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Modeling of the fringe shift in multiple beam interference for glass fibers

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Abstract. A quadratic model is suggested to describe the fringe shift occurred due to the phase variations of uncladded glass fiber introduced between the two plates of the liquid wedge interferometer. The fringe shift of the phase object is represented in the harmonic term which appears in the denominator of the Airy distribution formula of Fabry–Perot's interferometer. A computer program is written to plot the computed fringe shifts of the described model.

An experiment is conducted using liquid wedge interferometer where the fiber of a nearly quadratic thickness variation is introduced between the two plates of the interferometer. The obtained fringe shift shows a good agreement with the proposed quadratic model. Also, it is compared with the previous theoretical shift based on ray optics of semi-circular shape.

Keywords. Multiple beam interference; laser radiation; computerized images.

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1. Introduction

Tolansky [1] has obtained an approximate formula of Airy summation and pointed out the main features between the beams forming multiple-beam fringes at infinity by using a plane parallel plate and those forming multiple beam localized fringes. In the case of the wedge, the successively multiply reflected beams are not in phase in exact arithmetic series while in the case of plane parallel plates the phase difference between any two successive beams is $\lambda/2$. The optimum condition for producing multiple beam localized fringes, reached by Tolansky, necessitates small interferometric wedge angle α in order to fulfill the Airy's summation conditions. Barakat and Mokhtar [2] found that the permitted limit is $(3/8)\lambda$. In another study [3–10] interference was obtained using synthetic optical fibers. They considered ray optics approximation using monochromatic light emitted from mercury and other spectral lamps.

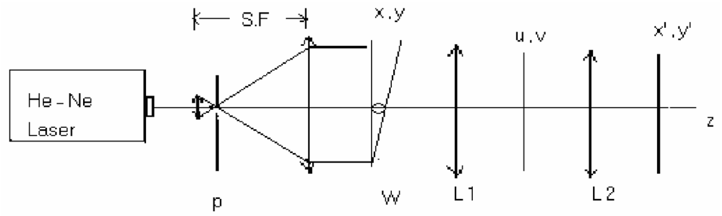

Figure 1. Imaging system of multiple beam Fizeau fringes using a liquid wedge interferometer (W) located in the plane (x, y) where SF is a spatial filter for rendering the beam parallel of certain cross-section with a pinhole p having diameter sufficient to pass the first order of diffraction pattern. L_1, L_2 are Fourier transform lenses each of focal length 20 cm. (u, v) is the spectral Fourier plane of the object plane (x, y). (x', y') is the imaging plane where the fringes are imaged.

Also, Barakat has obtained a right formula, which is based on ray optics, of multiple beam Fizeau fringes crossing a fiber of circular transverse cross-section immersed in a silvered liquid wedge. His work is followed by others [8] who extended the analysis to multilayer fibers. Hamza *et al* [9] determined the refractive indices and birefringence of fibers having irregular transverse sections of homogeneous fibers. Boggs *et al* [11] and Presby *et al* [12] described automated transverse interferometric method and described the index profile of graded index fiber. Recently, Hamed [13] has considered the effect of the wedge angle α on the arithmetic series using a Gaussian laser illumination. The modified Airy distribution is obtained using Fourier imaging applied on fibers. The effect of laser modulation on the contrast and sharpness of fringes have been investigated.

In this study, we suggest a model of quadratic variations to describe the thickness variations of fibers having circular transverse cross-sections. The Airy distribution is written in the case of fiber modulation giving nearly quadratic shift variations. An experiment [5] is done using liquid wedge interferometer where a fiber is enclosed between the two plates of the interferometer. The interferometer is illuminated by a He–Ne laser. The quadratic theoretical model is compared with the experimental shift of the fiber which shows good agreement. In the following section, theoretical analysis is presented including the quadratic model for the fringe shift. Finally, results and discussion are given in §4.

2. Theoretical analysis

A collimated laser beam emitted from He–Ne laser at $\lambda = 632.8$ nm is used to illuminate the wedge interferometer as shown in figure 1. The interferometer is formed by enclosing a matched liquid of suitable refractive index ($\mu_{\rm L}$) situated between two silvered optical plates forming a wedge gap (α). Hence, the examined phase object is located between the two optical plates. The recorded transmitted intensity of multiple beam interference is represented by the well-known expression of Airy distribution [1–5]

$$I(r') = \frac{1}{(1 - r_1^2) + 4r_1^2 \sin^2(k\Delta/2)}, \tag{1}$$

where $k = 2\pi/\lambda$ is the propagation constant, r_1 stands for the amplitude reflection coefficient of either of the interferometer plates, $r' \equiv (x', y')$ is the imaging plane where the fringes are located, and Δ is the optical path difference between the first two transmitted rays with ε the differential increment introduced in the successive transmitted rays.

From the theory of Fizeau fringes applied to fibers [4,5], using ray optics approximation, we get

$$(\mu_{\rm L} - \mu_{\rm f})t_{\rm f} = ({\rm d}z/\Delta z)\lambda/2, \tag{2}$$

where $t_{\rm f} = 2r_{\rm f}$ is the fiber diameter of refractive index $\mu_{\rm f}$, $\Delta z = \lambda/2$ is the distance between any two successive straight line fringes, and ${\rm d}z$ is the fringe shift introduced due to the fiber.

For the transverse section of uncladded fiber of constant refractive index $\mu_{\rm f}$ the variable thickness $t_{\rm f}$ varies in a quadratic form reaching the maximum value at the center of the fiber. Hence, we propose a model to describe the shift introduced by the fiber and compared it with the experimental shift. Also, it is compared with the theoretical shift that is based on ray optics. The parabolic function is introduced for the fringe shift of width equal to that of the fiber diameter and represented in the following paragraph.

3. A quadratic model for the fringe shift

A parabolic function is assumed to represent the fringe shift obtained in case of multiple beam interference occurred in a liquid wedge interferometer. It means utilization of the phase object of thickness $t_{\rm p}(y)$ varied in a quadratic parabolic shape keeping the refractive index fixed at $\mu_{\rm f}$. In this case, the quadratic function is represented as follows:

$$\begin{aligned} t_{\rm p}(y) &= t_0(y/t_{\rm f})^2; \quad y \le t_{\rm f}, \\ &= t; \quad y > t_{\rm f}, \end{aligned} \tag{3}$$

where t_0 is the maximum shift.

The phase of the object is calculated as follows:

$$\begin{aligned} \phi(y) &= \frac{2\pi}{\lambda}[2\mu_{\rm L}t + 4t_{\rm p}(y)(\mu_{\rm f} - \mu_{\rm L})] \\ &= \frac{2\pi}{\lambda}[2\mu_{\rm L}z\tan(\alpha) + 4t_0(y/t_{\rm f})^2(\mu_{\rm f} - \mu_{\rm L})]. \end{aligned} \tag{4}$$

The inter-fringe spacing Δz is given by $\Delta z = \lambda/2$.

For a fringe shift ${\rm d}z < \Delta z$, we obtain this inequality

$$\mu_{\rm f} - \mu_{\rm L} < \frac{\lambda t_{\rm f}^2}{4t_0 y^2 \sin(\alpha)}. \tag{5}$$

Also, no shift is occurred when $\mu_{\rm f} = \mu_{\rm L}$.

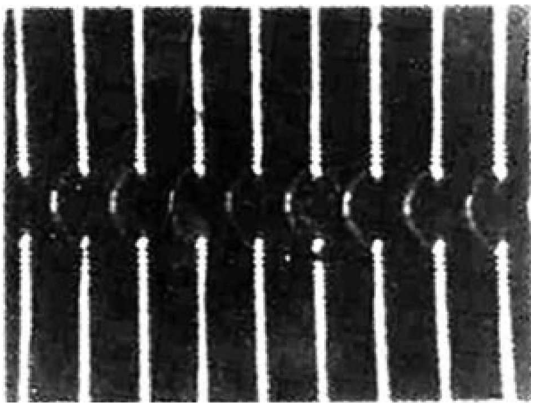

Figure 2. Experimental photograph of straight line fringes modulated by the phase shift introduced by the fiber, where $d = 109.03$ μm is the fiber diameter, $\mu_{\rm L} = 1.5154$ is the liquid refractive index, $\mu_{\rm fiber} = 1.5172$ is the fiber refractive index, $dz/\Delta z = 0.4907$ is the differential fringe shift, $\lambda = 632.8$ nm is the laser wavelength. Experiment is operated at room temperature $T = 25.5$°C.

The straight line fringes modulated by the quadratic function are represented by the following formula:

$$m\lambda = 2\mu_{\rm L} z \tan(\alpha) + 4(\mu_{\rm f} - \mu_{\rm L}) t_{\rm o} (y/t_{\rm f})^2. \tag{6}$$

The differential fringe shift is calculated as follows:

$$D_z(y) = {\rm d}z/\Delta z = \frac{4(\mu_{\rm f} - \mu_{\rm L}) t_{\rm o} (y/t_{\rm f})^2}{(\lambda/2)\sin(\alpha/2)} = \beta(y/t_{\rm f}), \tag{7}$$

where

$$\beta = \frac{8(\mu_{\rm f} - \mu_{\rm L}) t_{\rm o}}{\lambda \sin(\alpha/2)} \text{ is a constant.}$$

Hence, $D_z(y)$ represents an exact quadratic shift for the cited model.

4. Results and discussion

The experimental photograph of the straight line fringes modulated by a shift occurred due to the introduction of uncladded glass fiber between the two plates of the Fabry–Perot interferometer is shown in figure 2. This photo is obtained in the imaging plane (x', y') as shown in figure 1. The arrangement is considered as a liquid wedge interferometer illuminated with a collimated beam emitted from He–Ne laser at $\lambda = 632.8$ nm. The refractive index of the fiber $\mu_{\rm f} = 1.5172$ is bigger than the refractive index of the liquid $\mu_{\rm L} = 1.5158$. The differential fringe shift is computed as ${\rm d}z/\Delta z = 0.4907$ using traveling microscope of an accuracy 10 μm. Figures 3a–d represent the plot of fringes modulated by the parabolic function computed from eq. (6). The differential fringe shift is taken as ${\rm d}z/\Delta z = 0.25$,

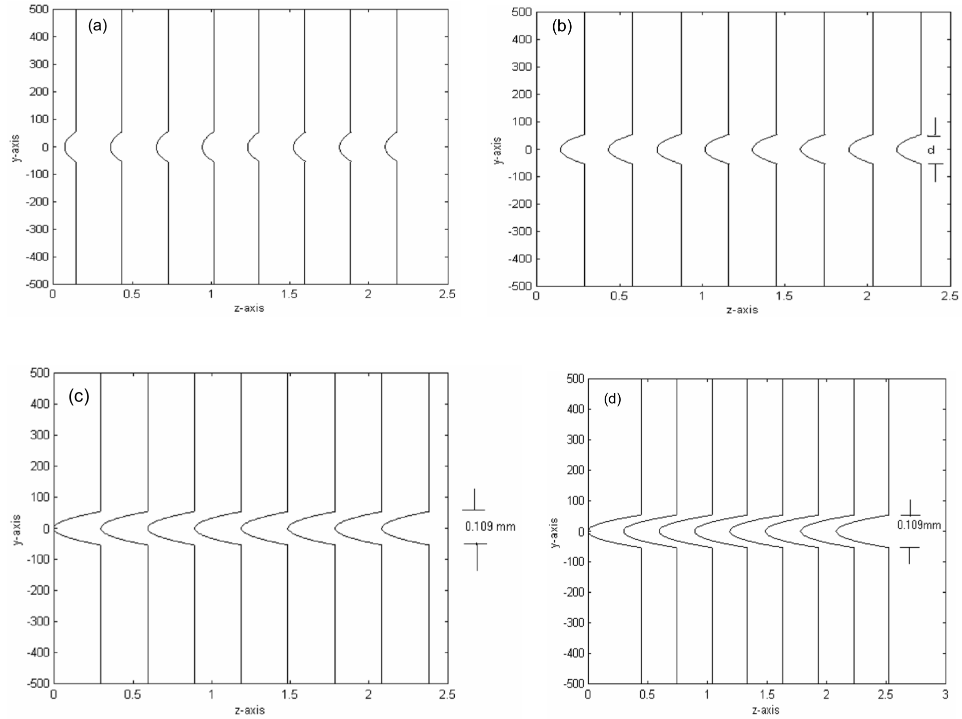

Figure 3. Plot of fringes modulated by a parabolic function of full width = 0.109 mm, y-axis and z-axis are in micrometric scale and the differential fringe shift is equal to (**a**) 0.25, (**b**) 0.5, (**c**) 1.0 and (**d**) 1.25.

0.5, 1, and 1.25 respectively and the full width is set equal to the fiber diameter at $t_{\mathrm{f}} = 109\ \mu$m. A comparative plot represented by semi-circular function of width equal to that of the fiber diameter at $t_{\mathrm{f}} = 109\ \mu$m is shown in figure 4. It is shown that the quadratic model shows a good agreement with both the experimental photograph and the semi-circular shape. A MATLAB program is written in order to plot the figures 3a–d using eq. (6). Other experimental results of the fringe shifts at different diameters and different refractive indices are presented in [5].

5. Conclusion

We have suggested a quadratic model to describe the fringe shift introduced in the phase term appeared in the Airy distribution formula. In this study, multiple beam interference is considered using Fabry–Perot interferometer where the phase object is introduced between the two plates of the interferometer. The quadratic profile is considered as a suitable model to represent the fringe shift that resembles the phase shift produced by fibers of transverse circular section. The potential interest of the presented research is to process the colored phase objects using liquid wedge interferometer. The He–Ne laser is mixed with an argon ion laser at $\lambda = 514.5$ nm

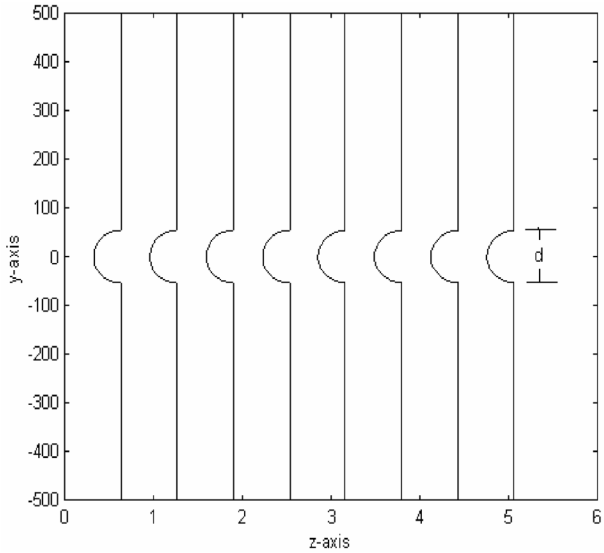

Figure 4. Plot of fringes modulated by a semi-circular function of diameter $d = 0.109$ mm, y-axis and z-axis are in micrometric scale.

to produce collimated polychromatic beam which is required for the investigation of colored phase objects.

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