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Perfect fluid Bianchi Type-I cosmological models with time varying G and Λ

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Abstract. Bianchi Type-I cosmological models containing perfect fluid with time varying G and Λ have been presented. The solutions obtained represent an expansion scalar θ bearing a constant ratio to the anisotropy in the direction of space-like unit vector λ^i. Of the two models obtained, one has negative vacuum energy density, which decays numerically. In this model, we obtain $\Lambda \sim H^2$, $\Lambda \sim R_{44}/R$ and $\Lambda \sim T^{-2}$ (T is the cosmic time) which is in accordance with the main dynamical laws for the decay of Λ. The second model reduces to a static solution with repulsive gravity.

Keywords. Bianchi Type-I Universe; varying G and Λ; cosmology.

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1. Introduction

Cosmological models with a cosmological constant are currently serious candidates to describe the dynamics of the Universe. Recent observations of Type-I$_\mathrm{a}$ supernovae with the red-shift up to about $z \leq 1$ provided evidence that we may be living in a low mass-density Universe, with the contribution of the non-relativistic matter (baryonic plus dark) to the total energy density of the Universe of order of $\Omega_m \sim 0.3$ [1–3]. The value of Ω_m is significantly less than unity [4]. Thus, a major part of the matter content in the Universe remains unobserved. This leads to the assumption that there is some additional energy density sufficient to reach the value $\Omega_{\text{total}} = 1$, predicted by inflationary theory. Several physical models have been proposed to give a consistent physical interpretation of these observational facts. The observational and theoretical features suggest that the most natural candidate for the missing energy is the vacuum energy density or the cosmological constant Λ [5–7]. But selection of the cosmological constant as vacuum energy faces a serious fine-tuning problem, which demands that the value of Λ must be 120 orders of magnitude greater than its presently observed value.

One possible explanation for a small Λ term is to assume that it is dynamically evolving and not constant, that is, as the Universe evolves from an earlier hotter and denser epoch, the effective cosmological term also evolves and decreases to its present value [8–11].

Since the pioneering work of Dirac [12], who proposed, motivated by the occurrence of large numbers in the Universe, a theory with a time variable gravitational coupling constant G, cosmological models with variable G and non-vanishing and variable cosmological term have been intensively investigated in the physical literature [13–30]. Bertolami [31] studied a cosmological model with a time-dependent cosmological term. Later, by considering a varying cosmological constant, Ozer and Taha [8] and others [5,14,20] obtained models of the Universe which are free from cosmological problem.

Another important quantity, which is supposed to be damped out in the course of cosmic evolution, is the anisotropy of the cosmic expansion. It is believed that the early Universe was characterized by a highly irregular mechanism which isotropized later [32]. The cosmological models, which are spatially homogeneous and anisotropic, have a significant role in the description of the Universe in the early stages of its evolution. Therefore, it makes sense to consider the models of the Universe with anisotropic background in the presence of dark energy.

The simplest of the anisotropic models are Bianchi Type-I homogeneous models whose spatial sections are flat but the expansion or contraction rate is direction-dependent. For studying the possible effects of anisotropy in the early Universe on present day observations, a number of researchers [27–30,33,34] have investigated Bianchi Type-I models from different points of view.

In this paper, we study homogeneous Bianchi Type-I space-time with variable G and Λ containing matter in the form of a perfect fluid. We obtain solutions of the Einstein equations assuming that the expansion scalar θ bears a constant ratio to the anisotropy in the direction of a unit space-like vector λ^i [35]. The paper is organized as follows. Basic equations of the model and solutions are given in §2. In §3, we discuss the models and conclude our results in §4.

2. Field equations and solutions

The line element for Bianchi Type-I space-time is taken in the orthogonal form as

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + A^2(t)\mathrm{d}x^2 + B^2(t)\mathrm{d}y^2 + C^2(t)\mathrm{d}z^2. \tag{1}$$

The distribution of matter in the space-time consists of perfect fluid given by the energy–momentum tensor

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij} \tag{2}$$

satisfying the equation of state

$$p = \omega\rho, \quad 0 \le \omega \text{ (constant) } \le 1. \tag{3}$$

Here, ρ, p and v_i are respectively the energy density, pressure and unit flow vector of the fluid satisfying $v_i v^i = -1$.

The Einstein field equations with time-dependent G and Λ are

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G(t) T_{ij} + \Lambda(t) g_{ij}. \tag{4}$$

For the line element (1) with perfect fluid distribution, field equations (4) in the comoving frame give rise to

$$8\pi G p - \Lambda = -\frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{B_4 C_4}{BC}, \tag{5}$$

$$8\pi G p - \Lambda = -\frac{A_{44}}{A} - \frac{C_{44}}{C} - \frac{A_4 C_4}{AC}, \tag{6}$$

$$8\pi G p - \Lambda = -\frac{A_{44}}{A} - \frac{B_{44}}{B} - \frac{A_4 B_4}{AB}, \tag{7}$$

$$8\pi G \rho + \Lambda = \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC}. \tag{8}$$

The suffix '4' stands for ordinary time-derivative of the concerned quantity.

Eliminating p and Λ from eqs (5)–(7), we obtain

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{C_4}{C}\left(\frac{A_4}{A} - \frac{B_4}{B}\right) = 0, \tag{9}$$

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = 0. \tag{10}$$

First integral of (9) and (10) are

$$\frac{A_4}{A} - \frac{B_4}{B} = \frac{k_1}{ABC} \tag{11}$$

and

$$\frac{B_4}{B} - \frac{C_4}{C} = \frac{k_2}{ABC}, \tag{12}$$

where k_1 and k_2 are the constants of integration.

An average scale factor $R(t)$ is defined by

$$R^3 = ABC. \tag{13}$$

The Hubble parameter H is given by $H = R_4/R$. Volume expansion θ, deceleration parameter q and shear σ for the metric (1) can be written as

$$\theta = 3H = 3\frac{R_4}{R}, \tag{14}$$

$$\sigma^2 = \frac{k_1^2 + k_2^2 + k_1 k_2}{3R^6}, \tag{15}$$

$$q = -\frac{R_{44}}{RH^2}. \tag{16}$$

Equation (15) implies that

$$\frac{\sigma_4}{\sigma} = -3H. \tag{17}$$

Equations (5)–(8) can be recast in terms of H, σ and q as

$$8\pi G p - \Lambda = H^2(2q-1) - \sigma^2, \tag{18}$$

$$8\pi G\rho + \Lambda = 3H^2 - \sigma^2. \tag{19}$$

From eq. (19), we obtain

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{24\pi G\rho}{\theta^2} - \frac{3\Lambda}{\theta^2} \tag{20}$$

implying that for $\Lambda \geq 0$,

$$0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}, \quad 0 < \frac{8\pi G\rho}{\theta^2} < \frac{1}{3}.$$

Thus the presence of a positive Λ lowers the upper limit of anisotropy whereas a negative Λ gives more room for anisotropy.

Equation (20) can also be written as

$$\frac{\sigma^2}{3H^2} = 1 - \frac{8\pi G\rho}{3H^2} - \frac{\Lambda}{3H^2} = 1 - \frac{\rho}{\rho_\mathrm{c}} - \frac{\rho_\mathrm{v}}{\rho_\mathrm{c}}, \tag{21}$$

where $\rho_\mathrm{c} = 3H^2/8\pi G$ is the critical density and $\rho_\mathrm{v} = \Lambda/8\pi G$ is the vacuum density. From eqs (18) and (19), we get

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -12\pi G p - \frac{\theta^2}{2} + \frac{3\Lambda}{2} = -12\pi G(\rho + p) - 3\sigma^2 \tag{22}$$

showing that the rate of volume expansion decreases during time evolution and the presence of a positive Λ slows down the rate of this decrease whereas a negative Λ would promote it. From eqs (18) and (19), we get

$$\Lambda = H^2(2-q) - 4\pi(1-\omega)G\rho \tag{23}$$

implying that $\Lambda \leq 0$ for $q \geq 2$.

For $p = 0$, eqs (18) and (19) give

$$\frac{\Lambda}{H^2} = 1 - 2q + \frac{\sigma^2}{H^2}, \tag{24}$$

$$\Omega = 1 - \frac{\Lambda}{3H^2} - \frac{\sigma^2}{3H^2}, \tag{25}$$

$$\Omega = \frac{2}{3}\left(q + 1 - \frac{\sigma^2}{H^2}\right), \tag{26}$$

where $\Omega = \rho/\rho_{\rm c}$ is the density parameter.

From eqs (24) and (25) one can deduce the present values of cosmological term Λ and density parameter Ω. We observe that in an anisotropic background, value of density parameter Ω is smaller in comparison to its value in the isotropic background.

In view of covariant divergence of the left-hand side of (4), we obtain

$$8\pi G\left\{\rho_4 + 3(\rho + p)\frac{R_4}{R}\right\} + 8\pi\rho G_4 + \Lambda_4 = 0. \tag{27}$$

From eq. (27), we observe that Λ is a constant in the absence of matter ($T_{ij} = 0$) implying that matter is essential for a time varying Λ. In the field eqs (4), Λ accounts for vacuum energy with its energy density $\rho_{\rm v}$ and isotropic pressure $p_{\rm v}$ satisfying the equation of state

$$p_{\rm v} = -\rho_{\rm v} = -\frac{\Lambda}{8\pi G}.$$

The usual conservation law for energy–momentum tensor

$$T^j_{i;j} = 0$$

leads to

$$\rho_4 + 3(\omega + 1)\rho\frac{R_4}{R} = 0 \tag{28}$$

leaving G and Λ as some kind of coupled fields

$$8\pi\rho G_4 + \Lambda_4 = 0 \tag{29}$$

implying that Λ is a constant whenever G is constant. Equation (28) on integration gives

$$\rho = \frac{k}{R^{3(\omega+1)}}, \quad k = \text{ constant } > 0 \tag{30}$$

The system of equations (3), (5)–(8) and (29) supply only six equations in seven unknowns. To have a determinate solution we require one more condition. For this purpose, we take the volume expansion θ to have a constant ratio to the anisotropy in the direction of unit space-like vector λ^i, i.e. $\theta/\sigma_{ij}\lambda^i\lambda^j$ is a constant [35]. Here we take $\lambda_i = (a_1A, a_2B, a_3C, 0)$, where a_1, a_2, a_3 are constants satisfying $a_1^2 + a_2^2 + a_3^2 = 1$.

In general the above condition gives rise to

$$A = B^m C^n, \tag{31}$$

where m, n are constants depending on a_1, a_2 and a_3.

Using eq. (31) in eqs (11) and (12), we obtain

$$\begin{aligned} C &= b_1(k_3 t + k_4)^{\frac{k_1-(m-1)k_2}{(m+n+2)k_1-(m-2n-1)k_2}}, \quad \text{for} \quad \frac{k_1}{k_2} \neq \frac{m-2n-1}{m+n+2} \\ &= k_5 \mathrm{e}^{\frac{-k_2(m+1)}{(m+n+2)}t}, \quad \text{for} \quad \frac{k_1}{k_2} = \frac{m-2n-1}{m+n+2} \end{aligned}$$

and

$$\begin{aligned} B &= b_2(k_3 t + k_4)^{\frac{k_1+k_2 n}{(m+n+2)k_1-(m-2n-1)k_2}}, \quad \text{for} \quad \frac{k_1}{k_2} \neq \frac{m-2n-1}{m+n+2} \\ &= b_3 \mathrm{e}^{\frac{k_2(n+1)}{(m+n+2)}t}, \quad \text{for} \quad \frac{k_1}{k_2} = \frac{m-2n-1}{m+n+2} \end{aligned}$$

provided $m+n \neq 1$. In the above k_3, k_4, k_5, b_1, b_2 and b_3 are constants of integration.

For these solutions, metric (1) assumes the following forms after suitable transformations:

$$\begin{aligned} \mathrm{d}s^2 = &-\mathrm{d}T^2 + T^{\frac{2(m+n)k_1+2nk_2}{(m+n+2)k_1-(m-2n-1)k_2}}\mathrm{d}X^2 + T^{\frac{2k_1+2k_2 n}{(m+n+2)k_1-(m-2n-1)k_2}}\mathrm{d}Y^2 \\ &+ T^{\frac{2k_1-2(m-1)k_2}{(m+n+2)k_1-(m-2n-1)k_2}}\mathrm{d}Z^2, \quad \text{for} \quad \frac{k_1}{k_2} \neq \frac{m-2n-1}{m+n+2}, \end{aligned} \tag{32}$$

$$\begin{aligned} \mathrm{d}s^2 = &-\mathrm{d}T^2 + \exp\left\{\frac{2k_2(m-n)t}{m+n+2}\right\}\mathrm{d}X^2 + \exp\left\{\frac{2k_2(n+1)t}{m+n+2}\right\}\mathrm{d}Y^2 \\ &+ \exp\left\{\frac{-2k_2(m+1)t}{m+n+2}\right\}\mathrm{d}Z^2, \quad \text{for} \quad \frac{k_1}{k_2} = \frac{m-2n-1}{m+n+2}. \end{aligned} \tag{33}$$

3. Discussion

Average scale factor R for the model (32) is given by

$$R = T^{1/3}.$$

Hubble parameter H, volume expansion θ and shear σ for this model are

$$\theta = 3H = \frac{1}{T},$$

$$\sigma^2 = \frac{a^2}{3T^2}, \quad \text{where } a^2 = k_1^2 + k_2^2 + k_1 k_2.$$

Matter density ρ, gravitational parameter G, cosmological term Λ, vacuum energy density ρ_{v} and critical density ρ_{c} are given by

$$\rho = kT^{-(\omega+1)},$$

$$G = \frac{(1-a^2)}{12\pi k(\omega+1)} T^{\omega-1},$$

$$\Lambda = \frac{(1-a^2)(\omega-1)}{3(\omega+1)} T^{-2},$$

$$\rho_{\rm v} = \frac{k(\omega-1)}{2} T^{-(\omega+1)},$$

$$\rho_{\rm c} = \frac{k(\omega+1)}{2(1-a^2)} T^{-(\omega+1)}.$$

In the model we observe that G is positive or negative depending on whether $a^2 < 1$ or $a^2 > 1$. The model has singularity at $T = 0$. The cosmic scenario starts from a Big Bang at $T = 0$ and continues till $T = \infty$. Since $\sigma/\theta = a/\sqrt{3}$, the anisotropy does not die out asymptotically. The model admits a negative Λ unless $\omega = 1$. Universe models with negative Λ are also admissible [36]. For $\omega = 1$, the model reduces to the standard Bianchi Type-I model with $\Lambda = 0$, $G =$ constant and $\rho \sim T^{-2}$. At $T = 0$, ρ, G, $|\Lambda|$, σ^2 and θ are all infinite and all of them tend to zero as $T \to \infty$. The ratio between vacuum and matter densities scales as

$$\frac{\Lambda}{8\pi G\rho} = \frac{\omega-1}{2}.$$

The density parameter

$$\Omega = \frac{8\pi G\rho}{3H^2} = \frac{2(1-a^2)}{\omega+1}$$

and

$$\Omega_\Lambda = \frac{\Lambda}{3H^2} = \frac{(1-a^2)(\omega-1)}{\omega+1}.$$

In the model, we obtain $\Lambda \sim H^2$, $\Lambda \sim R_{44}/R$ and $\Lambda \sim T^{-2}$ which is in accordance with the main dynamical laws one finds in literature proposed for the decay of Λ. The dynamical law $\Lambda \sim H^2$ has been proposed by Carvalho *et al* [19] and considered by Salim and Waga [37], Arbab and Abdel-Rahman [38], Wetterich [39] and Arbab [40]. In view of the present estimates Λ is of the order of H_0^2 [41]. The dynamical law $\Lambda \sim T^{-2}$ has been considered by several authors, e.g. Bertolami [31], Berman and Som [42], Berman [18], Beesham [17] to mention a few. The decay law of the form $\Lambda \sim R_{44}/R$ has been considered by Arbab [43]. We observe that $|\Lambda|$ decays faster than G whereas ρ, $\rho_{\rm v}$ and $\rho_{\rm c}$ scale as $T^{-(\omega+1)}$. In the model, we see that the quantity $G\rho$ satisfies the condition for a Machian cosmological solution, i.e. $G\rho \sim H^2$ [44].

For the radiation epoch, we have $\omega = 1/3$. In this case

$$\rho = kT^{-4/3},$$

$$G = \frac{(1-a^2)}{16\pi k} T^{-2/3},$$

$$\Lambda = \frac{-(1-a^2)}{6} T^{-2},$$

$$\rho_{\mathrm{v}} = \frac{-k}{3} T^{-4/3},$$

$$\rho_{\mathrm{c}} = \frac{2k}{3(1-a^2)} T^{-4/3}.$$

We observe that in this phase matter density is three times the vacuum density. The density parameter Ω in this phase is

$$\Omega = \frac{3}{2}(1-a^2) < \frac{3}{2}.$$

Also $|\Omega_\Lambda| = \frac{1}{2}(1-a^2) < \frac{1}{2}$. For the phase dominated by dust matter, we have $\omega = 0$.

In this epoch,

$$\rho = kT^{-1},$$

$$G = \frac{(1-a^2)}{12\pi k} T^{-1},$$

$$|\Lambda| = \frac{(1-a^2)}{3} T^{-2},$$

$$|\rho_{\mathrm{v}}| = \frac{k}{2} T^{-1},$$

$$\rho_{\mathrm{c}} = \frac{k}{2(1-a^2)} T^{-1},$$

$$\frac{|\Lambda|}{8\pi G\rho} = \frac{1}{2},$$

$$\Omega = 2(1-a^2) < 2$$

and

$$|\Omega_\Lambda| = (1-a^2) < 1.$$

The age of the Universe is given by

$$T_0 = \frac{1}{3} H_0^{-1}$$

which is smaller than the best estimation $T_0 = H_0^{-1}$ [45].

For the model (33), average scale factor R is given by $R = 1$. In this model

$$\theta = 3H = 0,$$

$$\sigma^2 = \frac{b^2}{3}, \quad b^2 = \frac{3k_2(m^2 + n^2 - mn + m + n + 1)}{m + n + 2} > 0,$$

$$\rho = k,$$

$$G = -\frac{b^2}{12\pi k(\omega + 1)},$$

$$\Lambda = \frac{b^2(1 - \omega)}{3(1 + \omega)}.$$

In this case, our model reduces to a static model with repulsive gravity.

4. Conclusion

Bianchi Type-I cosmological models with time varying G and Λ are obtained. The models obtained present an expansion scalar θ bearing a constant ratio to the anisotropy in the direction of space-like unit vector λ^i. One of the two models has a negative vacuum energy density, which decays numerically. In this model, we obtain $\Lambda \sim H^2$, $\Lambda \sim R_{44}/R$ and $\Lambda \sim T^{-2}$ which is in accordance with the main dynamical laws for the decay of Λ. The anisotropy in the model does not die out asymptotically. We also obtain that the model satisfies the condition for a Machian cosmological solution, i.e. $G\rho \sim H^2$. It is interesting to note that the metric (1) with condition (31) in the case of a perfect fluid distribution represents a stiff matter if and only if $\Lambda = 0$. Incidentally our derived results in view of condition (31) leads to solutions similar to Beesham [17] and Kalligas *et al* [22], ones which were obtained by assuming a particular form of G. The second model obtained reduces to a static Universe with repulsive gravity.

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References

[1] A G Riess *et al*, *Astron. J.* **116**, 1099 (1998)
[2] S Perlmutter *et al*, *Astrophys. J.* **517**, 565 (1999)
[3] S Perlmutter *et al*, *Nature (London)* **391**, 51 (1998)
[4] J P Ostriker and P J Steinhardt, *Nature (London)* **377**, 600 (1995)
[5] S Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989)

[6] S Weinberg, astro-ph/0005265
[7] M Gasperini, *Class Quantum Gravit.* **5**, 521 (1988)
[8] M Ozer and M O Taha, *Nucl. Phys.* **B287**, 776 (1987)
[9] K Freese *et al*, *Nucl. Phys.* **B287**, 797 (1987)
[10] J A Frieman *et al*, *Phys. Rev. Lett.* **75**, 2077 (1995)
[11] K Coble, S Dodelson and J A Frieman, *Phys. Rev.* **D55**, 1851 (1997)
[12] P A M Dirac, *Proc. R. Soc. London* **A165**, 199 (1938)
[13] A-M M Abdel-Rahman, *Nuovo Cimento* **B102**, 225 (1988)
[14] A-M M Abdel-Rahman, *Phys. Rev.* **D45**, 3497 (1992)
[15] J D Barrow, *Phys. Lett.* **B180**, 335 (1986)
[16] J D Barrow and P Parsons, *Phys. Rev.* **D55**, 1906 (1997)
[17] A Beesham, *Gen. Relativ. Gravit.* **26**, 159 (1994)
[18] M S Berman, *Phys. Rev.* **D43**, 1075 (1991)
[19] J C Carvalho, J A S Lima and I Waga, *Phys. Rev.* **D46**, 2404 (1992)
[20] W Chen and Y S Wu, *Phys. Rev.* **D41**, 695 (1990)
[21] D Kalligas, P S Wesson and C W F Everitt, *Gen. Relativ. Gravit.* **24**, 351 (1992)
[22] D Kalligas, P S Wesson and C W F Everitt, *Gen. Relativ. Gravit.* **27**, 645 (1995)
[23] Y K Lau, *Aust. J. Phys.* **38**, 547 (1985)
[24] Y K Lau and S J Prokhovnik, *Aust. J. Phys.* **39**, 339 (1986)
[25] J A S Lima and M Trodden, *Phys. Rev.* **D53**, 4280 (1996)
[26] J A S Lima and J M F Maia, *Phys. Rev.* **D49**, 5597 (1994)
[27] Bijan Saha, *Int. J. Theor. Phys.* **45**, 983 (2006)
[28] Bijan Saha, *Astrophys. Space Sci.* **302**, 83 (2006)
[29] A Pradhan and P Pandey, *Astrophys. Space Sci.* **301**, 221 (2006)
[30] A Pradhan and S K Singh, *Int. J. Mod. Phys.* **D13**, 503 (2004)
[31] O Bertolami, *Nuovo Cimento* **B93**, 36 (1986)
[32] C W Misner, *Ap. J.* **151**, 431 (1968)
[33] W Huang, *J. Math. Phys.* **31**, 1456 (1990)
[34] L P Chimento *et al*, *Class. Quantum Gravit.* **14**, 3363 (1997)
[35] S R Roy, S Narain and J P Singh, *Aust. J. Phys.* **38**, 239 (1985)
[36] R G Vishwakarma, *Gen. Relativ. Gravit.* **37**, 1305 (2005)
[37] L M Salim and I Waga, *Class. Quantum Gravit.* **10**, 1767 (1993)
[38] A I Arbab and A-M M Abdel-Rahman, *Phys. Rev.* **D50**, 7725 (1994)
[39] C Wetterich, *Astron. Astrophys.* **301**, 321 (1995)
[40] A I Arbab, *Gen. Relativ. Gravit.* **29**, 61 (1994)
[41] R G Vishwakarma, *Class. Quantum Gravit.* **18**, 1159 (2001)
[42] M S Berman and M M Som, *Int. J. Theor. Phys.* **29**, 1411 (1990)
[43] A I Arbab, *Class. Quantum Gravit.* **20**, 93 (2003)
[44] M S Berman, *Int. J. Theor. Phys.* **29**, 571 (1990)
[45] B M S Hansen *et al*, *Astrophys. J.* **574**, L155 (2002)