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Effects of ion-fluid temperature on dust-ion-acoustic solitons

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Abstract. The properties of dust-ion-acoustic (DIA) solitons in an unmagnetized dusty plasma, whose constituents are adiabatic ion-fluid, Boltzmann electrons, and static dust particles, are investigated by employing the reductive perturbation method. The Korteweg–de Vries equation is derived and its stationary solution is numerically analyzed. The parametric regimes for the existence of positive and negative solitons are found. It has been shown that ion-fluid temperature not only significantly modifies the basic features (width and amplitude) of DIA solitons, but also introduces some new features of DIA solitons.

Keywords. Dusty plasmas; dust-ion-acoustic solitons; reductive perturbation method.

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1. Introduction

Shukla and Silin [1] have first theoretically shown that due to the conservation of equilibrium charge density $n_{e0} + n_{d0}Z_d = n_{i0}$ and the strong inequality $n_{e0} \ll n_{i0}$ (where n_{s0} is the particle number density of the species s with $s = e(i)d$ for electrons (ions) dust particles, Z_d is the number of electrons residing on the dust grain surface, and $-e$ is the electronic charge) a dusty plasma (with negatively charged static dust grains) supports low-frequency dust-ion-acoustic (DIA) waves with phase speed much smaller (larger) than electron (ion) thermal speed. The dispersion relation (a relation between the wave frequency ω and the wave number k) of the linear DIA waves is [1] $\omega^2 = (n_{i0}/n_{e0})k^2C_i^2/(1 + k^2\lambda_{\rm De}^2)$, where $C_i = (k_{\rm B}T_e/m_i)^{1/2}$ is the ion-acoustic speed (with T_e the electron temperature, $k_{\rm B}$ the Boltzmann constant, and m_i the ion mass) and $\lambda_{\rm De} = (k_{\rm B}T_e/4\pi n_{e0}e^2)^{1/2}$ is the electron Debye radius. For a long wavelength limit (viz. $k\lambda_{\rm De} \ll 1$), the dispersion relation for the DIA waves becomes $\omega = (n_{i0}/n_{e0})^{1/2}kC_i$. This form of spectrum is similar to the usual ion-acoustic wave spectrum [2] for a plasma with $n_{i0} = n_{e0}$ and $T_i \ll T_e$ (where T_i is the ion-fluid temperature). The DIA waves have been observed in laboratory experiments [3,4]. The linear properties of the DIA waves in dusty plasmas are now well understood from both theoretical and experimental points of view [1,3–9]. The DIA solitary waves have been investigated by several authors [10–13]. However, all these investigations [10–13] are limited to a cold ion-fluid limit ($T_i = 0$).

To the best of our knowledge, there is no investigation to show how the minimum value of the dust charge density for which negative solitary potential structures exist, and the basic features (amplitude and width) of such positive and negative potential structures are significantly modified by the combined effects of the ion-fluid temperature and the dust charge density. Therefore, in our present work we have theoretically investigated the effects of ion-fluid temperature on DIA solitary structures in a dusty plasma containing adiabatic ion-fluid, Boltzmann electrons, and static dust particles.

2. Governing equations

We consider an unmagnetized dusty plasma system consisting of adiabatic ion-fluid, Boltzmann distributed electrons and static negatively charged dust particles. The dynamics of one-dimensional dust-ion-acoustic (DIA) waves propagating in such a dusty plasma system is governed by

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}(NU) = 0, \tag{1}$$

$$N\frac{\partial U}{\partial t} + NU\frac{\partial U}{\partial x} = -N\frac{\partial \psi}{\partial x} - \alpha\frac{\partial P}{\partial x}, \tag{2}$$

$$\frac{\partial P}{\partial t} + U\frac{\partial P}{\partial x} + 3P\frac{\partial U}{\partial x} = 0, \tag{3}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \mu \mathrm{e}^{\psi} - N + (1-\mu), \tag{4}$$

where N is the ion number density normalized by its equilibrium value n_{i0}, U is the ion-fluid speed normalized by $C_i = (k_{\mathrm{B}}T_e/m_i)^{1/2}$, ψ is the wave potential normalized by $k_{\mathrm{B}}T_e/e$, P is the ion-thermal pressure normalized by $n_{i0}k_{\mathrm{B}}T_i$, $\alpha = T_i/T_e$, and $\mu = n_{e0}/n_{i0} = 1 - Z_d n_{d0}/n_{i0}$. The time variable t is normalized by $\omega_{pi}^{-1} = (m_i/4\pi e^2 n_{i0})^{1/2}$, and the space variable x is normalized by $\lambda_{\mathrm{D}} = (k_{\mathrm{B}}T_e/4\pi e^2 n_{i0})^{1/2}$.

3. Small amplitude DIA solitary waves

To study small but finite amplitude DIA solitary waves, we now derive the Korteweg–de Vries (K-dV) equation from (1)–(4) by employing the reductive perturbation technique and the stretched coordinates [14] $\zeta = \epsilon^{1/2}(x - V_0 t)$ and $\tau = \epsilon^{3/2}t$, where ϵ is a smallness parameter measuring the weakness of the dispersion, and V_0 is the phase speed of the DIA waves normalized by C_i.

We can express (1)–(4) in terms of ζ and τ as

$$\epsilon^{3/2}\frac{\partial N}{\partial \tau} - V_0\epsilon^{1/2}\frac{\partial N}{\partial \zeta} + \epsilon^{1/2}\frac{\partial}{\partial \zeta}(NU) = 0, \tag{5}$$

$$\epsilon^{3/2}N\frac{\partial U}{\partial \tau} - V_0\epsilon^{1/2}N\frac{\partial U}{\partial \zeta} + \epsilon^{1/2}NU\frac{\partial U}{\partial \zeta} = -\epsilon^{1/2}N\frac{\partial \psi}{\partial \zeta} - \epsilon^{1/2}\alpha\frac{\partial P}{\partial \zeta}, \tag{6}$$

$$\epsilon^{3/2}\frac{\partial P}{\partial \tau} - V_0\epsilon^{1/2}\frac{\partial P}{\partial \zeta} + \epsilon^{1/2}U\frac{\partial P}{\partial \zeta} + 3\epsilon^{1/2}P\frac{\partial U}{\partial \zeta} = 0, \tag{7}$$

$$\epsilon\frac{\partial^2 \psi}{\partial \zeta^2} = \mu\left[1 + \psi + \frac{1}{2}\psi^2 + \cdots\right] - N + (1-\mu). \tag{8}$$

We can expand the variables N, U, P, and ψ in a power series of ϵ as

$$N = 1 + \epsilon N^{(1)} + \epsilon^2 N^{(2)} + \cdots, \tag{9}$$

$$U = 0 + \epsilon U^{(1)} + \epsilon^2 U^{(2)} + \cdots, \tag{10}$$

$$P = 1 + \epsilon P^{(1)} + \epsilon^2 P^{(2)} + \cdots, \tag{11}$$

$$\psi = 0 + \epsilon \psi^{(1)} + \epsilon^2 \psi^{(2)} + \cdots. \tag{12}$$

Now, substituting (9)–(12) into (5)–(8) and taking the coefficients of $\epsilon^{3/2}$ from (5)–(7) and ϵ from (8), we get

$$U^{(1)} = \frac{V_0\psi^{(1)}}{V_0^2 - 3\alpha}, \tag{13}$$

$$P^{(1)} = \frac{3\psi^{(1)}}{V_0^2 - 3\alpha}, \tag{14}$$

$$N^{(1)} = \frac{\psi^{(1)}}{V_0^2 - 3\alpha}, \tag{15}$$

$$V_0 = \sqrt{1/\mu + 3\alpha}. \tag{16}$$

Equation (16) is the linear dispersion relation for the DIA waves propagating in the dusty plasma under consideration. Similarly, substituting (9)–(12) into (5)–(8) and equating the coefficients of $\epsilon^{5/2}$ from (5)–(7) and ϵ^2 from (8), one obtains

$$\frac{\partial N^{(1)}}{\partial \tau} - V_0\frac{\partial N^{(2)}}{\partial \zeta} + \frac{\partial}{\partial \zeta}[U^{(2)} + N^{(1)}U^{(1)}] = 0, \tag{17}$$

$$\frac{\partial U^{(1)}}{\partial \tau} - V_0 N^{(1)}\frac{\partial U^{(1)}}{\partial \zeta} - V_0\frac{\partial U^{(2)}}{\partial \zeta} + U^{(1)}\frac{\partial U^{(1)}}{\partial \zeta} = -N^{(1)}\frac{\partial \psi^{(1)}}{\partial \zeta} - \frac{\partial \psi^{(2)}}{\partial \zeta} - \alpha\frac{\partial P^{(2)}}{\partial \zeta}, \tag{18}$$

$$\frac{\partial P^{(1)}}{\partial \tau} - V_0\frac{\partial P^{(2)}}{\partial \zeta} + U^{(1)}\frac{\partial P^{(1)}}{\partial \zeta} + 3P^{(1)}\frac{\partial U^{(1)}}{\partial \zeta} + 3\frac{\partial U^{(2)}}{\partial \zeta} = 0, \tag{19}$$

$$\frac{\partial^2 \psi^{(1)}}{\partial \zeta^2} = \mu\psi^{(2)} + \frac{1}{2}\mu[\psi^{(1)}]^2 - N^{(2)}. \tag{20}$$

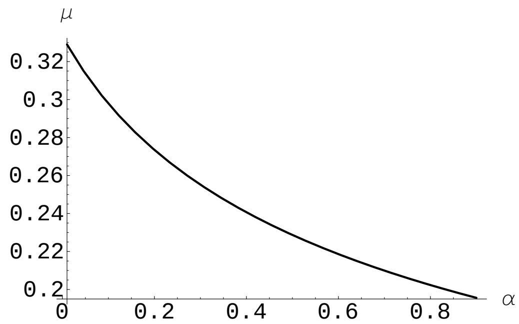

Figure 1. $A = 0$ (μ vs. α) curve. The parameters (α and μ), whose values lie above (below) the curve, correspond to the positive (negative) solitary potential profiles.

Now, using (13)–(16), and eliminating $N^{(2)}$, $U^{(2)}$, $P^{(2)}$, and $\psi^{(2)}$, we finally obtain

$$\frac{\partial \psi^{(1)}}{\partial \tau} + A\psi^{(1)}\frac{\partial \psi^{(1)}}{\partial \zeta} + B\frac{\partial^3 \psi^{(1)}}{\partial \zeta^3} = 0, \tag{21}$$

where the nonlinear coefficient A and the dispersion coefficient B are given by

$$A = (3\mu - 1 + 12\alpha\mu^2)B\mu^2, \tag{22}$$

$$B = \frac{1}{2\mu^2\sqrt{\mu(1+3\alpha\mu)}}. \tag{23}$$

Equation (21) is the K-dV equation describing the nonlinear propagation of the DIA waves in an unmagnetized dusty plasma consisting of adiabatic ion-fluid, Boltzmann distributed electrons and static negatively charged dust grains. The stationary solution of this K-dV equation is obtained by transforming the independent variables ζ and τ to $\xi = \zeta - U_0\tau$ and $\tau = \tau$, where U_0 is a constant velocity normalized by C_i, and imposing the appropriate boundary conditions, viz. $\psi^{(1)} \to 0$, $\partial\psi^{(1)}/\partial\xi \to 0$, $\partial^2\psi^{(1)}/\partial\xi^2 \to 0$ at $\xi \to \pm\infty$. Thus, one can express the stationary solution of the K-dV equation as

$$\psi^{(1)} = \Psi_m \,\mathrm{sech}^2(\xi/\Delta), \tag{24}$$

where the amplitude Ψ_m (normalized by $k_{\mathrm{B}}T_e/e$) and the width Δ (normalized by λ_{D}) are given by

$$\Psi_m = \frac{3U_0}{A}, \tag{25}$$

$$\Delta = \sqrt{4B/U_0}. \tag{26}$$

It is obvious from (24)–(26) that as U_0 increases, the amplitude (width) of the solitary waves increases (decreases). It is clear from (22), (24), and (25) that the solitary potential profiles are positive (negative) if $A >\ (<)\ 0$. To find the parametric regimes for which positive and negative solitary waves exist, we have numerically

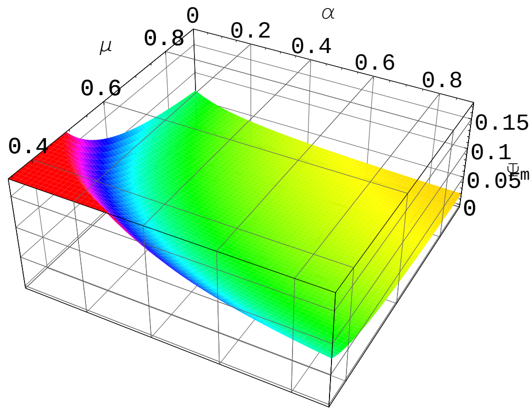

Figure 2. Variation of the amplitude of the positive solitary potential profiles with α and μ.

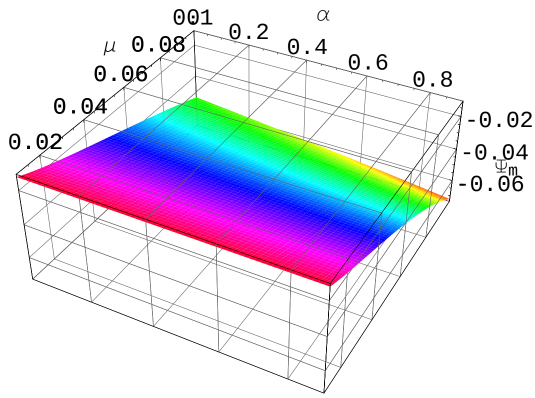

Figure 3. Variation of the amplitude of the negative solitary potential profiles with α and μ.

analyzed A and obtain $A = 0$ (μ vs. α) curve which is shown in figure 1. Figure 1 shows that we can have positive (negative) solitary potential profiles for the parameters whose values lie above (below) the $A = 0$ curve. We have also graphically shown how the amplitude and the width of the positive (corresponding to the parameters whose values lie above the $A = 0$ curve) and negative (corresponding to the parameters whose values lie below the $A = 0$ curve) solitary potential profiles vary with α and μ. These are displayed in figures 2–4.

Figure 2 shows that the amplitude of the positive solitary potential profiles decreases with α and μ. Figure 3 shows that the amplitude of the negative solitary potential profiles increases with α, but decreases with μ (we note that the magnitude of the amplitude of the negative solitary potential profiles decreases with α, but increases with μ). Figure 4 shows that the width of the solitary potential profiles decreases with the increase of α and μ.

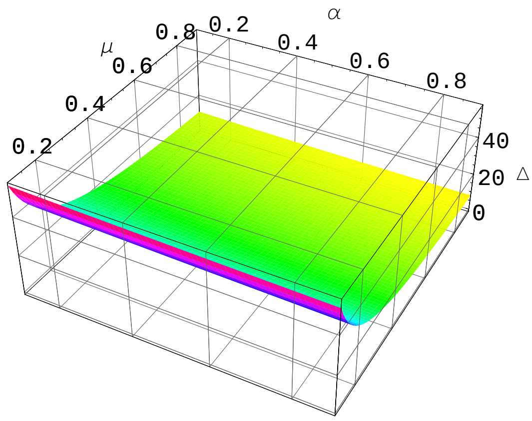

Figure 4. Variation of the width of the solitary potential profiles with α and μ.

4. Discussion

We consider an unmagnetized dusty plasma system consisting of adiabatic ion-fluid, Boltzmann distributed electrons and static negatively charged dust particles. We have investigated the basic properties of small but finite amplitude DIA solitary waves. To interpret our results or to show how our numerical analysis completely agree with our analytical calculations, we express (25) as $\Psi_m = 12/[\mu^2\Delta^2(3\mu - 1 + 12\alpha\mu^2)]$, where $\Delta = (\sqrt{2/U_0})\mu^{-3/4}(1+3\alpha\mu)^{-1/4}$. This means that for constant values of μ and α, the amplitude decreases by increasing the width, which is the well-known feature of the K-dV solitons. However, in our numerical analysis, we have varied the parameters μ and α, and graphically shown how the amplitude of positive and negative solitary potential profiles varies with μ and α (figures 2 and 3). Therefore, in addition to this well-known feature, we have found some new features which are shown in figures 2–4. The expression of Δ clearly indicates that the width always decreases by increasing the values of α and μ, which completely agrees with our numerical analysis (figure 4).

It is important to note that the DIA solitons are more suitable than the dust-acoustic solitons to study in laboratory dusty plasma conditions. We, therefore, propose to perform a laboratory experiment which can study such special new features of DIA solitons. We also stress that the present results may help to understand the salient features of DIA solitons in a dusty plasma (containing adiabatic ion-fluid, Boltzmann electrons and negatively charged static dust particles), which is relevant to space and laboratory experiments.

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