Psi' \end{pmatrix}, \tag{6}$$

where we can show that $\Psi' = \Psi^{-1}/[1-\beta^2(1-\alpha)^2]$, so that we must satisfy $\Omega^{-1}\Omega = I$.

Indeed we have $\Psi' \neq \Psi$ and therefore $\Omega^{-1} \neq \Omega^T$. This non-orthogonal aspect of Ω has an important physical implication. In order to understand such an implication, let us first consider the orthogonal (e.g. rotation) aspect of Lorentz matrix in SR. Under SR, we have $\alpha = 0$, so that $\Psi' \to \gamma' = \gamma = (1-\beta^2)^{-1/2}$. This symmetry ($\gamma' = \gamma$, $L^{-1} = L^T$) happens because the Galilean reference frames allow us to exchange the speed v (of S') for $-v$ (of S) when we are at rest at S'. However, under SSR, since there is no rest at S', we cannot exchange v (of S') for $-v$ (of S_V) due to that asymmetry ($\Psi' \neq \Psi$, $\Omega^{-1} \neq \Omega^T$). Due to this fact, S_V must be covariant, i.e., V remains invariant for any change of reference frame in such a space-time. Thus, we notice that the paradox of twins, which appears due to the symmetry by the exchange of v for $-v$ in SR should be naturally eliminated in SSR where only the reference frame S' can move with respect to S_V. So S_V remains covariant (invariant for any change of reference frame). Such a covariance will be verified soon.

We have $\det\Omega = \Psi^2[1-\beta^2(1-\alpha)^2] \Rightarrow [(\det\Omega)\Psi^{-2}] = [1-\beta^2(1-\alpha)^2]$. So we can alternatively write $\Psi' = \Psi^{-1}/[1-\beta^2(1-\alpha)^2] = \Psi^{-1}/[(\det\Omega)\Psi^{-2}] = \Psi/\det\Omega$. By inserting this result in (6) to replace Ψ', we obtain the relationship between the inverse matrix and the transposed matrix of Ω, namely $\Omega^{-1} = \Omega^T/\det\Omega$. Indeed Ω is a non-orthogonal matrix, since we have $\det\Omega \neq \pm 1$.

By dividing (1) by (2), we obtain the following speed transformation:

$$v_{\mathrm{Rel}} = \frac{v' - v + V}{1 - \frac{v'v}{c^2} + \frac{v'V}{c^2}}, \tag{7}$$

where we have considered $v_{\mathrm{Rel}} = v_{\mathrm{Relative}} \equiv \mathrm{d}x'/\mathrm{d}t'$ and $v' \equiv \mathrm{d}X/\mathrm{d}t$. v' and v are given with respect to S_V, and v_{Rel} is related between them. Let us consider $v' > v$ (see figure 2).

If $V \to 0$, the transformation (7) recovers the Lorentz velocity transformation where v' and v are given in relation to a certain Galilean frame S_0 at rest. Since (7) implements the ultra-referential S_V, the speeds v' and v are now given with respect

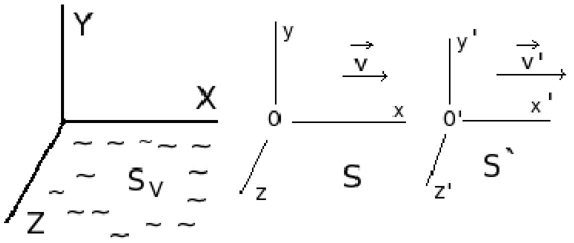

Figure 2. S_V is the covariant ultra-referential of background field. S represents the reference frame for a massive particle with speed v in relation to S_V, where $V < v < c$. S' represents the reference frame for a massive particle with speed v' in relation to S_V. In this case, we consider $V < v \leq v' \leq c$.

to S_V, which is covariant (absolute). Such a covariance is verified if we assume that $v' = v = V$ in (7). Thus, for this case, we obtain $v_{\rm Rel} =$ '$V - V$' $= V$.

Let us also consider the following cases in (7):

(a) $v' = c$ and $v \leq c \Rightarrow v_{\rm Rel} = c$. This just verifies the well-known invariance of c.

(b) if $v' > v(=V) \Rightarrow v_{\rm Rel} =$ '$v' - V$' $= v'$. For example, if $v' = 2V$ and $v = V$ $\Rightarrow v_{\rm Rel} =$ '$2V - V$' $= 2V$. This means that V really has no influence on the speed of the particles. So V works as if it were an *absolute zero of movement*, being invariant and having the same value in all directions of space of the isotropic background field.

(c) if $v' = v \Rightarrow v_{\rm Rel} =$ '$v - v$'$(\neq 0) = \frac{V}{1-\frac{v^2}{c^2}(1-\frac{V}{v})}$. From (c) let us consider two specific cases, namely,

– (c_1) assuming $v = V \Rightarrow v_{\rm Rel} =$ '$V - V$' $= V$ as verified before.

– (c_2) if $v = c \Rightarrow v_{\rm Rel} = c$, where we have the interval $V \leq v_{\rm Rel} \leq c$ for $V \leq v \leq c$.

The last case (c) shows us in fact that it is impossible to find the rest for the particle on its own reference frame S', where $v_{\rm Rel}(v)$ $(\equiv \Delta v(v))$ is a function that increases with the increase in v. However, if $V \to 0$, then we would have $v_{\rm Rel} \equiv \Delta v = 0$ and therefore it would be possible to find the rest for S', which would become simply a Galilean reference frame of SR.

By dividing (4) by (5), we obtain

$$v_{\rm Rel} = \frac{v' + v - V}{1 + \frac{v'v}{c^2} - \frac{v'V}{c^2}}. \tag{8}$$

In (8), if $v' = v = V \Rightarrow$ '$V + V$' $= V$. Indeed V is invariant, working like an *absolute zero state* in SSR. If $v' = c$ and $v \leq c$, this implies $v_{\rm Rel} = c$. For $v' > V$ and considering $v = V$, this leads to $v_{\rm Rel} = v'$. As a specific example, if $v' = 2V$ and assuming $v = V$, we would have $v_{\rm Rel} =$ '$2V + V$' $= 2V$. And if we make $v' = v$ we get $v_{\rm Rel} =$ '$v + v$' $= \frac{2v-V}{1+\frac{v^2}{c^2}(1-\frac{V}{v})}$. In Newtonian regime $(V \ll v \ll c)$, we recover $v_{\rm Rel} =$ '$v + v$' $= 2v$. In relativistic (Einsteinian) regime $(v \to c)$, we reinstate Lorentz transformation for this case $(v' = v)$, i.e., $v_{\rm Rel} =$ '$v + v$' $= 2v/(1 + v^2/c^2)$.

By joining both transformations (7) and (8) into just one, we write the following compact form:

$$v_{\rm Rel}=\frac{v'\mp\epsilon v}{1\mp\frac{v'\epsilon v}{c^2}}=\frac{v'\mp v(1-\alpha)}{1\mp\frac{v'v(1-\alpha)}{c^2}}=\frac{v'\mp v\pm V}{1\mp\frac{v'v}{c^2}\pm\frac{v'V}{c^2}}, \tag{9}$$

where $\alpha=V/v$ and $\epsilon=(1-\alpha)$. For $\alpha=0$ ($V=0$) or $\epsilon=1$, we recover Lorentz speed transformations.

Transformations for $(3+1)D$ and also a new group algebra for SSR will be sought elsewhere.

3. Covariance of the Maxwell wave equation in the presence of the ultra-referential S_V

Let us assume a light ray emitted from the frame S' (figure 1). Its equation of electrical wave at this reference frame is

$$\frac{\partial^2\vec{E}(x',t')}{\partial x'^2}-\frac{1}{c^2}\frac{\partial^2\vec{E}(x',t')}{\partial t'^2}=0. \tag{10}$$

As it is well-known, when we make the following exchange: $X\rightarrow\partial/\partial t$ and $t\rightarrow\partial/\partial X$; also $x'\rightarrow\partial/\partial t'$ and $t'\rightarrow\partial/\partial x'$, the transformations (1) and (2) for differential operators are written in the form:

$$\frac{\partial}{\partial t'}=\Psi\left[\frac{\partial}{\partial t}-\beta c(1-\alpha)\frac{\partial}{\partial X}\right], \tag{11}$$

$$\frac{\partial}{\partial x'}=\Psi\left[\frac{\partial}{\partial X}-\frac{\beta}{c}(1-\alpha)\frac{\partial}{\partial t}\right], \tag{12}$$

where $\beta=v/c$ and $\alpha=V/v$.

By squaring (11) and (12), inserting into (10) and after performing the calculations, we will finally obtain

$$\det\Omega\left(\frac{\partial^2\vec{E}}{\partial X^2}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}\right)=0, \tag{13}$$

where $\det\Omega=\Psi^2[1-\beta^2(1-\alpha)^2]$ (see (3)).

As the ultra-referential S_V is definitely inaccessible for any particle, we always have $\alpha<1$ (or $v>V$), which always implies $\det\Omega=\Psi^2[1-\beta^2(1-\alpha)^2]>0$. And as we already have shown in the last section, such a result is in agreement with the fact that we must have $\det\Omega>0$. Therefore, this will always assure

$$\frac{\partial^2\vec{E}}{\partial X^2}-\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}=0. \tag{14}$$

By comparing (14) with (10), we verify the covariance of the electromagnetic wave equation propagating in the background field of the ultra-referential S_V. Indeed we conclude that the space-time transformations in SSR also preserve the covariance of the Maxwell equations in vacuum as well as Lorentz transformations. This leads us to think that S_V works like a relativistic background field that is compatible with electromagnetism, other than it was the Galilean ether of pre-Einsteinian physics, breaking the covariance of the electromagnetism under the exchange of reference frames.

4. The flat space-time and the ultra-referential S_V

Let us consider the ultra-referential S_V as a uniform background field that fills the whole flat space-time as a perfect fluid, playing the role of a kind of de-Sitter (dS) space-time [7] shown in the next section ($\Lambda > 0$). So let us define the following metric:

$$\mathrm{d}s^2 = \Theta g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu, \tag{15}$$

where $g_{\mu\nu}$ is the well-known Minkowski metric. Θ is a scale factor that increases for very large wavelengths (cosmological scales) governed by vacuum (dS), that is to say for much lower energies, where we have $\Theta \rightarrow \infty$. On the other hand, Θ decreases to 1 for smaller scales of length, namely for higher energies ($\Theta \rightarrow 1$) where dS space-time approximates to the Minkowski metric as a special case. Θ breaks the invariance of $\mathrm{d}s$ strongly only for very large distances governed by vacuum of the ultra-referential S_V. For smaller scales of length governed by matter, we naturally restore Lorentz symmetry and the invariance of $\mathrm{d}s$. Following such considerations, let us consider Θ to be a function of speed v with respect to the background field S_V, that is,

$$\Theta = \Theta(v) = \frac{1}{(1 - \frac{V^2}{v^2})}, \tag{16}$$

such that $\Theta \approx 1$ for $v \gg V$ (Lorentz symmetry regime) and $\Theta \rightarrow \infty$ for $v \rightarrow V$ (regime of ultra-referential S_V that breaks $\mathrm{d}s$ invariance strongly, so that $\mathrm{d}s \rightarrow \infty$).

The total energy E of a particle in S_V is

$$E = \theta(\gamma mc^2) = \Psi mc^2 = mc^2 \frac{\sqrt{1 - \frac{V^2}{v^2}}}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{17}$$

where $\theta = \Theta^{-1/2} = \sqrt{1-\alpha^2}$ and $\gamma = 1/\sqrt{1-\beta^2}$, $\alpha = V/v$ and $\beta = v/c$. v is given in relation to S_V.

In (17), we observe that $E \rightarrow 0$ for $v \rightarrow V$ (S_V). For the case $v = v_0 = \sqrt{cV}$, we obtain $\theta\gamma = \Psi(v_0) = 1 \Rightarrow E = mc^2$. Actually, as a massive particle always has motion v ($V(S_V) < v < c$) with respect to the unattainable ultra-referential S_V, its proper energy mc^2 requires a non-zero motion $v(=v_0)$ in relation to S_V (see figure 3).

The momentum of the particle in relation to S_V is

$$\vec{P} = m\vec{v} \frac{\sqrt{1 - \frac{V^2}{v^2}}}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{18}$$

From (17) and (18), we show the following energy–momentum relation: $c^2\vec{P}^2 = E^2 - m^2c^4(1 - \frac{V^2}{v^2})$.

The de-Broglie wavelength of the particle in S_V is due to its motion v with respect to S_V, that is,

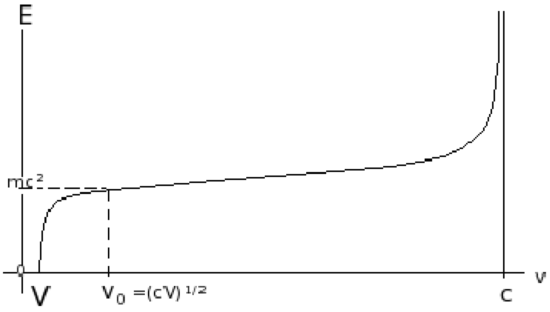

Figure 3. v_0 represents the speed in relation to S_V, from where we get the proper energy of the particle ($E_0 = mc^2$), being $\Psi_0 = \Psi(v_0) = 1$. For $v \ll v_0$ or closer to S_V ($v \to V$), a new relativistic correction on energy arises, so that $E \to 0$.

$$\lambda = \frac{h}{P} = \frac{h}{mv}\frac{\sqrt{1-\frac{v^2}{c^2}}}{\sqrt{1-\frac{V^2}{v^2}}}, \tag{19}$$

from where we have used the momentum (18) given with respect to S_V.

If $v \to c \Rightarrow \lambda \to 0$ (spatial contraction), and if $v \to V$ (S_V) $\Rightarrow \lambda \to \infty$ (spatial dilation to the infinite, breaking strongly Lorentz symmetry in SSR), it means we have cosmological wavelengths. This leads to $\Theta \to \infty$ (see (16)).

5. Cosmological implications

5.1 *Energy–momentum tensor in the presence of the ultra-referential S_V*

Let us write the 4-velocity in the presence of S_V, as follows:

$$U^\mu = \left[\frac{\sqrt{1-\frac{V^2}{v^2}}}{\sqrt{1-\frac{v^2}{c^2}}}, \frac{v_\alpha\sqrt{1-\frac{V^2}{v^2}}}{c\sqrt{1-\frac{v^2}{c^2}}}\right], \tag{20}$$

where $\mu = 0, 1, 2, 3$ and $\alpha = 1, 2, 3$. If $V \to 0$, we recover the 4-velocity of SR.

The well-known energy–momentum tensor to deal with perfect fluid is of the form

$$T^{\mu\nu} = (p+\epsilon)U^\mu U^\nu - pg^{\mu\nu}, \tag{21}$$

where U^μ is given in (20), p represents pressure and ϵ the energy density.

From (20) and (21), performing the new component T^{00}, we obtain

$$T^{00} = \frac{\epsilon(1-\frac{V^2}{v^2}) + p(\frac{v^2}{c^2}-\frac{V^2}{v^2})}{(1-\frac{v^2}{c^2})}. \tag{22}$$

If $V \to 0$, we recover the old component T^{00}.

Now, as we are interested only in obtaining T^{00} in the absence of matter, i.e., the vacuum limit connected to the ultra-referential S_V, we perform the limit of (22) as follows:

$$\lim_{v\rightarrow V} T^{00} = T^{00}_{\text{vacuum}} = \frac{p(\frac{V^2}{c^2}-1)}{(1-\frac{V^2}{c^2})} = -p. \tag{23}$$

From (22), we notice that the term $\epsilon\gamma^2(1-V^2/v^2)$ representing the matter vanishes naturally in the limit of vacuum-S_V ($v \rightarrow V$), and therefore just the contribution of vacuum prevails. As we must always have $T^{00} > 0$, we get $p < 0$ in (23). This implies a negative pressure for the vacuum energy density of the ultra-referential S_V. So we verify that a negative pressure emerges naturally from such a new tensor in the limit of S_V.

We can obtain $T^{\mu\nu}_{\text{vacuum}}$ by calculating the limit of vacuum-S_V for (21), by considering (20), as follows:

$$T^{\mu\nu}_{\text{vacuum}} = \lim_{v\rightarrow V} T^{\mu\nu} = -pg^{\mu\nu}, \tag{24}$$

where we conclude that $\epsilon = -p$. In (20), we see that the new 4-velocity vanishes in the limit of the vacuum-S_V ($v \rightarrow V$), namely $U^{\mu}_{\text{vac.}} = (0,0)$. So $T^{\mu\nu}_{\text{vac.}}$ is in fact a diagonal tensor as we hope it to be. The vacuum-S_V inherent to such a space-time works like a *sui generis* fluid in equilibrium with negative pressure, leading to a cosmological anti-gravity.

5.2 *The cosmological constant* Λ

The well-known relation [8] between the cosmological constant Λ and the vacuum energy density $\rho_{(\Lambda)}$ is

$$\rho_{(\Lambda)} = \frac{\Lambda c^2}{8\pi G}. \tag{25}$$

Let us consider a simple model of spherical universe with Hubble radius filled by a uniform vacuum energy density. On the surface of such a sphere (frontier of the observable universe), the bodies (galaxies) experience an accelerated expansion (anti-gravity) due to the whole ‘dark mass’ of vacuum inside the sphere. So we could think that each galaxy is a proof of body interacting with that big sphere of ‘dark mass’, like in the simple case of two bodies interaction. However, we need to show that there is an anti-gravitational interaction between the ordinary proof mass m and the big sphere with a ‘dark mass’ of vacuum (M_{Λ}), but let us first start from the well-known simple model of a massive proof particle m that escapes from a classical gravitational potential ϕ on the surface of a big sphere of matter, namely $E = mc^2(1-v^2/c^2)^{-1/2} \equiv mc^2(1+\phi/c^2)$, where E is its relativistic energy. Here the interval of escape velocity $0 \leq v < c$ is associated with the interval of potential $0 \leq \phi < \infty$, where we stipulate $\phi > 0$ to be the attractive (classical) gravitational potential.

Now we can show that the influence of the background field (vacuum energy inside the sphere) connected to the ultra-referential S_V (see (23)) leads to a strong repulsive (negative) gravitational potential ($\phi \ll 0$) for very low energies ($E \rightarrow 0$). In order to see this non-classical aspect of gravitation [9], we use eq. (17) just taking into account the new approximation given for very low energies ($v(\approx V) \ll c$), as follows:

$$E \approx mc^2\sqrt{1-\frac{V^2}{v^2}} \equiv mc^2\left(1+\frac{\phi}{c^2}\right), \tag{26}$$

where $\phi < 0$ (repulsive). For $v \rightarrow V$, this implies $E \rightarrow 0$, which leads to $\phi \rightarrow -c^2$. So the non-classical (most repulsive) minimum potential $\phi(V)$ ($=-c^2$) connected to the vacuum energy of S_V ($v = V$) is responsible for the cosmological anti-gravity (see also (23) and (24)). We interpret this result assuming that only an exotic 'particle' of the vacuum energy at S_V could escape from the anti-gravity ($\phi = -c^2$) generated by the vacuum energy inside the sphere (consider $v = V$ in (26)). Therefore, ordinary bodies like galaxies and any matter on the surface of such a sphere cannot escape from its anti-gravity, being accelerated far away.

According to (26), we should note that such an exotic 'particle' of vacuum (at S_V) has an infinite mass m since we should consider $v = V$ ($\theta = 0$) in order to have a finite value of E, other than the photon ($v = c$), that is a massless particle (see (17)). So we conclude that an exotic 'particle' of vacuum works like a counterparty of the photon, namely an infinitely massive boson.

We consider that the most negative (repulsive) potential ($\phi = -c^2$ for $v = V$, see (26)) is directly related to the cosmological constant (vacuum energy density) since we have shown (in (23) and (24)) that the fundamental reference frame S_V ($v = V$) plays the role of the vacuum energy density with a negative pressure, working like the cosmological constant Λ ($p = -\epsilon = -\rho_{(\Lambda)}$). So we write

$$\phi_\Lambda = \phi(\Lambda) = \phi(V) = -c^2. \tag{27}$$

Let us consider the simple model of spherical universe with a radius R_{u}, being filled by a uniform vacuum energy density $\rho_{(\Lambda)}$, so that the total vacuum energy inside the sphere $E_\Lambda = \rho_{(\Lambda)}V_{\mathrm{u}} = -pV_{\mathrm{u}} = M_\Lambda c^2$. V_{u} is its volume and M_Λ is the total dark mass associated with the 'dark energy' for Λ (vacuum energy: $w = -1$ [8]). Therefore, the repulsive gravitational potential on the surface of such a sphere

$$\phi_\Lambda = -\frac{GM_\Lambda}{R_{\mathrm{u}}} = -\frac{G\rho_{(\Lambda)}V_{\mathrm{u}}}{R_{\mathrm{u}}c^2} = \frac{4\pi GpR_{\mathrm{u}}^2}{3c^2}, \tag{28}$$

where $p = -\rho_{(\Lambda)}$, with $w = -1$ [8].

By introducing (25) into (28), we find

$$\phi_\Lambda = \phi(\Lambda) = -\frac{\Lambda R_u^2}{6}. \tag{29}$$

Finally, by comparing (29) with (27), we extract

$$\Lambda = \frac{6c^2}{R_{\mathrm{u}}^2}, \tag{30}$$

where $\Lambda S_{\mathrm{u}} = 24\pi c^2$, $S_{\mathrm{u}} = 4\pi R_{\mathrm{u}}^2$.

By comparing (28) with (27), we have

$$\rho_{(\Lambda)} = -p = \frac{3c^4}{4\pi G R_{\mathrm{u}}^2}, \tag{31}$$

where $\rho_{(\Lambda)} S_{\mathrm{u}} = -pS_{\mathrm{u}} = 3c^4/G$. We can verify that (31) and (30) satisfy (25).

In (30), Λ is a kind of cosmological scalar field, extending the old concept of Einstein about the cosmological constant for stationary universe. From (30), by considering the Hubble radius, with $R_{\mathrm{u}} = R_{\mathrm{H}_0} \sim 10^{26}$ m, we obtain $\Lambda = \Lambda_0 \sim (10^{17}\ \mathrm{m}^2\ \mathrm{s}^{-2}/10^{52}\ \mathrm{m}^2) \sim 10^{-35}\ \mathrm{s}^{-2}$. To be more accurate, we know the age of the universe $T_0 = 13.7$ Gyr, being $R_{\mathrm{H}_0} = cT_0 \approx 1.3 \times 10^{26}$ m, which leads to $\Lambda_0 \approx 3 \times 10^{-35}\ \mathrm{s}^{-2}$. This tiny positive value is very close to the observational results [10–14]. The vacuum energy density [15,16] given in (31) for R_{H_0} is $\rho_{(\Lambda_0)} \approx 2 \times 10^{-29}\ \mathrm{g/cm}^3$, which is also in agreement with observations. For scale of the Planck length, where $R_{\mathrm{u}} = l_{\mathrm{P}} = (G\hbar/c^3)^{1/2}$, from (30) we find $\Lambda = \Lambda_{\mathrm{P}} = 6c^5/G\hbar \sim 10^{87}\ \mathrm{s}^{-2}$, and from (31) $\rho_{(\Lambda)} = \rho_{(\Lambda_{\mathrm{P}})} = T^{00}_{\mathrm{vac.P}} = \Lambda_{\mathrm{P}} c^2/8\pi G = 3c^7/4\pi G^2\hbar \sim 10^{113}$ $\mathrm{J/m}^3$($=3c^4/4\pi l_{\mathrm{P}}^2 G \sim 10^{43}$ kgf/$\mathrm{S_P} \sim 10^{108}$ atm $\sim 10^{93}\ \mathrm{g/cm}^3$). So just at that past time, Λ_{P} or $\rho_{(\Lambda_{\mathrm{P}})}$ played the role of an inflationary vacuum field with 122 orders of magnitude [8] beyond the ones (Λ_0 and $\rho_{(\Lambda_0)}$) for the present time.

It must be stressed that our assumption for obtaining a tiny positive value of Λ starts from first principles.

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