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Generation of whistler mode in a relativistic plasma

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Abstract. This paper contains the plasma maser interaction between high frequency nonresonant whistler R-mode and low frequency resonant ion acoustic mode in a relativistic plasma. It shows that the whistler R-mode grows through the plasma maser interaction between the relativistic electrons and the ion acoustic fluctuation.

Keywords. Energy up-conversion process; weak turbulence theory.

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1. Introduction

Whistlers are low frequency, circularly polarised electromagnetic waves in the audiofrequency range. Whistler modes excited by energetic electrons are often observed in the outer Earth's radiation belt [1,2] and in the auroral kilometric radiation (AKR) [3–6] zone with larger value of $f_{\rm p}/f_{\rm c}$, where $f_{\rm p}$ and $f_{\rm c}$ respectively denote the plasma frequency and the electron cyclotron frequency. They travel almost along the Earth's magnetic field when their frequencies are well below the electron cyclotron frequency. Whistler can be generated in quiescent plasma owing to numerous macroscopic instabilities; the free source of energy being the anisotropic velocity space distribution [7], beam of electrons [8] etc. Wave particle interaction can also produce whistler [9] in the magnetosphere. Nambu [10] has proposed the generation of whistler waves in the process of induced bremsstrahlung in the presence of background turbulence in nonrelativistic neutral plasma. Besides, the subject of generation of electromagnetic waves through nonlinear interaction between different types of waves in non-neutral plasma has been studied extensively in the last decade [11,12].

In view of the above facts, we study the generation of whistler wave in the presence of ion acoustic turbulence in a relativistic plasma through the process of plasma maser effect in plasma. In this effect we have considered the relativistic interaction of plasma particle with two kinds of waves: one is resonant low frequency ion acoustic wave which satisfies the Cherenkov resonance condition and the other is a nonresonant high frequency electro-magnetic whistler wave which satisfies neither the Cherenkov resonance condition nor the resonant scattering condition. It is found that the above process leads to the up-conversion of energy from the resonant ion acoustic mode to the nonresonant whistler mode as described by Tsytovich *et al* [13]. We show that the nonlinear dielectric constant is composed of two parts: direct and polarisation mode coupling terms. We derive the growth rates of the whistler mode for the direct and the polarisation terms and we discuss the study and its potential importance and applicability. It has been found that all the earlier studies are based on the investigations in nonrelativistic cases only [14–17]

2. Mathematical model

We derive the dipersion relation of the whistler mode in the presence of ion acoustic wave turbulence propagating along the external magnetic field B_0 in the z-direction with the wave vector $(0,0,k_{||})$. The ion acoustic wave turbulence is assumed to be driven by a relativistic electron beam [18]. The beam-type electron distribution function is

$$f_{0\mathrm{e}}(p_{||},p_\perp) = \frac{1}{\pi^{3/2}a_{||}a_\perp^2}\exp\left\{-\frac{p_\perp^2}{a_\perp^2}\right\}\exp\left\{-\frac{(p_{||}-p_0)^2}{a_{||}^2}\right\}, \tag{1}$$

where $a_{||} = (2\langle p_{||}^2\rangle)^{1/2}, a_\perp = (\langle p_\perp^2\rangle)^{1/2}$ and the measure of anisotropy is described by $A = (a_\perp^2/a_{||}^2) - 1$. The symbols in the above equation have their usual meaning. In the kinetic theory the problem is described by the following set of Vlasov–Poisson equation:

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla} - e\{\mathbf{E}(\mathbf{r},t) + \upsilon\cdot\mathbf{B}(\mathbf{r},t)\}\cdot\frac{\partial}{\partial \mathbf{p}}\right] f_\mathrm{e}(\mathbf{r},\mathbf{p},t) = 0,$$

$$\boldsymbol{\nabla}\times\mathbf{E}(\mathbf{r},t) = -\frac{\partial}{\partial t}\mathbf{B}(\mathbf{r},t),$$

$$\boldsymbol{\nabla}\times\mathbf{B}(\mathbf{r},t) = -\frac{n_0 e}{\epsilon_0 c^2}\int \mathbf{v} f_\mathrm{e}(\mathbf{r}\,,\mathbf{p},t)\mathrm{d}p + \frac{1}{c^2}\frac{\partial}{\partial t}\mathbf{E}(\mathbf{r},t), \tag{2}$$

where $f_\mathrm{e}(\mathbf{r},\mathbf{p},t)$ denotes the electron distribution function, e is the magnitude of the electronic charge, c is the velocity of light, ϵ_0 is the vacuum electric permittivity, n_0 is the equilibrium plasma concentration, $\mathbf{E}(\mathbf{r},t)$ is the electric field, $\mathbf{B}(\mathbf{r},t)$ is the magnetic field and p is the relativistic momentum of electron, i.e. $\mathbf{p} = \Gamma m_0\upsilon$, $\Gamma = 1/(1-\upsilon^2/c^2)^{1/2} = (1+p^2/m_0^2c^2)^{1/2}$ with m_0 the rest mass of the electron. According to the linear response theory of turbulent plasma, the unperturbed electron distribution function and fields are $f_\mathrm{e} = f_{0\mathrm{e}} + \epsilon f_{1\mathrm{e}} + \epsilon^2 f_{2\mathrm{e}}; \epsilon < 1$ and $\mathbf{E} = \epsilon\mathbf{E}_l, \mathbf{B} = \mathbf{B}_0$ where $f_{0\mathrm{e}}$ is the space time averaged part described by eq. (1), $f_{1\mathrm{e}}$ and $f_{2\mathrm{e}}$ are the low frequency fluctuating parts of the electron distribution function. ϵ is the ordering of the low frequency ion acoustic mode fluctuations. The interaction between the low frequency ion acoustic wave turbulence and the high frequency whistler mode is considered through the relativistic resonant electrons.

The linear response of the electron distribution function $f_{1\mathrm{e}}$ due to ion acoustic wave reduces to

$$f_{1\mathrm{e}}(\omega, \mathbf{k}) = \frac{ie\Gamma E_{l||}(\omega, \mathbf{k})}{\Gamma\omega - (p_{||}k_{||}/m_0) + i0} \frac{\partial}{\partial p} f_{0\mathrm{e}}. \quad (3)$$

Equation (3) is valid only if the ion waves have sufficiently small amplitude so that the particle trapping [19] does not play a significant role. Moreover, the second-order electric field vanishes under random phase approximation. The result gives the foundation of the linear response theory of a turbulent plasma in which we neglect the ensembled averaged second-order electric field due to the low frequency turbulence. We now perturbed the quasi-steady state by a high frequency electromagnetic test whistler mode wave field $\delta\mathbf{E}_h$ with frequency Ω. Here $\mu \ll \epsilon$. Thus, the total perturbed electron distribution function and electric and magnetic fields are $\delta f = \mu\delta f_h + \mu\epsilon\delta f_{lh} + \mu\epsilon^2\Delta f$, $\delta\mathbf{E} = \mu\delta\mathbf{E}_h + \mu\epsilon\delta\mathbf{E}_{lh} + \mu\epsilon^2\Delta\mathbf{E}$ and $\delta\mathbf{B} = \mu\delta\mathbf{B}_h + \mu\epsilon\delta\mathbf{B}_{lh} + \mu\epsilon^2\Delta\mathbf{B}$ where $\delta\mathbf{E}_{lh}$, $\delta\mathbf{B}_{lh}$, $\Delta\mathbf{E}$ and $\Delta\mathbf{B}$ are the modulation fields. Linearising the Vlasov eq. (2) to the orders of μ, $\mu\epsilon$ and $\mu\epsilon^2$ we obtain

$$\mathbf{P}\delta f_h = e\left(\delta\mathbf{E}_h + \upsilon \times \mathbf{B}_h\right) \cdot \frac{\partial}{\partial \mathbf{p}} f_{0\mathrm{e}}, \quad (4)$$

$$\begin{aligned}\mathbf{P}\delta f_{lh} = {} & e\mathbf{E}_l \cdot \frac{\partial}{\partial \mathbf{p}}\delta f_h + e\left(\delta\mathbf{E}_{lh} + \upsilon \times \mathbf{B}_{lh}\right) \cdot \frac{\partial}{\partial \mathbf{p}} f_{0\mathrm{e}} \\ & + e\left(\delta\mathbf{E}_h + \upsilon \times \mathbf{B}_h\right) \cdot \frac{\partial}{\partial \mathbf{p}} f_{1\mathrm{e}},\end{aligned} \quad (5)$$

$$\mathbf{P}\Delta f = e\left\langle \mathbf{E}_l \cdot \frac{\partial}{\partial \mathbf{p}}\delta f_{lh} \right\rangle + e\left\langle \left(\delta\mathbf{E}_{lh} + \upsilon \times \mathbf{B}_{lh}\right) \cdot \frac{\partial}{\partial \mathbf{p}} f_{1\mathrm{e}} \right\rangle, \quad (6)$$

where $\langle\cdots\rangle$ means the averaging over the low frequency fluctuations and the operator $\mathbf{P}$ is given by $\mathbf{P} = \partial/\partial t + \upsilon \cdot \partial/\partial\mathbf{r} - e(\upsilon \times \mathbf{B}_0) \cdot \partial/\partial p$. The high frequency, perturbed distribution function δf_h can now be split into two parts $\delta f_h = \delta f_{h+}\mathrm{e}^{-i\theta} + \delta f_{h-}\mathrm{e}^{i\theta}$ [16], where δf_{h+} and δf_{h-} are the perturbed distribution functions corresponding to the left- (L) and right-handed (R) circularly polarised waves, respectively. The high frequency electric fields δE_{h+} and δE_{h-} associated with left- and right-handed circularly polarised waves are represented as $\delta E_{h+} = (\delta E_{hx} + i\delta E_{hy})/2$ and $\delta E_{h-} = (\delta E_{hx} - i\delta E_{hy})/2$. Using eqs (4)–(6) the R-wave components of δf_h, δf_{lh} and Δf are obtained as

$$\begin{aligned}\delta f_{h-}(\mathbf{K}, \Omega) = {} & ie\pounds^{-1}(\mathbf{K}, \Omega)\delta E_{h-}(\mathbf{K}, \Omega) \\ & \times \left\{ \frac{\partial}{\partial p_\perp} + \frac{K}{m\Omega}\left(p_\perp \frac{\partial}{\partial p_z} - p_z \frac{\partial}{\partial p_\perp}\right) \right\} f_{0\mathrm{e}},\end{aligned} \quad (7)$$

$$\begin{aligned}& \delta f_{lh-}(\mathbf{K}, \Omega) \\ & = ie\pounds^{-1}(\mathbf{K}, \Omega)\left[\sum_{k', \omega'} \left\langle E_{lz}(\mathbf{k}', \omega') \frac{\partial}{\partial p_z} f_{h-}(\mathbf{K} - \mathbf{k}') \right.\right.\end{aligned}$$

$$+\delta E_{h-}(\mathbf{K}-\mathbf{k}')\times\left\{\frac{\partial}{\partial p_\perp}+\frac{K-k'}{m(\Omega-\omega')}\left(p_\perp\frac{\partial}{\partial p_z}-p_z\frac{\partial}{\partial p_\perp}\right)\right\}$$
$$\left.\times f_{1\mathrm{e}}(k')\right\rangle+\delta E_{lh-}(\mathbf{K})\left\{\frac{\partial}{\partial p_\perp}+\frac{K}{m\Omega}\left(p_\perp\frac{\partial}{\partial p_z}-p_z\frac{\partial}{\partial p_\perp}\right)\right\}f_{0\mathrm{e}}\Bigg] \quad (8)$$

$$\Delta f_-(\mathbf{K},\Omega)=ie\pounds^{-1}(\mathbf{K},\Omega)\Bigg[\sum_{k',\omega'}\left\langle E_{lz}(k',\omega')\frac{\partial}{\partial p_z}f_{lh-}(\mathbf{K}-\mathbf{k}')+\delta E_{lh-}(\mathbf{K}-\mathbf{k}')\right.$$
$$\left.\times\left\{\frac{\partial}{\partial p_\perp}+\frac{K-k'}{m(\Omega-\omega')}\left(p_\perp\frac{\partial}{\partial p_z}-p_z\frac{\partial}{\partial p_\perp}\right)\right\}f_{1\mathrm{e}}(k')\right\rangle$$
$$+\sum_{k'',\omega''}\left\langle\delta E_{h-}(\mathbf{K}-\mathbf{k}'')\times\left\{\frac{\partial}{\partial p_\perp}+\frac{K-k''}{m(\Omega-\omega'')}\left(p_\perp\frac{\partial}{\partial p_z}-p_z\frac{\partial}{\partial p_\perp}\right)\right\}\right.$$
$$\left.\times f_{2\mathrm{e}}(k'')\right\rangle+\mathbf{\Delta E}_{h-}(\mathbf{K})\left\{\frac{\partial}{\partial p_\perp}+\frac{K}{m\Omega}\left(p_\perp\frac{\partial}{\partial p_z}-p_z\frac{\partial}{\partial p_\perp}\right)\right\}f_{0\mathrm{e}}\Bigg], \quad (9)$$

where $\pounds(\mathbf{K},\Omega)=\Omega-\frac{K_{||}p_z}{m}-\frac{\Omega_\mathrm{e}}{\Gamma}$, $\Omega_\mathrm{e}=\frac{eB_0}{m_0}$ is the nonrelativistic electron cyclotron frequency and Ω and K are the frequency and wave number of the high frequency wave.

The electric fields for the whistler and the mixed modes are obtained by means of Maxwell's equations. In the Fourier space, we have

$$\delta E_{h-}(\mathbf{K})=\frac{\Omega^2}{C^2K^2}\delta E_{h-}(\mathbf{K})-\frac{i\omega_{pe}^2\Omega}{2\Gamma eC^2K^2}\int p_\perp\delta f_{h-}(\mathbf{K})\mathrm{d}\mathbf{p} \quad (10)$$

$$\delta E_{lh-}(\mathbf{K})=\frac{\Omega^2}{C^2K^2}\delta E_{lh-}(\mathbf{K})-\frac{i\omega_{pe}^2\Omega}{2\Gamma eC^2K^2}\int p_\perp\delta f_{lh-}(\mathbf{K})\mathrm{d}\mathbf{p}. \quad (11)$$

Using eqs (8) and (11), we get

$$\delta E_{lh-}(\mathbf{K})=-\frac{i\Omega\omega_{pe}^2}{\Gamma eC^2R(K)}\int p_\perp\pounds^{-1}(K)\Bigg[\frac{2e^2p_\perp}{a_\perp^2}\sum_{k',\omega'}\left\langle E_{lz}(\mathbf{k}',\omega')\frac{\partial}{\partial p_z}\right.$$
$$\left.\times\left[\pounds^{-1}(K-k')\left\{1-\frac{K-k'}{m(\Omega-\omega')}(ap_0-Ap_z)\right\}\delta E_{h-}(\mathbf{K}-\mathbf{k}')f_{0\mathrm{e}}\right]\right\rangle$$
$$-\frac{2iep_\perp}{a_\perp^2}\sum_{k',\omega'}\left\langle\left\{1-\frac{K-k'}{m(\Omega-\omega')}(ap_0-Ap_z+b(k'))\right\}\right.$$
$$\left.\times\delta E_{h-}(\mathbf{K}-\mathbf{k}')f_{1\mathrm{e}}(k')\right\rangle\Bigg]\mathrm{d}\mathbf{p}, \quad (12)$$

where

$$R(\mathbf{K}) = K^2 - \frac{\Omega^2}{C^2} + \frac{\omega_{pe}^2 \Omega}{\Gamma C^2 a_\perp^2} \int p_\perp^2 \pounds^{-1}(K) \times \left\{1 - \frac{K}{m\Omega}(ap_0 - Ap_z)\right\} f_{0\mathrm{e}} \mathrm{d}\mathbf{p}, \tag{13}$$

$$\begin{aligned}\delta E_{lh-}(\mathbf{K}-\mathbf{k}') = &-\frac{i(\Omega-\omega')\omega_{pe}^2}{2\Gamma e C^2 R(\mathbf{K}-\mathbf{k}')} \\ &\times \int p_\perp \Bigg[\Bigg\langle ie\pounds^{-1}(K-k')\Bigg[E_{lz}(-\mathbf{k}',-\omega')\frac{\partial}{\partial p_z} \\ &\times \Bigg\{-\frac{2iep_\perp}{a_\perp^2}\pounds^{-1}(K,\Omega)\left(1-\frac{K}{m\Omega}(ap_0-Ap_z)\right) f_{0\mathrm{e}}\Bigg\} - \frac{2p_\perp}{a_\perp^2} \\ &\times \Bigg\{1-\frac{K}{m\Omega}(ap_0-Ap_z+b(k'))\Bigg\} f_{1\mathrm{e}}(-k',-\omega')\Bigg\rangle\Bigg]\delta E_{h-}(\mathbf{K})\Bigg]\mathrm{d}\mathbf{p}\end{aligned} \tag{14}$$

where

$$R(\mathbf{K}-\mathbf{k}') = (K-k')^2 - \frac{(\Omega-\omega')^2}{C^2} + \frac{\omega_{pe}^2(\Omega-\omega')}{\Gamma C^2 a_\perp^2}\int p_\perp^2 \pounds^{-1}(K-k') \times \left\{1 - \frac{K-k'}{m(\Omega-\omega')}(ap_0 - Ap_z)\right\} f_{0\mathrm{e}}\mathrm{d}p, \tag{15}$$

where $a = a_\perp^2/a_{||}^2$, $b(k') = a_\perp^2 k'_{||}/2(m\omega' - k'_{||}p_z)$ and $b(k'') = a_\perp^2 k''_{||}/2(m\omega'' - k''_{||}p_z)$. In eq. (9) we have neglected the $\mu\epsilon^2$ order perturbation field quantities. Here the modulation electric field and magnetic field involving terms $\mu\epsilon^2\mathbf{\Delta E}$ and $\mu\epsilon^2\mathbf{\Delta B}$ are negligible, since the ensembled averaged $\mu\epsilon^2$ fields gives small nonlinear frequency shift in the final analysis [20]. Here $\omega_{pe}^2 = n_0 e^2/m_0\epsilon_0$ is the nonrelativistic electron plasma frequency and $\mathrm{d}\mathbf{p} = 2\boldsymbol{\pi} p_\perp \mathrm{d}p_\perp \mathrm{d}p_z$.

Combining eqs (3), (9), (10) and (15) we obtain the nonlinear dielectric constant of the whistler mode in the presence of ion acoustic wave turbulence as

$$D(\mathbf{K},\Omega) = D_0(\mathbf{K},\Omega) + D_\mathrm{d}(\mathbf{K},\Omega) + D_\mathrm{p}(\mathbf{K},\Omega), \tag{16}$$

where $D_0(\mathbf{K},\Omega)$, $D_\mathrm{d}(\mathbf{K},\Omega)$ and $D_\mathrm{p}(\mathbf{K},\Omega)$ are respectively the linear, direct coupling and polarisation term of the nonlinear dielectric constant emerging from the new maser interaction. They are

$$D_0(\mathbf{K},\Omega) = 1 - \frac{C^2K^2}{\Omega^2} - \frac{\omega_{pe}^2}{\Gamma\Omega a_\perp^2}\int p_\perp^2 \pounds^{-1}(K) \times \left(1 - \frac{K}{m\Omega}(ap_0 - Ap_z)\right) f_{0\mathrm{e}}\mathrm{d}\mathbf{p} \tag{17}$$

$$D_\mathrm{d}(\mathbf{K},\Omega) = \frac{e^2\omega_{pe}^2}{\Omega\Gamma a_\perp^2}\sum_{k',\omega'} |E_{lz}(\mathbf{k}')|^2 \int p_\perp^2 \pounds^{-1}(K)\left[\frac{\partial}{\partial p_z}\left[\pounds^{-1}(K-k')\frac{\partial}{\partial p_z}\right.\right.$$

$$\times\left(1-\frac{K}{m\Omega}\left(ap_0-Ap_z\right)\right)\Big] + \pounds^{-1}(K-k')$$
$$\times\left(1-\frac{K}{m\Omega}\left(ap_0-Ap_z+b(k')\right)\right)\frac{\Gamma}{\frac{k'_{||}p_z}{m_0}-\Gamma\omega'}\frac{\partial}{\partial p_z}\Big] f_{0\mathrm{e}}\mathrm{d}\mathbf{p}, \tag{18}$$

$$D_{\mathrm{p}}(\mathbf{K},\Omega) = -\frac{e^2\omega_{pe}^4(\Omega-\omega')}{a_\perp^4\Gamma^2C^2\Omega}\sum_{k',\omega'}|E_{lz}(\mathbf{k}')|^2[(F+G)\times H], \tag{19}$$

where

$$F = \int\frac{p_\perp^2}{\pounds(K)R(K-k')}\frac{\partial}{\partial p_z}$$
$$\times\left\{\pounds^{-1}(K-k')\left(1-\frac{K-k'}{m(\Omega-\omega')}\left(ap_0-Ap_z\right)\right)\right\} f_{0\mathrm{e}}\mathrm{d}\mathbf{p}$$

$$G = \int\frac{p_\perp^2}{\pounds(K)R(K-k')}\left\{1-\frac{K-k'}{m(\Omega-\omega')}\left(ap_0-Ap_z+b(k')\right)\right\}$$
$$\times\frac{\Gamma}{\Gamma\omega'-\frac{k'_{||}p_z}{m_0}}\frac{\partial}{\partial p_z}f_{0\mathrm{e}}\mathrm{d}\mathbf{p}$$

$$H = \int\frac{p_\perp^2}{\pounds(K-k')}\left[\frac{\partial}{\partial p_z}\left\{\pounds^{-1}(K)\left(1-\frac{K}{m\Omega}\left(ap_0-Ap_z\right)\right)\right\}\right.$$
$$\left.+\left\{1-\frac{K-k'}{m(\Omega-\omega')}(ap_0-Ap_z+b(k'))\right\}\frac{1}{\frac{k'_{||}p_z}{m_0}-\Gamma\omega'}\frac{\partial}{\partial p_z}\right] f_{0\mathrm{e}}\mathrm{d}\mathbf{p}.$$

3. Nonlinear dielectric constant and growth rate

We consider the plasma maser interaction between relativistic electron and ion acoustic turbulence. The conditions for plasma maser are $\omega = k_{||}p_{||}/m$, $\Omega > K_{||}p_{||}/m$. We first estimate the linear part of the dielectric constant of whistler R-mode. This can be expressed as

$$D_0(\mathbf{K},\Omega) = 1-\frac{C^2K_{||}^2}{\Omega^2}+\frac{\omega_{pe}^2}{\Gamma\Omega a_{||}}Z(\zeta)$$
$$\times\left[\frac{m}{K_{||}}\left(1-\frac{K_{||}}{m\Omega}ap_0\right)+\frac{Ap_0}{\Omega}-\frac{a_{||}A}{2\Omega}Z(\zeta)\right], \tag{20}$$

where $Z(\zeta)$ is the usual plasma dispersion function with the argument $\zeta = (\Omega - \frac{p_0K_{||}}{m} - \frac{\Omega_e}{\Gamma})/\frac{a_{||}K_{||}}{m}$. For $|\zeta| \gg 1$, we have $Z(\zeta) \cong -1/\zeta$, and the linear dispersion relation becomes

$$D_0(\mathbf{K},\Omega)=1-\frac{C^2K_{||}^2}{\Omega^2}-\frac{K_{||}\omega_{pe}^2}{\Gamma m\Omega\left(\Omega-\frac{p_0K_{||}}{m}-\frac{\Omega_e}{\Gamma}\right)}\left[\frac{m}{K_{||}}\left(1-\frac{K_{||}}{m\Omega}ap_0\right)\right.$$
$$\left.+\frac{Ap_0}{\Omega}+\frac{a_{||}^2AK_{||}}{2m\Omega\left(\Omega-\frac{p_0K_{||}}{m}-\frac{\Omega_e}{\Gamma}\right)}\right]. \tag{21}$$

We note that large argument expansion of the plasma dispersion function is equivalent to the consideration of waves whose phase velocity is much larger than the electron thermal velocity.

For $a=1$, $A=0$, i.e., $T_{||}=T_{\perp}=T_{\mathrm{e}}$, we find

$$D_0(\mathbf{K},\Omega)=1-\frac{C^2K^2}{\Omega^2}-\frac{\omega_{pe}^2}{\Gamma\Omega\left(\Omega-\frac{p_0K_{||}}{m}-\frac{\Omega_e}{\Gamma}\right)}\left(1-\frac{K_{||}}{m\Omega}p_0\right). \tag{22}$$

Hence the relativistic linear dispersion relation of whistler mode in isotropic plasma is

$$C^2K^2=\Omega^2-\frac{\Omega\omega_{pe}^2}{\Gamma\left(\Omega-\frac{p_0K_{||}}{m}-\frac{\Omega_e}{\Gamma}\right)}\left(1-\frac{K_{||}}{m\Omega}p_0\right). \tag{23}$$

For $p_0=0$ and $\omega_{pe}^2\gg\Omega_{\mathrm{e}}^2,\ \Omega^2$ the relativistic linear dispersion relation of whistler mode becomes $\Omega=C^2K_{||}^2\Omega_{\mathrm{e}}/[\Gamma(C^2K_{||}^2+\omega_{pe}^2/\Gamma)]$. The nonlinear contribution to the dielectric constant are given by eqs (18) and (19). For $\frac{p_0K_{||}}{m}\ll|(\Omega-\frac{\Omega_{\mathrm{e}}}{\Gamma})|$ we find

$$D_{\mathrm{d}}(\mathbf{K},\Omega)=-2i\sqrt{\pi}\frac{\omega_{pe}^2}{\Gamma\Omega\left(\Omega-\frac{\Omega_{\mathrm{e}}}{\Gamma}\right)^3}e^2\sum_{k',\omega'}|E_{lz}(k')|^2\left(\frac{K_{||}}{k'_{||}}\right)\frac{p_0}{a_{||}^3}$$
$$\times\left[1-\frac{K_{||}}{m\Omega}(ap_0+b(k'))-\frac{K_{||}A\omega'}{k'_{||}\Omega}\right]\mathrm{e}^{-\zeta^2}, \tag{24}$$

$$D_{\mathrm{p}}(\mathbf{K},\Omega)=-\frac{4i\sqrt{\pi}e^2\omega_{pe}^2m_0^2p_0}{a_{||}a_{\perp}^4C^2}\left(\frac{a_{\perp}}{a_{||}}\right)^4\sum_{k',\omega'}|E_{lz}(k')|^2\frac{(\Omega-\omega')}{\Omega\left(\Omega-\frac{\Omega_{\mathrm{e}}}{\Gamma}\right)^2}$$
$$\times\frac{1}{R\,(K-k')\,(k')^2}\left[1-\frac{K_{||}}{m\Omega}(ap_0+b(k'))-\frac{K_{||}A\omega'}{k'_{||}\Omega}\right]$$
$$\times\left[1-\frac{K-k'}{m(\Omega-\omega')}(ap_0+b(k'))-\frac{(K_{||}-k')A\omega'}{k'_{||}(\Omega-\omega')}\right]\mathrm{e}^{-\zeta^2}. \tag{25}$$

The growth rate γ of the whistler mode is given by $\gamma(\Omega,K)=-\mathrm{Im}\,D(\Omega,K)/[(\partial/\partial\Omega)\mathrm{Re}D_0(\Omega,K)]$, where Re and Im denote the real and imaginary parts of the respective dielectric constant of the whistler mode. From eq. (22)

we get $(\partial/\partial\Omega)\mathrm{Re}D_0(\Omega, K) \cong -(\omega_{pe}^2/\Omega_\mathrm{e}\Omega^2)$. Hence the growth rates due to the direct coupling term and the polarisation term are respectively

$$\gamma_\mathrm{d} = -2\sqrt{\pi}\frac{\Omega\Omega_\mathrm{e}}{\Gamma(\Omega - \frac{\Omega_\mathrm{e}}{\Gamma})^3} e^2 \sum_{k',\omega'} |E_{lz}(\mathbf{k}')|^2 \left(\frac{K_{||}}{k'_{||}}\right) \frac{p_0}{a_{||}^3} \times \left[1 - \frac{K_{||}}{m\Omega}(ap_0 + b(k')) - \frac{K_{||}A\omega'}{k'_{||}\Omega}\right] \mathrm{e}^{-\zeta^2}, \tag{26}$$

$$\gamma_\mathrm{p} = -4\sqrt{\pi}\frac{m^2 p_0}{\Gamma^2 a_{||}^5 C^2} e^2 \sum_{k',\omega'} |E_{lz}(k')|^2 \frac{\Omega\Omega_\mathrm{e}\,(\Omega - \omega')\,\omega_{pe}^2}{\left(\Omega - \frac{\Omega_\mathrm{e}}{\Gamma}\right)^2 R(\mathbf{K} - \mathbf{k}')} \frac{1}{(k'_{||})^2} \times \left[1 - \frac{K_{||}}{m\Omega}(ap_0 + b(k')) - \frac{K_{||}A\omega'}{k'_{||}\Omega}\right] \times \left[1 - \frac{K - k'}{m(\Omega - \omega')}(ap_0 + b(k')) - \frac{(K_{||} - k')A\omega'}{k'_{||}(\Omega - \omega')}\right] \mathrm{e}^{-\zeta^2}. \tag{27}$$

4. Result and discussion

In this paper, we have studied the relativistic plasma maser interaction of the high frequency electromagnetic right-handed circularly polarised whistler mode (R-mode) in the presence of low frequency ion-acoustic turbulence. A general formula has been derived for the growth rate of electromagnetic whistler R-mode. The electrons are modeled by relativistic electron beam distribution exhibiting temperature anisotropy. As an illustration, we apply the result of our investigation to the Earth's radiation belt. Accordingly, we take the typical parameters, viz., $p_0 \sim 1$ MeV, $T_{||} \sim 1$ MeV, $\omega_{pe}/\Omega_\mathrm{e} \sim 20$, $K_{||} \sim 2\pi \times 10^{-4}$ m^{-1} $\sim k_{||}$, $\Omega_\mathrm{e} \sim 2\pi C/10\lambda$, $\Omega \sim 3.2$ kHz (eq. (27)), $\Gamma \sim 2$, $A \sim 2$, $\omega \sim 210$ Hz. For these parameters the growth rates due to the direct coupling term and the polarisation term are found as $\gamma_\mathrm{d} \sim 5.6 \times 10^{-2}$ and $\gamma_\mathrm{p} \sim 1.52 \times 10^{-2}$ respectively.

It may be noted that the temperature anisotropy induced by electrostatic (ES) turbulence is the origin of the angular momentum source for the electromagnetic (EM) wave generation by plasma maser process. The angular momentum source of EM radiation exists in the temperature anisotropy of space induced by ES turbulence. Therefore, the mechanism of plasma maser instability for EM radiation is quite similar to that of Stark effect [21] induced by ES field. In other words, both the plasma maser instability and Stark effect come from the broken symmetry of space.

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