PRAMANA — journal of physics

Vol. 70, No. 4
April 2008
pp. 649–656

The analytical investigation of temperature distribution in off-central diode-pumped lasers

P ELAHI*, A TAGHAVI and A GHARAATI
Department of Physics, Payame Noor University, Shiraz, Iran
*Corresponding author. E-mail: pelahi@spnu.ac.ir

MS received 2 July 2007; revised 21 September 2007; accepted 4 October 2007

Abstract. The influence of displacement of the pump source with respect to the crystal center on the thermal behavior of the laser crystal is studied analytically. We consider the pump energy to be deposited into the pump region which has been slightly displaced with respect to the crystal center. An analytical expression for temperature distribution for such off-central diode-pumped laser is investigated. The results are then applied to the Nd : YAG and Nd : YVO_4 laser crystals and compared with the conventional diode-pumped lasers. We showed that in this special case, the temperature distribution equation in the off-central pumping convert to the conventional central pump scheme.

Keywords. Off-central diode-pumped; diode-pumped; temperature distribution.

PACS Nos 42.55.f; 42.55.Xi

1. Introduction

Diode-pumped solid-state lasers have many advantages and have now replaced other traditional flash-lamped-pumped lasers. End-pumped solid-state lasers can operate with high efficiency (>50%), high output power, good spatial beam profile, and good stability. This form of pumping allows the diode pump energy to be deposited into the central region of the rod [1–8].

End-pumping has more advantages with respect to side pumping, but thermal effects will appear more in the end-pumped configuration specially in high power regime. The thermal effects influence the optical behavior of the laser crystal. Therefore, the usual way of neglecting thermal effects can no longer be justified. All previous works for deriving the analytical expression for temperature distribution in high power regime diode-pumped lasers was based on the concentric of the pump region and crystal surface [9–12].

In this paper we consider the heat to be deposited in the pump region which has been displaced with respect to the crystal center. The heat equation was solved analytically and temperature distribution has been derived. The expressions are applied to Nd : YAG and Nd : YVO_4 laser crystals.

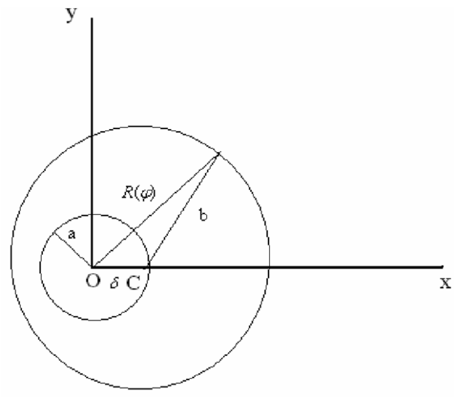

Figure 1. O is the center of the pump region and coordinate axis, C is the center of the crystal.

2. The analytical model

In our model we consider a laser crystal of radius b and length L. Heat is deposited in the pump region with radius a. The center of the pump region has been displaced by δ with respect to the center of the crystal, C. For derivation and then solving the heat conducting equation, we consider the center of the coordinate, O, on the center of the pump region (figure 1).

In the cylindrical coordinate the heat conduction equation in the pump and unpump regions can be written as

$$\frac{\partial^2 T_1(r,\varphi)}{\partial r^2} + \frac{1}{r}\frac{\partial T_1(r,\varphi)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T_1(r,\varphi)}{\partial \varphi^2} = -\frac{Q}{K}, \quad 0 \le r \le a, \tag{1}$$

$$\frac{\partial^2 T_2(r,\varphi)}{\partial r^2} + \frac{1}{r}\frac{\partial T_2(r,\varphi)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T_2(r,\varphi)}{\partial \varphi^2} = 0, \quad a \le r \le R(\varphi), \tag{2}$$

where $T_1(r,\varphi)$ and $T_2(r,\varphi)$ are temperature distributions in the pump region and outside respectively, Q is the heat power density and K is the thermal conductivity of the medium. $R(\varphi)$ is the boundary equation of the surface of the crystal with respect to O and is

$$R(\varphi) = \sqrt{b^2 - \delta^2 \sin^2\varphi} + \delta\cos\varphi \approx b + \delta\cos\varphi - \frac{\delta^2}{2b}\sin^2\varphi + O(\delta^3). \tag{3}$$

The solution of eqs (1) and (2) can be written as [13]

$$T_1(r,\varphi) = \sum_{n=1}^{\infty}[A_n \sin(n\varphi) + B_n\cos(n\varphi)]\, r^n\delta^n + \sum_{n=1}^{\infty}[C_n\sin(n\varphi) + D_n\cos(n\varphi)]\, r^{-n}\delta^n + A_0 + B_0 \ln\left(\frac{r}{a}\right) - \frac{Q}{4K}r^2 \tag{4}$$

for the pumped region and

$$T_2(r,\varphi) = \sum_{n=1}^{\infty}[A'_n \sin(n\varphi) + B'_n \cos(n\varphi)]r^n\delta^n$$
$$+\sum_{n=1}^{\infty}[C'_n \sin(n\varphi) + D'_n \cos(n\varphi)]\, r^{-n}\delta^n$$
$$+A'_0 + B'_0 \ln\left(\frac{r}{a}\right) \tag{5}$$

for the outside region. We consider the case in which the displacement of the pump center with respect to crystal center is small, and so in our calculation we neglect the terms of δ^3 and higher orders. So we have

$$\begin{aligned} T_1(r,\varphi) = {} & [A_1 \sin(\varphi) + B_1 \cos(\varphi)]r\delta + t[A_2 \sin(2\varphi) + B_2 \cos(2\varphi)]r^2\delta^2 \\ & +[C_1 \sin(\varphi) + D_1 \cos(\varphi)]r^{-1}\delta \\ & +[C_2 \sin(2\varphi) + D_2 \cos(2\varphi)]r^{-2}\delta^2 \\ & +A_0 + B_0 \mathrm{ln}\left(\frac{r}{a}\right) - \frac{Q}{4K}r^2, \end{aligned} \tag{6}$$

$$\begin{aligned} T_2(r,\varphi) = {} & [A'_1 \sin(\varphi) + B'_1 \cos(\varphi)]r\delta + [A'_2 \sin(2\varphi) + B'_2 \cos(2\varphi)]r^2\delta^2 \\ & +[C'_1 \sin(\varphi) + D'_1 \cos(\varphi)]r^{-1}\delta \\ & +[C'_2 \sin(2\varphi) + D'_2 \cos(2\varphi)]r^{-2}\delta^2 \\ & +A_0 + B'_0 \mathrm{ln}\left(\frac{r}{a}\right). \end{aligned} \tag{7}$$

Equations (6) and (7) describe the analytic function of temperature distribution in the pump region and outside respectively.

3. Boundary conditions

The temperature at the center is finite. So, $C_{1,2} = D_{1,2} = B_0 = 0$. By considering $A_0 = T_0$, eq. (6) will be

$$\begin{aligned} T_1(r,\varphi) = {} & [A_1 \sin(\varphi) + B_1 \cos(n\varphi)]r\delta + [A_2 \sin(2\varphi) + B_2 \cos(2\varphi)]r^2\delta^2 \\ & -\frac{Q}{4K}r^2 + T_0. \end{aligned} \tag{8}$$

By using the continuity of the temperature and derivatives at the boundary [4], means

$$T_1|_{r=a} = T_2|_{r=a}, \quad \vec{\nabla}T_1|_{r=a} = \vec{\nabla}T_2|_{r=a}, \tag{9}$$

and using the Newtonian boundary conditions [4] as

$$-K\vec{\nabla}T_1 \cdot \hat{n}|_{r=R} = H(T_c - T_{\mathrm{out}})|_{r=R}, \tag{10}$$

and by using the following approximation relations

$$\ln\left[\frac{b+\delta\cos(\varphi)}{a}\right] \cong \ln\left(\frac{b}{a}\right) + \frac{\delta}{b}\cos(\varphi) - \frac{\delta^2}{2\,b^2}\cos^2(\varphi) \tag{11}$$

and

$$\left[b+\delta\cos(\varphi) - \frac{\delta^2}{2b}\sin^2(\varphi)\right]^{-1} \cong \frac{1}{b} - \frac{\delta}{b^2}\cos(\varphi) + \frac{\delta^2}{2b^3}\sin^2(\varphi) \tag{12}$$

after some calculations, we find the following relations for the coefficients:

$$\begin{aligned}
T_0 = T_c + \frac{Qa^2}{2K}\Bigg[\ln\left(\frac{b}{a}\right) + \frac{K}{Hb} \\
+\delta^2\,(K-bH)\left(\frac{1}{4Hb^3} + \frac{1}{2b^2(K+Hb)}\right) + \frac{1}{2}\Bigg],
\end{aligned}$$

$$B_1 = B_1' = \frac{Qa^2}{2b^2K}\left(\frac{Hb-K}{Hb+K}\right),$$

$$A_1 = A_1' = 0, \quad A_2 = A_2' = 0, \quad C_1' = C_2' = D_1' = D_2' = 0,$$

$$B_2 = B_2' = \frac{Qa^2}{4b^3K}\left(\frac{Hb-K}{Hb+2K}\right)\left(\frac{1}{2b} - \frac{H}{Hb+K}\right),$$

$$\begin{aligned}
A_0' = T_c + \frac{Qa^2}{2K}\Bigg[\ln\left(\frac{b}{a}\right) + \frac{K}{Hb} \\
+\delta^2(K-bH)\left(\frac{1}{4Hb^3} + \frac{1}{2b^2(K+Hb)}\right)\Bigg],
\end{aligned}$$

$$B_0' = -\frac{1}{2}\frac{Q}{K}a^2. \tag{13}$$

So by inserting the coefficients which we obtained in eq. (13), into eqs (6) and (7) we have

$$\begin{aligned}
T_1(r,\varphi) = \delta\frac{Qa^2}{2b^2K}\left(\frac{Hb-K}{Hb+K}\right)r\cos(\varphi) + \delta^2\frac{Qa^2}{4b^3K} \\
\times\left(\frac{Hb-K}{Hb+2K}\right)\left(\frac{1}{2b} - \frac{H}{Hb+K}\right)r^2\cos(2\varphi) \\
+T_c + \frac{Qa^2}{2K}\Bigg[\ln\left(\frac{b}{a}\right) + \frac{K}{Hb} + \delta^2(K-bH) \\
\times\left(\frac{1}{4Hb^3} + \frac{1}{2b^2(K+Hb)}\right) + \frac{1}{2}\Bigg] - \frac{Q}{4K}r^2,
\end{aligned} \tag{14}$$

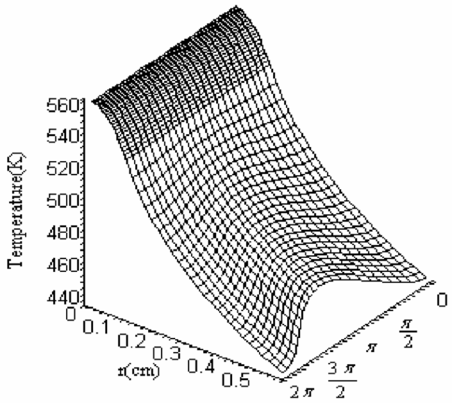

Figure 2. Temperature distribution in Nd : YAG crystal for 1 mm pump radius, 5 mm crystal radius and $\delta = 0.5$ mm.

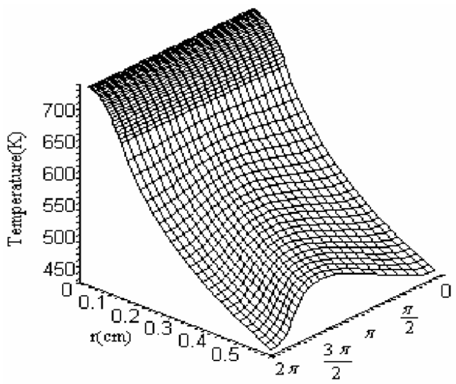

Figure 3. Temperature distribution in Nd : YVO_4 crystal for 1 mm pump radius, 5 mm crystal radius and $\delta = 0.5$ mm.

$$\begin{aligned} T_2(r,\varphi) = {} & \delta \frac{Qa^2}{2b^2K}\left(\frac{Hb-K}{Hb+K}\right) r\,\cos(\varphi) + \delta^2 \frac{Qa^2}{4b^3K} \\ & \times \left(\frac{Hb-K}{Hb+2K}\right)\left(\frac{1}{2b} - \frac{H}{Hb+K}\right) r^2\,\cos(2\varphi) \\ & + T_c + \frac{Qa^2}{2K}\left[\ln\left(\frac{b}{a}\right) + \frac{K}{Hb} + \delta^2(K-bH)\right. \\ & \left. \times \left(\frac{1}{4Hb^3} + \frac{1}{2b^2(K+Hb)}\right)\right] - \frac{1}{2}\frac{Q}{K}a^2 \ln\left(\frac{r}{a}\right). \end{aligned} \tag{15}$$

4. Results

In this section we used the expressions (14) and (15) for typical Nd : YAG and Nd : YVO_4 laser crystals. The following parameters are used for calculations. The length of the crystal $L = 1$ cm. The ambient temperature $T_c = 300$ K. The heat transfer coefficient is considered as $h = 0.1$ W cm^{-2} K^{-1}. Typically we take Nd : YAG and Nd : YVO_4 laser crystals with the thermal conductivity $K = 0.13$ W cm^{-1} K^{-1} and $K = 0.052$ W cm^{-1} K^{-1} respectively. Figure 2 shows the temperature distribution in the Nd : YAG crystal for $a = 1$ mm pump radius and $b = 5$ mm crystal radius. In this figure we consider $\delta = 0.5$ mm. As shown, the effect of off-central pumping is more pronounced near the surface of the crystal. Figure 3 shows the same graph for Nd : YVO_4 laser crystal.

Figures 4 and 5 show the temperature distribution on the surface of the crystal mean at $r = b$ for Nd : YAG and Nd : YVO_4 laser crystals. As we can see from these figures, the temperature on the surface of the crystal is maximum at $\varphi = \pi$ and the variations of temperature with respect to φ are more pronounced with increasing δ. The maximum temperature difference on the surface of Nd : YVO_4 crystal is

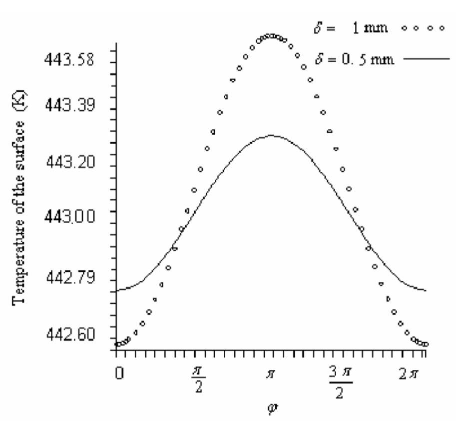

Figure 4. Temperature of the surface vs. φ for Nd : YAG crystal for 1 mm pump radius, 5 mm crystal radius for $\delta = 0.5$ mm (solid) and $\delta = 1$ mm (points).

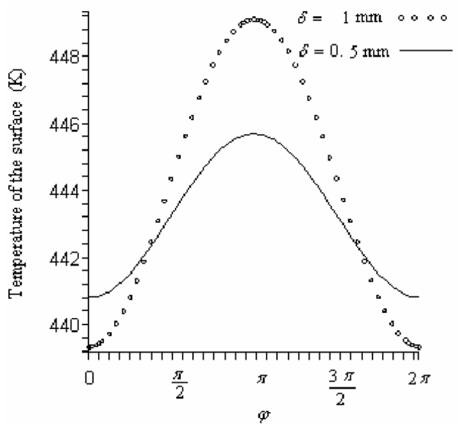

Figure 5. Temperature of the surface vs. φ for Nd : YVO_4 crystal for 1 mm pump radius, 5 mm crystal radius for $\delta = 0.5$ mm (solid) and $\delta = 1$ mm (points).

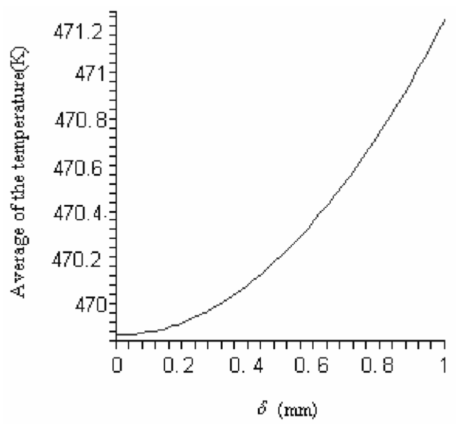

Figure 6. The average temperature of the crystal for 187 W pump power for Nd : YAG laser crystal.

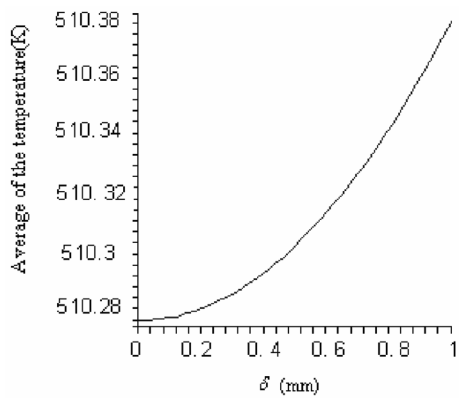

Figure 7. The average temperature of the crystal for 187 W pump power for Nd : YVO_4 laser crystal.

noticeable while it can be neglected for Nd : YAG crystal. So, for Nd : YAG with higher thermal conductivity, the influence of off-central pumping will be more.

Figures 6 and 7 show the average temperature of the crystal vs. δ. As we can see from these figures, the average temperature increases with respect to δ. The average temperature varies from 469.88 to 471.2 K when δ varies from zero to 1 mm for YAG, while this variation is from 510.24 to 510.38 K for vanadate. So for vanadate with poor thermal conductivity the average temperature is greater and the variation with respect to δ is less than YAG. An important result of these figures is the small variation of average temperature with respect to δ for both crystals.

By considering the approximations that have been used in our calculation, we can remove the off-central pumping effect in a special case. An important result from eqs (6) and (7) is that for $K = bH$ the temperature distribution is independent of δ and will be

$$T_1(r) = T_c + \frac{Qa^2}{2K}\left[\ln\left(\frac{b}{a}\right) + \frac{K}{Hb} + \frac{1}{2}\right] - \frac{Q}{4K}r^2, \tag{16}$$

$$T_2(r) = T_c + \frac{Qa^2}{2K}\left[\ln\left(\frac{b}{a}\right) + \frac{K}{Hb}\right] - \frac{1}{2}\frac{Q}{K}a^2\ln\left(\frac{r}{a}\right). \tag{17}$$

So for such condition, the effect of off-central pumping can be removed.

5. Discussions

In this paper the effect of off-central pumping on the temperature distribution of laser crystal has been investigated analytically. Firstly we consider that the center of pump region has been displaced by δ with respect to the crystal center. The effect of displacement appears in the azimuthal dependence of temperature distribution. We show that this effect is more pronounced near the surface of the rod and average temperature of the crystal will be greater rather than concentric pumping. So it is necessary to have a heavy coolant in such pumping scheme. In spite of dependency of temperature with respect to φ, we show that in a special case for which $K = bH$, the temperature distributions in the crystal are the same as in concentric pumping. It shows that we can remove the azimuthal dependence of temperature in off-central pumping by considering such conditions.

References

[1] Y F Chen, T M Huang, C F Kao, C L Wang and S C Wang, *IEEE J. Quantum Electron.* **71**, 1424 (1997)
[2] H Nadgaran and P Elahi, The analysis of temperature distribution in an end-pumped solid-state laser for spectroscopic applications, *Proceedings of the Eighteenth Colloquium on High-resolution Molecular Spectroscopy* edited by H Berger, Dijon, France, September, **L15** (2003)
[3] W Koechner and D K Rice, *IEEE J. Quantum Electron.* **6(9)**, 557 (1970)
[4] W Koechner, *Solid state laser engineering*, 5th edn (Springer-Verlag, New York, 1999) p. 408
[5] M Schmid, R Weber, Thomas Graf, M Rooz and Henz P Weber, *IEEE J. Quantum Electron.* **36**, 620 (2000)
[6] H Nadgaran and P Elahi, *Int. J. Pure Appl. Phys.* **2(4)**, 215 (2006)
[7] R S Abbott and P J King, *Rev. Sci. Instrum.***72**, 1346 (2001)
[8] K P Petrov, R F Curl and F K Tittel, *Appl. Phys.* **B66**, 531 (1998)
[9] Q Lue, N Kugler, H Weber, S Dong, N Muller and U Wittrock, *Opt. Quantum Electron.* **28**, 57 (1996)

[10] W A Clarkson, N S Felgate and D C Hanna, *Simple method for compensation of thermally induced birefringence in high power solid-state lasers*, presented at the CLEO/EUROPE-EQEC 98 (1998)
[11] P Elahi and H Nadgaran, *Pramana – J. Phys.* **66(3)**, 513 (2006)
[12] H Nadgaran and P Elahi, *Pramana – J. Phys.* **65(1)**, 95 (2005)
[13] M Abramawitz and I A Stegun (eds), *Handbook of mathematical function* (Dover, New York, 1965) p. 227