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Remarks on Hawking radiation as tunneling from a uniformly accelerating black hole

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Abstract. Motivated by the Hamilton–Jacobi method of Angheben *et al*, we investigate the Hawking tunneling radiation from a uniformly accelerating rectilinear black hole for which the horizons and entropy are functions of θ. After several coordinate transformations, we conclude that when the self-gravitational interaction and energy conservation are taken into account, the actual radiation spectrum deviates from the thermal one and the tunneling rate is the function of θ though it is still related to the change of the Bekenstein–Hawking entropy.

Keywords. Hamilton–Jacobi method; energy conservation; Bekenstein–Hawking entropy; tunneling rate; spatial distance.

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1. Introduction

Since Stephen Hawking, calculating the Bogoliubov transformation coefficients between the initial and final vacuum states of the emitted particles in the collapse geometry of the formation of black hole, proved that black holes have true temperature and that it radiates thermally [1,2], many physicists have devoted themselves to studying Hawking radiation and its corresponding temperature [3–7]. However, most of their works relied on the quantum theory on a fixed background. Recently, Parikh and Wilczek proposed to visualize the Hawking radiation as a tunneling event [8–10]. According to their observation, when a pair of virtual particles is created spontaneously near the periphery of the black hole horizon, the negative energy particle will be absorbed by the negative energy orbit of the black hole while the positive energy virtual particle tunnels out of the horizon and turns into true particle. Due to the back-reaction of emitted particles, the mass of the black hole will decrease and the radius of the horizon, which becomes a function of the missing mass, will shrink correspondingly. Based on this scenario, they assumed that the tunneling potential barrier is created by the outgoing particle itself. Recently, many extensions about this have been done [11–18]. However, for this method we must make Painlevé coordinate transformation and solve the null geodesic equation. Inspired, Angheben *et al* [19], based on the work of Srinivasan and Padmanabhan [20], put forward a more refined paradigm to depict the tunneling event soon after. The key trick of this method is also to find out the imaginary part of the emitted particle's action. Notably, it involves consideration of the action of the radiated quantum satisfies the relativistic Hamilton–Jacobi equation. According to their arguments, one can avoid performing Painlevé coordinate transformation and exploring the geodesic equation. However, they ignored the self-gravitational interaction and back-reaction of the emitted particle which leads to the assumption that the derived radiation spectrum is not an exact one but the leader term. Fortunately, Medved and Vagenas [21] soon after elaborated this method by considering these effects. Their result agrees with the initial viewpoint of Parikh and Wliczek.

The main aim of our present work is to investigate the Hawking tunneling radiation from a uniformly accelerating rectilinear black hole by the modified the Hamilton–Jacobi method. Due to the horizon of this hole is the function of θ and the metric does not take the form as the general stationary space-time, its Hawking tunneling radiation has not been done till now. In this paper, to overcome these difficulties, we make a series of coordinate transformations, which reduce the four-dimensional stationary space-time to a three-dimensional hyper-surface where the event horizon and infinite red-shift surface are completely coincident with each other. Eventually, by the Hamilton–Jacobi method, the tunneling rate which depends on θ is expressed explicitly.

Our paper is arranged as follows. In the next section, we will introduce two available coordinate transformations to impel the event horizon to be coincident with the infinite red-shift surface of the uniformly accelerating rectilinear black hole. Then in §3, we study its tunneling rate by the Hamilton–Jacobi method. Finally, a brief discussion, particularly about contact between black hole horizon and Rindler horizon, is given in §4.

2. Available coordinate transformations

The line elements of the uniformly accelerating rectilinear black hole in the advanced Eddington–Finkelstein coordinate can be written as [22]

$$\begin{aligned} \mathrm{d}s^2 = &-(\Xi - r^2 a^2 \sin\theta^2)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r \\ &-2a\sin\theta r^2 \mathrm{d}v\mathrm{d}\theta + r^2\mathrm{d}\theta^2 + r^2\sin^2\mathrm{d}\varphi^2, \end{aligned} \tag{1}$$

where a is the value of uniform acceleration and

$$\Xi = 1 - 2ar\cos\theta - 2mr^{-1}, \quad f = a\sin\theta. \tag{2}$$

This metric is inconvenient for us to investigate the tunneling process, and so first we do the following coordinate transformation:

$$\mathrm{d}v = \mathrm{d}t_{UA} + \frac{1}{\Xi}\mathrm{d}r, \quad \mathrm{d}r = \mathrm{d}r, \quad \mathrm{d}\theta = \mathrm{d}\theta - \frac{A}{\Xi}\mathrm{d}r, \quad \mathrm{d}\varphi = \mathrm{d}\varphi. \tag{3}$$

Then the line element (1) is simplified as

$$\begin{aligned} \mathrm{d}s^2 = &-(\Xi - r^2a^2\sin\theta^2)\mathrm{d}t_{UA}^2 + \frac{1}{\Xi}\mathrm{d}r^2 \\ &-2a\sin\theta r^2\mathrm{d}t\mathrm{d}\theta + r^2\mathrm{d}\theta^2 + r^2\sin^2\mathrm{d}\varphi^2. \end{aligned} \tag{4}$$

Equation (4) is the general form of stationary space-time, and the merit of this line element will be explained in our next discussion. In addition, the null super-surface equation $g^{\mu\nu}\frac{\partial f}{\partial x^\mu}\frac{\partial f}{x^\nu} = 0$ tell us the horizon equation as

$$\Xi = 1 - 2ar\cos\theta - 2mr^{-1} = 0. \tag{5}$$

Solving eq. (5), we obtain the inner horizon and outer horizon, respectively as

$$r_\pm = \frac{1 \pm \sqrt{1 - 16am\cos\theta}}{4a\cos\theta}. \tag{6}$$

The determinant of this metric and the area of the event horizon are

$$g = -r^4\sin^2\theta, \quad A = 4\pi r_+^2. \tag{7}$$

Moreover, from eq. (4), we obtain the infinite red-shift surface equation

$$\Xi - r^2a^2\sin\theta^2 = 0. \tag{8}$$

Obviously, the event horizon and the infinite red-shift surface are not coincident with each other, and so the s-wave does not satisfy the infinite blue condition and geometrical optics limit is not reliable here. Furthermore, because of the rotation, materials in the ergosphere near the event horizon may be dragged simultaneously. These are adverse for us to investigate the tunneling event. So for metric (4), we make the following coordinate transformation:

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{g_{02}}{g_{22}} = f, \tag{9}$$

which yields

$$\begin{aligned} \mathrm{d}s^2 &= -\Xi\mathrm{d}t_{UA}^2 + \frac{1}{\Xi}\mathrm{d}r^2 + r^2\sin^2\mathrm{d}\varphi^2 \\ &= \hat{g}_{00}\mathrm{d}t_{UA}^2 + \hat{g}_{11}\mathrm{d}r^2 + \hat{g}_{22}\mathrm{d}\varphi^2. \end{aligned} \tag{10}$$

Equation (10) stands for a three-dimensional hyper-surface in the four-dimensional space-time. Here, we learn that the event horizon is coincident with the infinite red-shift surface completely, which means the geometrical optics limit is reliable and the WKB approximation can be adopted thereafter.

3. Hamilton–Jacobi method

Now, we concentrate on discussing the Hawking tunneling radiation from the uniformly accelerating rectilinear black hole by making use of the Hamilton–Jacobi equation. Near the event horizon, the line element (10) can be expressed as

$$\mathrm{d}s^2 = -\frac{\Delta_{,r}(r_h)(r-r_h)}{r_h}\mathrm{d}t^2 + \frac{r_h}{\Delta_{,r}(r_h)(r-r_h)}\mathrm{d}r^2 + r_h^2 \sin^2\theta \mathrm{d}\theta^2, \quad (11)$$

where

$$\Delta_{,r}(r_h) = \left.\frac{\partial \Delta}{\partial r}\right|_{r=r_h} = 1 - 4ra\cos\theta. \quad (12)$$

We assume the action of the outgoing particle is given by the classical action I that satisfies the relativistic Hamilton–Jacobi equation

$$g^{\mu\nu}\partial_\mu I \partial_\nu I + u^2 = 0, \quad (13)$$

where u and $g^{\mu\nu}$ are the mass of the emitted particles and the inverse metric tensor derived from the metric (10) respectively. As a matter of fact, the Hamilton–Jacobi equation has been a very useful equation in black hole physics since Carter's works [23,24]. It played a prominent role in clarifying many important local as well as global properties of black hole [25,26]. Incorporating eq. (11) with eq. (13) yields

$$-\frac{r_h}{\Delta_{,r}(r_h)(r-r_h)}(\partial_t I)^2 + \frac{\Delta_{,r}(r_h)(r-r_h)}{r_h}(\partial_r I)^2 + \frac{1}{r_h^2 \sin^2\theta}(\partial_\varphi I)^2 + u^2 = 0. \quad (14)$$

Solving the action from eq. (14) directly is not easy. However, taking into account the time-like killing vector $(\partial/\partial t)^a$ and the space-like killing vector $(\partial/\partial\varphi)^a$, we carry out the following separation variable:

$$I = -\omega t + j\varphi + W(r,\theta). \quad (15)$$

Then we find

$$W(r,\theta) = \frac{r_h}{\Delta_{,r}(r_h)}\int \frac{\mathrm{d}r}{r-r_h} \times \sqrt{(\omega - j\Omega_h)^2 - \frac{\Delta_{,r}(r_h)(r-r_h)}{r_h}\left[\frac{[\partial_\varphi W(r,\theta)]^2}{r_h^2\sin^2\theta} + u^2\right]}. \quad (16)$$

The imaginary part of the action can only come from the pole at the horizon. It is important to introduce proper spatial distance to make sure that the result is correct. The proper spatial distance between any two points at the fixed time t is defined as

$$\mathrm{d}\sigma^2 = \frac{\rho^2(r_h)}{\Delta_{,r}(r_h)(r-r_h)}\mathrm{d}r^2 + \rho^2(r_h)\mathrm{d}\theta^2. \quad (17)$$

After the consideration of the radial motion of the emission, we have

$$\sigma = 2\sqrt{\frac{\rho^2(r_h)(r-r_h)}{\Delta_{,r}(r_h)}}. \quad (18)$$

Furthermore, in view of the non-rotation of the uniformly accelerating rectilinear space-time, angular momentum j must be zero. Hence, the expression of eq. (16) can be further reduced as

$$W(\sigma) = \frac{2r_h}{\Delta_{,r}(r_h)} \int \frac{\mathrm{d}\sigma}{\sigma} \sqrt{\omega^2 - \frac{\Delta_{,r}^2(r_h)\sigma^2}{4(r_h^2+a^2)^2}\left[\frac{[\partial_\theta W(r,\theta)]^2}{\rho^2(r_h)} + u^2\right]}. \quad (19)$$

To proceed further we need to estimate the last integral. Note that there is a pole at the horizon where $\sigma = 0$. Deforming the integration contour from the real σ-axis to the lower complex σ-plane, and inserting the result from eq. (18), the imaginary part of the radiated particle's action, ignoring the real part, can be written as

$$\mathrm{Im}\, I = \frac{2\pi r_h \omega}{\Delta_{,r}(r_h)}, \quad (20)$$

and the corresponding Hawking temperature takes the form as

$$T_{\mathrm{BH}} = \frac{\Delta_{,r}(r_h)}{4\pi r_h}. \quad (21)$$

Here, we find that the derived radiation spectrum is purely thermal. However, the real radiation is not like this. The origin of this is the ignorance of the self-gravitational interaction and back reaction. Therefore, to precisely picture the Hawking radiation, we have to take these effects into account.

Fixing the ADM mass of the total background space-time and allowing them to fluctuate, when a particle with a shell of energy ω tunnels out, the mass of the black hole would reduce to $m-\omega$ accordingly. Thus the radius of the horizon will shrink and become the function of $m-\omega$. So the imaginary part of the actual action is

$$\mathrm{Im}\, I = \int_0^\omega \frac{2\pi r_h' \mathrm{d}\omega'}{\Delta_{,r}'(r_h')}, \quad (22)$$

where

$$\Delta_{,r}'(r_h') = 1 - 4a\cos\theta r_h', \quad r_h' = \frac{1 \pm \sqrt{1-16a(m-\omega)\cos\theta}}{4a\cos\theta}. \quad (23)$$

Finishing the integral yields

$$\begin{aligned}\mathrm{Im}\, I &= -\frac{\pi}{2a\cos\theta}\int_m^{m-\omega}\left(1+\frac{1}{\sqrt{1-16a(m-\omega)\cos\theta}}\right)\mathrm{d}(1-16a(m-\omega)\cos\theta) \\ &= -\pi\left[\frac{\begin{array}{c}-16a(m-\omega)\cos\theta + 2\sqrt{1-16a(m-\omega)\cos\theta} \\ +16am\cos\theta - 2\sqrt{1-16am\cos\theta}\end{array}}{32a^2\cos^2\theta}\right] \\ &= -\frac{1}{2}[S_{\mathrm{BH}}(m-\omega) - S_{\mathrm{BH}}(m)]. \quad (24)\end{aligned}$$

We find

$$\Gamma \sim \mathrm{e}^{-2\mathrm{Im}I} = \mathrm{e}^{\Delta S_{\mathrm{BH}}}, \tag{25}$$

where $\Delta S_{\mathrm{BH}} = S_{\mathrm{BH}}(m-\omega) - S_{\mathrm{BH}}(m)$ is the change of Bekenstein–Hawking entropy. Equation (25) tells us that the tunneling probability from the uniformly accelerating rectilinear black hole, which is a function of θ, is related to the change of Bekenstein–Hawking entropy and the actual radiation spectrum is no longer purely thermal but has some corrections.

4. Discussion

As we mentioned, the horizon and the entropy of the uniformly accelerating rectilinear black hole is a function of θ. When $\theta = \pi/2$, from eq. (5) we find the horizon reduces to the Schwarzschild radius $r = 2m$. Then employing eq. (24) we also get the tunneling probability of the Schwarzschild metric as

$$\Gamma \sim \mathrm{e}^{-2\mathrm{Im}S} = \mathrm{e}^{-8\pi\omega(m-\frac{\omega}{2})}, \tag{26}$$

which is consistent with the initial work of Parikh and Wliczek that satisfy Bohr's correspondence principle. Moreover while $ma \ll 1$, the inner horizon and outer horizon in eq. (6) respectively reduces to, $r_1 \sim 2m$ and $r_2 \sim (2a\cos\theta)^{-1}$. Obviously, r_1 is the event horizon and r_2 is the Rindler horizon [27]. When the two horizons contact with each other, due to the symmetry of background space-time, the contact points can only take place at $\theta = 0$ and $\theta = \pi$. For $\theta = 0$, employing eq. (6), we get,

$$r = r_1 = \frac{1+\sqrt{1-16am}}{4a} = r_2 = \frac{1-\sqrt{1-16am}}{4a}. \tag{27}$$

That is

$$r = \frac{1}{4a}, \quad a = \frac{m}{r^2}, \tag{28}$$

while $16am = 1$. Inserting eq. (28) into eq. (21), we find that the Hawking temperature is approximately zero, which will breach the third law of thermodynamics. On the other hand, at the other pole point $(\theta = \pi, r = r_2)$, the Hawking temperature

$$T = -\frac{1}{2\pi}\left(a\cos\theta - \frac{m}{r^2} + rf^2\right) \tag{29}$$

will increase. This phenomenon can be regarded as the simulation of the collision of two black holes. That is to say, when two black holes collide with each other, the temperature of the contact point will drop to zero while the temperature of the tail of two black holes will increase accordingly.

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