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Dependence of NaI(Tl) detector intrinsic efficiency on source–detector distance, energy and off-axis distance: Their implications for radioactivity measurements

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Abstract. In this work the dependence of intrinsic efficiency of a NaI(Tl) detector of radius 3.82 cm and height 7.62 cm on source–detector distance (d), source–off-axis distance (d_0) and γ-photon energy have been investigated using analytical and Monte Carlo methods. The results showed that, for a given off-axis distance, there exists a value of the ratio of source–detector distance (d) to detector radius (R) where intrinsic efficiency is minimum. This d/R value at which minimum efficiency occurs approaches zero as off-axis distance increases and it is almost constant with increase in energy. In the region where $d/R < 0.01$, a criteria given by Jehouani *et al* [1] for good photon detection, intrinsic efficiency decreases with increasing off-axis distance. The implications of the results for radioactivity measurement and radiation protection are discussed. Chacteristics of intrinsic efficiency in the regions $d/R < 0.01$ and $d/R > 10$ are also compared.

Keywords. Detections; γ-photons; analytical formula; Monte Carlo; intrinsic efficiency.

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1. Introduction

In γ-ray spectrometry and radiation transmission experiments, bare-surface and well-type HpGe and NaI(Tl) detectors are widely used for γ-photon detection [1,2]. When better efficiency is the most important parameter, then a NaI(Tl) detector is usually preferred for γ-photon detection [3] due to its high efficiency [4]. High detection efficiency is an important requirement in γ-ray spectrometry for good signal-to-noise ratio. Signal-to-noise ratio increases as detection efficiency increases.

Many analytical and Monte Carlo methods that can be used to predict the efficiency of a detector for different source and detector geometries have been reported [1,2,5]. Jehouani *et al* [1] studied the dependence of efficiency of a NaI(Tl) detector on its geometrical parameters for a radiation source placed on the axis of the detector. Based on their studies, they suggested that for good photon detection the ratio of source–detector distance (d) to detector radius (R) should be less than 0.01. It should be noted at this point that from the point of view of radiation protection these criteria cannot be justified in certain circumstances, for instance, when the area to be monitored presents high background radiation level or contaminated by sources of high radiation level [6,7]. Radiation protection regulation requires that exposure of radiation to personnel should be kept as low as reasonably achievable. This possibly is the reason why Scheffing and Krichinsky [6] used d values of 192 inches (4876.8 mm) and 254 inches (6451.6 mm) for a detector of radius 25 mm in their measurements. This corresponds to $d/R = 195$ and 258 respectively. Therefore, there is the need to suggest another criterion that will not violate this radiation protection regulation in cases where sources of very high activity are to be monitored. Furthermore, since the study of Jehouani *et al* [1] was only for the case where the source axis and that of the detector coincides, there is the need to study the dependence of intrinsic efficiency on off-axis distance (d_0) (i.e. the distance between the lines that pass through the centres of both the source and the detector). This may be useful for measurements where it may not be possible to align the two axes to coincide.

In the present work, we present the dependence of intrinsic efficiency of NaI(Tl) detector on source–detector distance, off-axis distance and energy. From the results we hope to suggest another criterion for good photon detection for γ-spectrometry in cases where ratio of source–detector distance (d) to detector radius (R) should be greater than 0.01. The cases considered in this work are those for which $d_0 < R$. The results from this work are important for low-level radioactivity measurements and also for monitoring area or radiation source of very high activity.

2. Methods

2.1 *Analytical formula*

The analytical formula used in this work to calculate the intrinsic efficiency (ε) of a bare-surface detector at different off-axis distances is adapted from the analytical formulae presented by Mahmoud [2] for total efficiency ($\varepsilon_{\text{total}}$) calculation of a well-type NaI(Tl). These formulae for a well-type detector become that of a bare-surface detector shown in figure 1 if k is set to 0 and R_{i} to R_{o}. In the paper of Mahmoud [2] k, R_{i} and R_{o} stand for well depth, inner radius and outer radius of the well-type detector respectively. By setting $k = 0$ and $R_{\text{i}} = R_{\text{o}}$ in the analytical formula presented by Mahmoud [2] it becomes

$$\varepsilon_{\text{total}} = \frac{W_1 + W_2 + W_3}{4\pi}. \tag{1}$$

The geometric solid angle [5] is also obtained to be

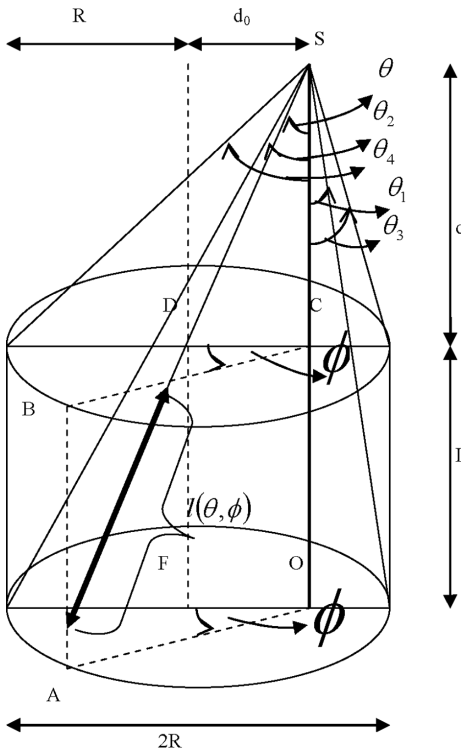


Figure 1. Geometrical configuration of the punctual source at off-axial distance above the bare-surface NaI(Tl) detector.

$$\Omega = B_1 + B_2 \tag{2}$$

with

$$W_1 = 2\pi \int_0^{\theta_1} (1 - \exp(-\mu(E) l_1)) \sin\theta \mathrm{d}\theta, \tag{3}$$

$$\begin{aligned} W_2 = {} & 2\int_{\theta_1}^{\theta_3} \int_0^{\pi} (1 - \exp(-\mu(E) l_2)) \sin\vartheta \mathrm{d}\phi \mathrm{d}\theta \\ & +2\int_{\theta_1}^{\theta_2} (\phi_{\max(d+L)} (1 - \exp(-\mu(E) l_1)) \sin\theta \\ & - \int_0^{\phi_{\max(d+L)}} (1 - \exp(-\mu(E) l_2)) \sin\theta \mathrm{d}\phi) \mathrm{d}\theta, \end{aligned} \tag{4}$$

$$W_3 = 2\int_{\theta_3}^{\theta_4} \int_0^{\phi_{\max(d)}} (1 - \exp(-\mu(E) l_2)) \sin\theta \mathrm{d}\phi \mathrm{d}\theta, \tag{5}$$

$$B_1 = 2\pi \int_0^{\theta_3} \sin\theta \mathrm{d}\theta, \tag{6}$$

$$B_2 = 2 \int_{\theta_3}^{\theta_4} \int_0^{\phi_{\max(d)}} \sin\theta \mathrm{d}\phi \mathrm{d}\theta, \tag{7}$$

where $\mu(E)$ is the γ-photon attenuation coefficient at the energy E. θ_1, θ_2, θ_3 and θ_4 are as shown in figure 1 and are given by

$$\theta_1 = \arctan\left(\frac{R - d_0}{d + L}\right), \tag{8}$$

$$\theta_2 = \arctan\left(\frac{R + d_0}{d + L}\right), \tag{9}$$

$$\theta_3 = \arctan\left(\frac{R - d_0}{d}\right), \tag{10}$$

$$\theta_4 = \arctan\left(\frac{R + d_0}{d}\right) \tag{11}$$

and

$$\phi_{\max(x)} = \arccos\left(\frac{d_0^2 - R^2 + x^2 \tan^2\theta}{2d_0 x \tan\theta}\right). \tag{12}$$

A relation between intrinsic efficiency, which is of interest in this work, and total efficiency is obtained by the following two expressions [5]:

$$\varepsilon_{\text{total}} = \frac{\Omega_{\text{ef}}}{4\pi} \tag{13}$$

and

$$\varepsilon = \frac{\Omega_{\text{ef}}}{\Omega}, \tag{14}$$

where Ω_{ef} is the effective solid angle and Ω is the geometric solid angle.

From eqs (13) and (14), it implies that the intrinsic efficiency (ε) is

$$\varepsilon = \frac{4\pi\varepsilon_{\text{total}}}{\Omega}. \tag{15}$$

Using eqs (1) and (2) in eq. (15), the intrinsic efficiency is given by

$$\varepsilon = \frac{W_1 + W_2 + W_3}{B_1 + B_2}. \tag{16}$$

The configuration of the source and detector used in this work is shown in figure 1. The arbitrarily positioned point source is defined by the quantities (d_0, d). The direction of the incidence of a γ-ray photon entering the detector's top is defined by the polar (θ) and the azimuthal (ϕ) angles. $l(\theta, \phi)$ is the path length of the photon through the detector's active volume until it emerges from the crystal. The effective rays may enter the top of the detector and

(1) emerge from the bottom, and for this case the path length

$$l_1 = \frac{L}{\cos\theta}, \tag{17}$$

(2) emerge from the detector's sides and for this the path length

$$l_2 = \frac{Y}{\sin\theta} - \frac{d}{\cos\theta}, \tag{18}$$

where Y is the distance from the source axis on the detector to the edge of the detector ($\overline{\mathrm{OA}}$ and $\overline{\mathrm{CB}}$ in figure 1) and is defined as

$$Y = d_0 \cos\phi + \sqrt{R^2 - d_0^2 \sin^2\phi}. \tag{19}$$

Equation (16) was solved using the 10-point gauss-quadrature numerical integration formula. Attenuation coefficients used in the work are calculated with the XCOM program and database of Berger and Seltzer [8].

2.2 *Monte Carlo method*

The Monte Carlo simulation approach used in this work is hereby discussed. γ-photon emission is sampled over a hypothetical detector of radius $R + d_0$ centred at O^1 and with the punctual γ-photon source S centred at O^1 as shown in figure 2. The real detector is considered to be part of this hypothetical detector with a radius R and centre O as shown in figure 2. Only γ-photons that impinge on the surface of the real detector are considered detected.

The polar angle (θ) and the azimuthal angle (ϕ) which define γ-photon emission direction with respect to the hypothetical detector of radius $R+d_0$ take their values from 0 to θ_4 and 2π respectively, while with respect to the real detector, the polar angle (θ) takes values from 0 to $\theta_{2\,\max}$ and the azimuthal angle (ϕ) from 0 to 2π. For $\phi = 0$, $\theta_{2\,\max} = \theta_4$.

$$\theta_{2\,\max} = \arctan\left(\frac{Y}{d}\right) \tag{20}$$

and

$$\theta_{1\,\max} = \arctan\left(\frac{Y}{d+L}\right). \tag{21}$$

The γ-photon emission direction (θ, ϕ) towards detector's surface is sampled by

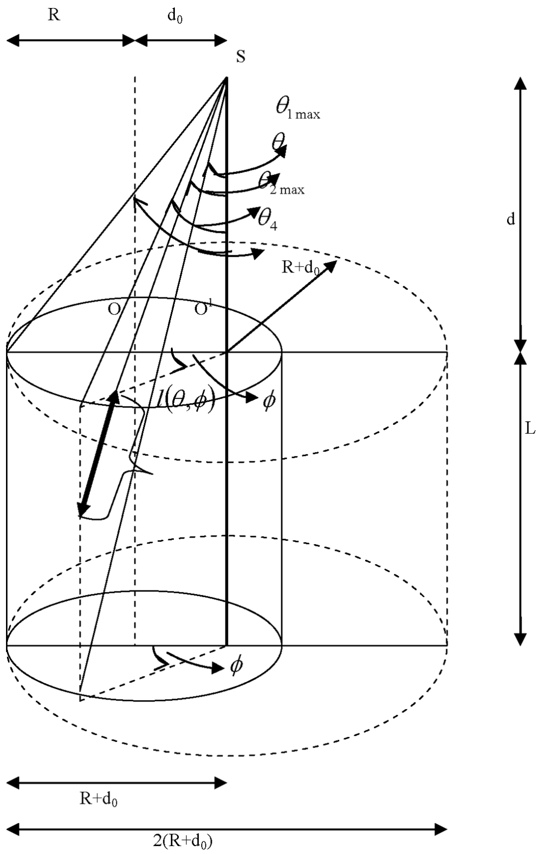


Figure 2. Geometrical configuration for the hypothetical detector for punctual γ-photon sampling.

$$\theta = \arccos(1 - r_2(1 - \cos\theta_4)), \tag{22}$$

$$\phi = \pi(2r_1 - 1), \tag{23}$$

where r_1 and r_2 are random numbers in [0,1]. The γ-photon path length $l(R, d, \theta, \phi)$ inside the detector are as follows:

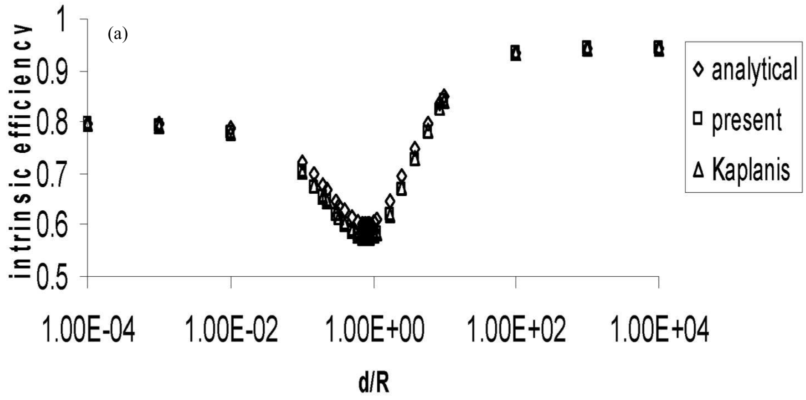
$$l = \frac{L}{\cos\theta}, \quad \text{if } \theta \leq \theta_{1\,\max} \tag{24}$$

$$l = \frac{Y - d\tan\theta}{\sin\theta}, \quad \text{if } \theta_{1\,\max} < \theta \leq \theta_{2\,\max}. \tag{25}$$

If $\theta > \theta_{2\,\max}$ the γ-photon is not detected by the real detector and so not included in the efficiency calculation. The efficiency is calculated using

$$\varepsilon = \frac{\sum_{i=1}^{N}(1 - \exp(-\mu(E)l(\theta,\phi)_i))}{N}, \tag{26}$$

where N is the number of histories.



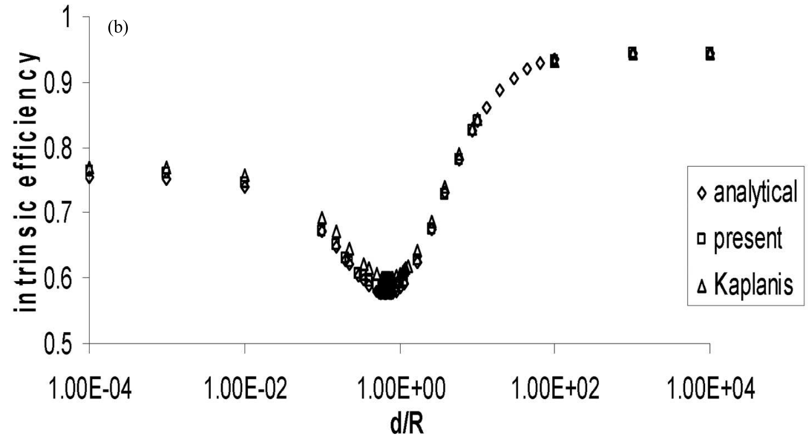


Figure 3. Comparison of the analytical method and the Monte Carlo method with the approach of Kaplanis [5] for γ-photon energy 662 keV for two off-axis distances of (**a**) 0 cm and (**b**) 1.5 cm.

3. Results and discussion

The results from the analytical formula and the Monte Carlo approach employed in this work were compared with those obtained using a Monte Carlo approach described by Kaplanis [5] in order to check the validity of approaches used in this work. The results from the two methods used in this work compare very well with the results from the Monte Carlo approach in Kaplanis [5]. The agreement among the three approaches, shown in figure 3, indicates that the analytical formula and the Monte Carlo approach employed in this work are valid for the calculation of intrinsic efficiency.

Figure 4 shows the variation of intrinsic efficiency with change in d/R for different energies and off-axis distances. As shown in the figure, for all off-axis distances and energies considered in this work, as d/R increases intrinsic efficiency first decreases and then passes through a minimum after which it starts to increase and saturates for all $d/R > 100$. Similar results have been reported by Jehouani *et al* [1] but just

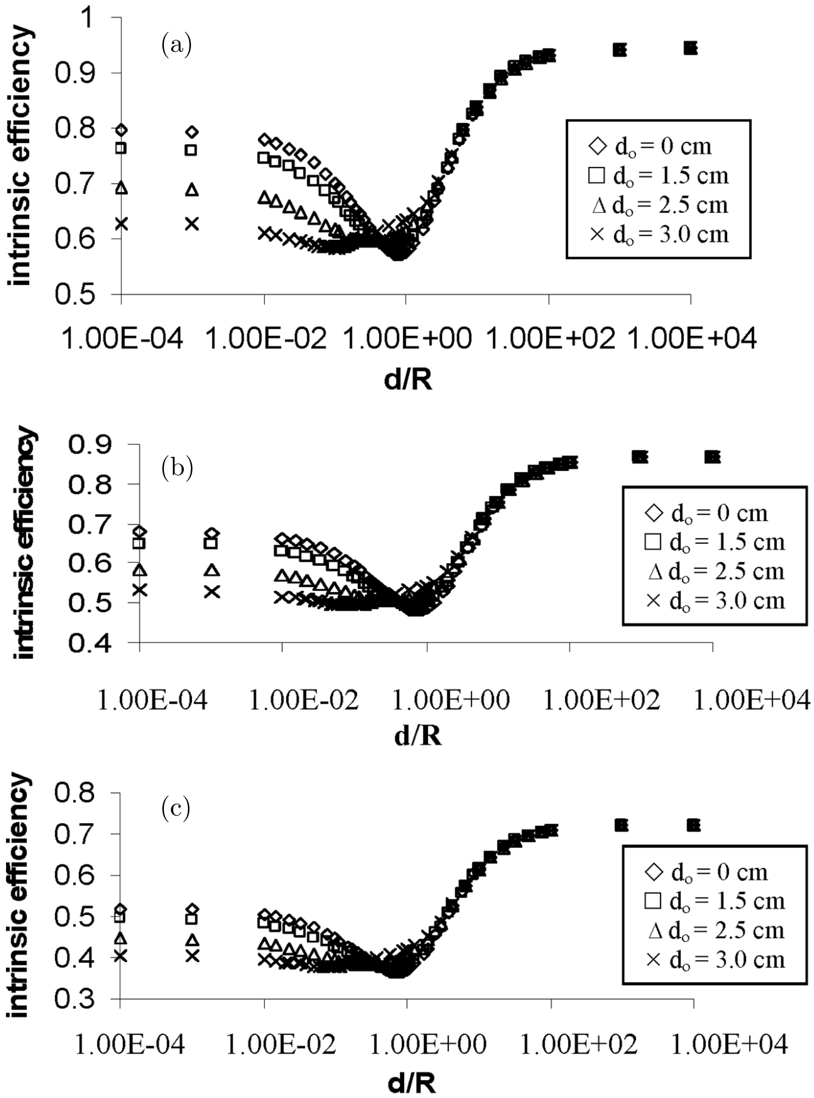


Figure 4. Variation of intrinsic efficiency with the ratio of source–detector distance to detector radius at four different off-axis distances for γ-photon with energies (**a**) 662 keV, (**b**) 1500 keV and (**c**) 2750 keV.

for the case of an axial source. On both sides of the minimum efficiency there are two regions ($d/R < 0.01$ and $d/R > 10$) where intrinsic efficiency is relatively high. The fact that intrinsic efficiency is high in the region $d/R < 0.01$ informed the suggestion made by Jehouani *et al* [1] that for good photon detection, the source and the detector should be arranged such that $d/R < 0.01$. Jehouani *et al* [1] did not comment on the region $d/R > 10$ where intrinsic efficiency is also high and even

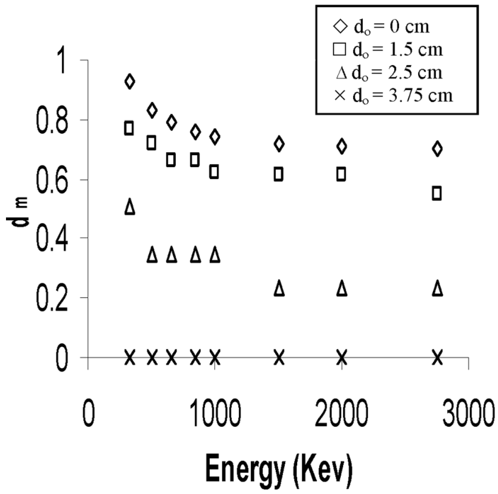


Figure 5. Dependence of ratio of source–detector distance to detector radius at which minimum intrinsic efficiency occur on γ-photon energy.

higher than for $d/R < 0.01$. In some situations, however, the use of the criteria that $d/R > 10$ becomes more justifiable than $d/R < 0.01$. One such situation is when the area or the radiation source to be monitored presents a high radiation risk to the personnel. Examples are *in situ* measurements of high background radiation level area and of contaminated surfaces in industries such as nuclear power plants. Radiation protection regulation requires that for any operation, radiation exposure of personnel should be kept as low as reasonably achievable. In these cases, the use of the criteria $d/R > 10$ instead of $d/R < 0.01$ will ensure compliance with regulation. Another advantage which is in favour of the use of $d/R > 10$ instead of $d/R < 0.01$ is that in the region where $d/R > 10$ intrinsic efficiency is independent of off-axis distance. A possible explanation for this is that at these large source–detector distances, the directions of all the photons getting to the surface of the detector are parallel to the axis of the detector. This means that all the photons that get to the surface of the detector will travel the same distance inside the detector. Therefore, for a given energy the intrinsic efficiency calculated using eq. (16) or (26) becomes independent of off-axis distance. Where possible, the best choice is $d/R > 100$ where intrinsic efficiency is almost a constant. $d/R > 10$ is suggested mainly because it will be easier to arrange, intrinsic efficiency in this region is usually greater than in the region $d/R < 0.01$ as suggested by Jehouani *et al* [1] and also because intrinsic efficiency in the region is independent of off-axis distance.

For the region where $d/R < 0.01$, intrinsic efficiency decreases as d_0 increases. From practical point of view, the implication of this is that when monitoring a large surface area that requires measurements at several points in the area, with reference to the centre of the detector, off-axis distances of the points that constitute the area to be monitored will differ, and more than one efficiency value (depending on the number of points with different off-axis distance) will be needed for activity calculation if source and detector are positioned such that $d/R < 0.01$. Using one

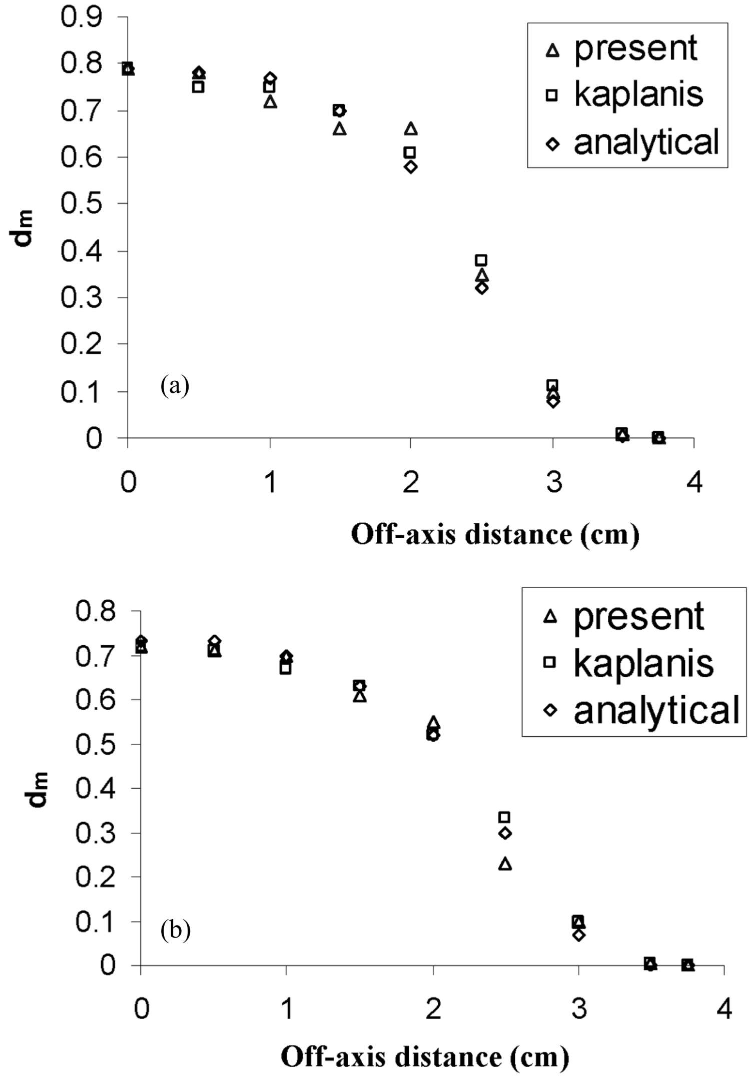


Figure 6. Variation of ratio of source–detector distance to detector radius at which minimum intrinsic efficiency occur with off-axis distance for different γ-photon energies (**a**) 662 keV and (**b**) 1500 keV.

efficiency value in this situation will not yield accurate result. However, if the source and the detector are positioned such that $d/R > 10$, only one efficiency value is needed irrespective of the number of points where measurements are made. For low-level radioactivity measurements in the laboratory where the use of the criteria $d/R < 0.01$ is more justified, it means that for accurate results the off-axis distance of the standard source container during measurements for the determination of efficiency should be the same as the off-axis distance of sample container during

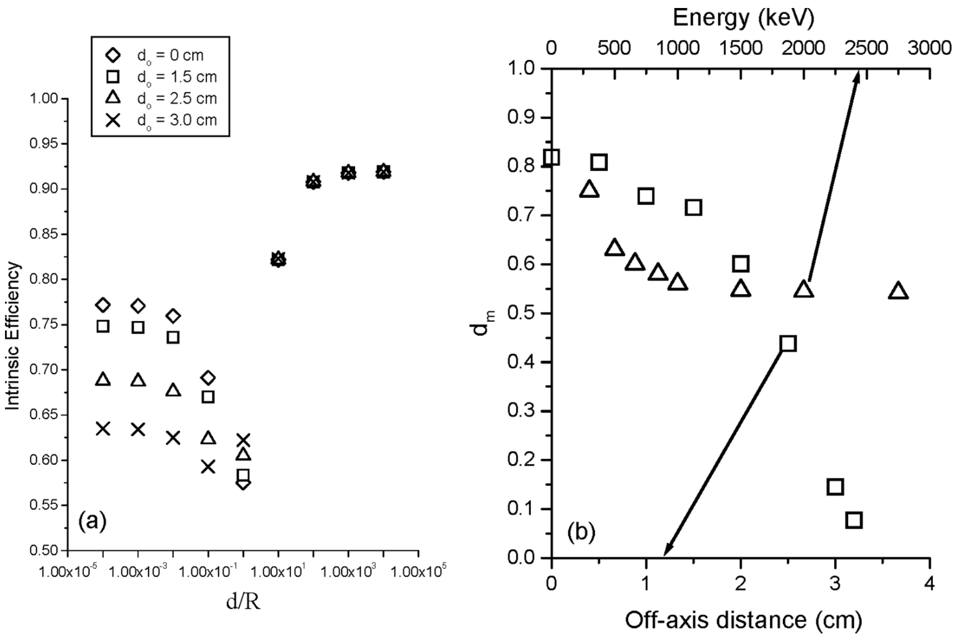


Figure 7. Variation of (**a**) intrinsic efficiency with ratio of source–detector distance to detector radius and (**b**) ratio of source–detector distance to detector radius at which minimum intrinsic efficiency occur with off-axis distance and energy for a disk source of radius 0.38 cm and thickness 0.1 cm.

measurements. This is to make sure that the off-axis distances in both cases are the same; otherwise the efficiency determined using the standard source would not adequately represent the efficiency of photon detection from the sample. This will not be a problem if standard containers (e.g. Marinelli beaker), which have been designed in such a way that its position on the detector is always the same, are used. However, the containers that are being used in many laboratories [9–11] are not designed this way. In such laboratories it should be known that for accurate results, off-axis distance must always be the same from sample to sample.

Figure 5 shows the variation in the value of d/R at which minimum intrinsic efficiency occurs ($d_{\rm m}$) as a function of energy for four different off-axis distances. Knowledge of this parameter is important in γ-spectrometry because it gives information about where to expect minimum efficiency. As can be seen in figure 5, $d_{\rm m}$ is generally constant with increasing energy except for a slight decrease at small energies. The dependence of $d_{\rm m}$ on off-axis distance is clearly shown in figure 6. The figure shows that as off-axis distance increases, $d_{\rm m}$ first decreases slowly and then at a faster rate. The fact that $d_{\rm m}$ approaches zero as d_0 increases indicate that care must be taken when using the criterion $d/R < 0.01$. This is because this minimum efficiency can shift into this region. Probably the only way to ensure good photon detection efficiency in this region is to set $d/R = 0$.

All the results presented so far were based on a point source. In figure 7, we present similar calculations for a disc source of radius 0.38 cm and thickness 0.1 cm.

As shown in figure 7a the variation of the intrinsic efficiency with d/R using a disk source is qualitatively similar to that of a point source. The data in figure 7a were obtained with the source placed 2.0 cm off-detector axis. Variations of $d_{\rm m}$ with off-axis distance and energy, as shown in figure 7a, were also similar to that of a point source. For variation of $d_{\rm m}$ with energy the source was placed 2.0 cm off-detector axis. This shows that the results presented in this work are also valid for distributed sources.

4. Conclusion

The dependence of intrinsic efficiency on source–detector distance, off-axis distance and γ-photon energy are reported in this work. The d/R value at which minimum intrinsic efficiency occurs approaches zero as γ-photon energy and/or off-axis distance increases. The implication of this is that, when following the criteria that $d/R < 0.01$, the source should be placed on the detector (i.e. $d = 0$) to ensure that this point of minimum efficiency is avoided. Also, for accurate determination of activity and dose rate from radioactivity measurements, it was found that the off-axis distance of the standard source container during measurements for efficiency determination should be the same as that of the sample to be measured. For compliance with radiation protection regulation when the radiation source to be monitored can cause over-exposure of the personnel, the source–detector distance should be $d/R > 10$ instead of $d/R < 0.01$.

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