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Berry phases in the three-level atoms driven by quantized light fields

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Abstract. A theoretical analysis of Berry's phases is given for the three-level atoms interacting with external one-mode and two-mode quantized light fields. Three main results are obtained: (i) There is a Berry phase which vanishes in the classical limit or this Berry phase is completely induced by the field quantization; (ii) Berry's phases for the one-mode field and the two-mode field can be equal so long as the photon numbers of the two-mode field are properly chosen; (iii) In the two-mode case, Berry phases of the atom interacting with one mode are affected by the other mode even if the photon number of the other mode is zero.

Keywords. Berry phase; quantized light field; three-level atom.

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1. Introduction

When a quantum state evolves slowly and eventually brought back to its initial form, the state acquires a quantum adiabatic geometrical phase, now known as the Berry phase, as pointed out by Berry in 1984 [1]. Writing the quantum state as $|n, \vec{R}(t)\rangle$ with n being the quantum number and $\vec{R}(t) \equiv (R_1(t), R_2(t), \ldots)$ the time-dependent parameters, the Berry phase is

$$\gamma = i \oint \left\langle n, \vec{R}(\tau) \right| \frac{\partial}{\partial \tau} \left| n, \vec{R}(\tau) \right\rangle \cdot \mathrm{d}\tau = i \oint \left\langle n, \vec{R} \right| \frac{\partial}{\partial \vec{R}} \left| n, \vec{R} \right\rangle \cdot \mathrm{d}\vec{R}. \quad (1.1)$$

Later, many studies have been done on this geometrical phase [2–8]. The original adiabatic and periodic evolutions [1] are also generalized to, for instances, non-adiabatic evolution [9], non-adiabatic and non-cyclic evolution [10], mixed states [11–14], open systems [15] and composite systems [16]. For all these investigations, the driving field is classical. The effects of field quantization are studied in [17–19], where two-level atom is addressed. Besides the theoretical interest, the geometrical phase has potential applications, such as the fault tolerant quantum computation [20–23].

In this study, we deal with the Berry phases of three-level atoms interacting with quantized light fields. It is found that the effects of field quantization are richer. For example, besides the vacuum-induced Berry phase, there is a Berry phase that vanishes in the classical limit. The next section is a brief discussion of the method to derive the Berry phase when the field is quantized. The third section deals with the calculation of the exact Berry phase for the Ξ-type atom [24]. The fourth section is about the V-type atom [24]. The final section is the conclusion.

2. Method to derive the Berry phase

In the semiclassical theory, the Hamiltonian for a two-level atom interacting with a time-dependent classical field can be written as [17]

$$H_c = (\Delta/2)\sigma_z + \lambda(\alpha\sigma_+ \mathrm{e}^{-i\phi} + \sigma_-\alpha\mathrm{e}^{i\phi}), \tag{2.1}$$

where ϕ is a time-dependent phase of the classical field and α is the amplitude. Meanings of other quantities are respectively: $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma_+ = |e\rangle\langle g| = (\sigma_x + i\sigma_y)/2, \sigma_- = |g\rangle\langle e| = (\sigma_x - i\sigma_y)/2$ with $|g\rangle$ and $|e\rangle$ being the ground and excited states of the two-level atom, λ is the atom-field coupling coefficient, Δ is given below (2.5). The Hamiltonian (2.1) is effectively written in the form

$$H_c = \vec{B}\cdot\vec{\sigma}, \tag{2.2}$$

where $\vec{B} = (\lambda\alpha\cos\phi, \lambda\alpha\sin\phi, \Delta/2)$ is an effective time-dependent field. To calculate the Berry phase for the present system, the time τ in (1.1) should be replaced by ϕ. The Berry phases are [17]

$$\gamma = \pi(1 \pm \cos\theta), \quad \cos\theta = \Delta/(2\lambda\alpha). \tag{2.3}$$

In ref. [17], a fully quantized version of the Berry phase for the system of a two-level atom interacting with a quantized light field has been given that considers the vacuum-induced effects. Under the rotating wave approximation, the Hamiltonian

$$H = (\omega/2)\sigma_z + \omega_\mathrm{f} a^+ a + \lambda(a\sigma_+ + \sigma_- a^+), \tag{2.4}$$

where a^+ and a are the creation and annihilation operators for the light field. The Hamiltonian (2.4) can be split into two parts, $H = H_0 + H_\mathrm{int}$, where

$$H_0 = (\omega - \Delta)\sigma_z/2 + \omega_\mathrm{f} a^+ a, \quad H_\mathrm{int} = (\Delta/2)\sigma_z + \lambda(a\sigma_+ + \sigma_- a^+), \tag{2.5}$$

where $\Delta = \omega - \omega_\mathrm{f}$ is the detuning between the atom and the field. One can prove that H_0 and H_int communicate with each other. So, in the interaction picture, the interaction Hamiltonian is just H_int. The eigenstate of H_int may be written in the form (the specific forms of the coefficients are not given because they are unimportant for the following discussions)

$$|\psi_n\rangle = C_1|n\rangle|e\rangle + C_2|n+1\rangle|g\rangle. \tag{2.6}$$

In the fully quantized context, the same procedure to generate an analogous phase change in the state of the field is needed. To meet this need, a phase shift operator $U(\phi) = \exp(-i\phi a^+ a)$ is applied adiabatically to the quantum state in ref. [17]. After the phase shift operator is applied, the state (2.6) becomes

$$U(\phi)|\psi_n\rangle = \mathrm{e}^{-in\phi}|\psi_n(\phi)\rangle, \tag{2.7}$$

where

$$|\psi_n(\phi)\rangle = C_1|n\rangle|e\rangle + C_2\mathrm{e}^{-i\phi}|n+1\rangle|g\rangle. \tag{2.8}$$

Using the state (2.7), the Berry phases are [17]

$$\gamma_n = 2\pi n + \pi(1 \pm \cos\,\theta_n), \quad \cos\theta_n = \frac{\Delta}{\sqrt{\Delta^2 + 4\lambda^2(n+1)}}. \tag{2.9}$$

When $n = 0$, the Berry phase is non-zero, which is the vacuum-induced phase. In ref. [17], a second mode of the field is introduced to omit the term $2\pi n$ and to observe the physical effects of the rest part.

To understand the method given in ref. [17], let us have a look at the Hamiltonian corresponding to the eigenstate (2.7), which is

$$H_\phi = UH_{\mathrm{int}}U^+ = (\Delta/2)\sigma_z + \lambda(a\mathrm{e}^{-i\phi}\sigma_+ + \sigma_- a^+\mathrm{e}^{i\phi}). \tag{2.10}$$

The Berry phases in (2.9) correspond to this dynamical system. One may notice that (2.10) and (2.1) have clear quantum-classical correspondence. In the classical limit $n \to \infty$, the operator a or a^+ becomes c-number (α in the present system. If α is assumed to be complex initially, there will be the relations $a \to \alpha$ and $a^+ \to \alpha^*$) and (2.10) returns to (2.1). But the Berry phases in (2.9) cannot go to (2.3) in the classical limit because there is an extra term $2\pi n$ in (2.9). As the eigenstates (2.7) and (2.8) only differ in a total phase factor $\exp(-in\phi)$, they both are the eigenstates of Hamiltonian (2.10). If the state (2.8) is used to calculate the Berry phase, there will be no $2\pi n$ term (of course, with or without the term $2\pi n$, the phases are equivalent). Under this condition, (2.3) is the classical correspondence of (2.9) and meanwhile we have $\alpha = \sqrt{n}$. In the following calculations, we choose to write the eigenstate of H_ϕ in the form (2.8), which agrees with the usual way of writing the wave function. For example, when writing the wave function for the semiclassical Hamiltonian, the phase factor $\exp(-in\phi)$ is usually not multiplied [17]. Under such treatment, terms such as $2\pi n$ vanish in the Berry phases.

3. Berry phases for Ξ-type atom

We assume that the energies satisfy $E_1 > E_2 > E_3$ for the three levels of the atom. For the Ξ-type atom, the allowed dipole transitions are $1 \leftrightarrow 2$ and $2 \leftrightarrow 3$, but not $1 \leftrightarrow 3$. The Hamiltonian of the system is

$$\begin{aligned} H = {} & \omega_{\mathrm{f}} a^+ a + E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2| + E_3|3\rangle\langle 3| \\ & + \lambda(a|1\rangle\langle 2| + a|2\rangle\langle 3| + a^+|2\rangle\langle 1| + a^+|3\rangle\langle 2|) \end{aligned} \tag{3.1}$$

which can be rewritten as

$$\begin{aligned} H &= H_0 + H_{\rm int}, \\ H_0 &= \omega_{\rm f}(a^+a + |1\rangle\langle 1| - |3\rangle\langle 3|) + E_2(|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3|), \\ H_{\rm int} &= \Delta_1|1\rangle\langle 1| - \Delta_2|3\rangle\langle 3| + \lambda(a|1\rangle\langle 2| + a|2\rangle\langle 3| \\ &\quad + a^+|2\rangle\langle 1| + a^+|3\rangle\langle 2|), \end{aligned} \tag{3.2}$$

where $\Delta_1 = E_1 - E_2 - \omega_{\rm f}$ and $\Delta_2 = E_2 - E_3 - \omega_{\rm f}$ are the detunings. By some calculations, one can show that $[H_0, H_{\rm int}] = 0$ or H_0 and $H_{\rm int}$ commute with each other. So, in the interaction picture, the interaction Hamiltonian is

$$\begin{aligned} H_{\rm int}^{\rm (I)} = H_{\rm int} &= \Delta_1|1\rangle\langle 1| - \Delta_2|3\rangle\langle 3| + \lambda(a|1\rangle\langle 2| + a|2\rangle\langle 3| \\ &\quad + a^+|2\rangle\langle 1| + a^+|3\rangle\langle 2|). \end{aligned} \tag{3.3}$$

The Hamiltonian corresponding to (2.9) takes the form

$$\begin{aligned} H_\phi^{\rm (I)} = UH_{\rm int}U^+ &= \Delta_1|1\rangle\langle 1| - \Delta_2|3\rangle\langle 3| + \lambda[a{\rm e}^{i\phi}(|1\rangle\langle 2| + |2\rangle\langle 3|) \\ &\quad + a^+{\rm e}^{-i\phi}(|2\rangle\langle 1| + |3\rangle\langle 2|)]. \end{aligned} \tag{3.4}$$

In the adiabatic case, we have the eigenvalue equation

$$H_\phi^{\rm (I)}|\psi(\phi)\rangle = \varepsilon|\psi(\phi)\rangle, \tag{3.5}$$

where ε is the eigenvalue. Through some calculations, it is found that the eigenvalues satisfy

$$\varepsilon^3 - (\Delta_2 - \Delta_1)\varepsilon^2 - q\varepsilon + p = 0, \tag{3.6}$$

where

$$q = \Delta_1\Delta_2 + \lambda^2(2n+3), \quad p = \lambda^2(n+2)\Delta_1 - \lambda^2(n+1)\Delta_2. \tag{3.7}$$

The exact solution to (2.6) is

$$\begin{aligned} \varepsilon_+ &= \frac{\Delta_2 - \Delta_1}{3} + 2R\cos\left(\frac{\varphi}{3}\right), \\ \varepsilon_0 &= \frac{\Delta_2 - \Delta_1}{3} - 2R\cos\left(\frac{\varphi}{3} + \frac{\pi}{3}\right), \\ \varepsilon_- &= \frac{\Delta_2 - \Delta_1}{3} - 2R\cos\left(\frac{\varphi}{3} - \frac{\pi}{3}\right), \end{aligned} \tag{3.8}$$

where

$$\begin{aligned} R &= \frac{1}{3}\sqrt{3q + (\Delta_2 - \Delta_1)^2} \\ \varphi &= \cos^{-1}\frac{9q(\Delta_2 - \Delta_1) + 2(\Delta_2 - \Delta_1)^3 - 27p}{2[3q + (\Delta_2 - \Delta_1)^2]^{3/2}}. \end{aligned} \tag{3.9}$$

The corresponding eigenfunctions are found to be

$$|\psi_\varepsilon(\phi)\rangle = \frac{1}{\sqrt{N}}\left[\frac{\lambda\sqrt{n+1}}{\varepsilon-\Delta_1}\mathrm{e}^{i\phi}|1,n\rangle + |2,n+1\rangle + \frac{\lambda\sqrt{n+2}}{\Delta_2+\varepsilon}\mathrm{e}^{-i\phi}|3,n+2\rangle\right]$$

$$N = 1 + \frac{\lambda^2(n+1)}{(\Delta_1-\varepsilon)^2} + \frac{\lambda^2(n+2)}{(\Delta_2+\varepsilon)^2}. \tag{3.10}$$

Substituting the eigenvalues (3.8) into (3.10), one will get the eigenfunction for the specific eigenvalue. For the state (3.10), the Berry phase is not difficult to get

$$\gamma_\varepsilon = \frac{2\pi}{N}\left[\frac{\lambda^2(n+2)}{(\Delta_2+\varepsilon)^2} - \frac{\lambda^2(n+1)}{(\Delta_1-\varepsilon)^2}\right]. \tag{3.11}$$

For $n = 0$, the Berry phase is non-zero as in the two-level atom case [17]. Considering a special case $\Delta_1 = \Delta_2 = \Delta$, the Berry phase reduces to

$$\gamma_\varepsilon = \frac{2\pi}{N}\left[\frac{\lambda^2(n+2)}{(\Delta+\varepsilon)^2} - \frac{\lambda^2(n+1)}{(\Delta-\varepsilon)^2}\right]. \tag{3.12}$$

In the classical limit (or large photon number limit), there is the relation $n+2 \to n+1 \to n$. For the state $\varepsilon_0 \to 0$, the Berry phase tends to zero, which can be seen from (3.12) without much difficulty. That is to say, the Berry phase for the state with eigenvalue ε_0 does not exist classically and is purely induced by the field quantization. The other two eigenvalues are now $\varepsilon_+ = -\varepsilon_- = \sqrt{q}$ and the corresponding Berry phases satisfy $\gamma_+ = -\gamma_-$.

Now let us turn to the two-mode case. The Hamiltonian of the system takes the form

$$\begin{aligned} H = {} & \omega_{1\mathrm{f}}a_1^+a_1 + \omega_{2\mathrm{f}}a_2^+a_2 + E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2| + E_3|3\rangle\langle 3| \\ & +\lambda(a_1|1\rangle\langle 2| + a_2|2\rangle\langle 3| + a_1^+|2\rangle\langle 1| + a_2^+|3\rangle\langle 2|) \end{aligned} \tag{3.13}$$

which can be further changed into

$$\begin{aligned} H &= H_0 + H_{\mathrm{int}} \\ H_0 &= \omega_{1\mathrm{f}}(a_1^+a_1 + |1\rangle\langle 1|) + \omega_{2\mathrm{f}}(a_2^+a_2 - |3\rangle\langle 3|) \\ &\quad +E_2(|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3|) \\ H_{\mathrm{int}} &= \Delta_1|1\rangle\langle 1| - \Delta_2|3\rangle\langle 3| + \lambda(a_1|1\rangle\langle 2| + a_2|2\rangle\langle 3| \\ &\quad +a_1^+|2\rangle\langle 1| + a_2^+|3\rangle\langle 2|). \end{aligned} \tag{3.14}$$

Now the detunings are $\Delta_1 = E_1 - E_2 - \omega_{1\mathrm{f}}$ and $\Delta_2 = E_2 - E_3 - \omega_{2\mathrm{f}}$. It can also be shown that $[H_0, H_{\mathrm{int}}] = 0$. So, the interaction Hamiltonian in the interaction picture is

$$\begin{aligned} H_{\mathrm{int}}^{(\mathrm{I})} = H_{\mathrm{int}} = {} & \Delta_1|1\rangle\langle 1| - \Delta_2|3\rangle\langle 3| + \lambda(a_1|1\rangle\langle 2| + a_2|2\rangle\langle 3| \\ & +a_1^+|2\rangle\langle 1| + a_2^+|3\rangle\langle 2|). \end{aligned} \tag{3.15}$$

The Hamiltonian corresponding to (2.9) is

$$H_{\phi}^{(\mathrm{I})} = UH_{\mathrm{int}}U^{+} = \Delta_1|1\rangle\langle 1| - \Delta_2|3\rangle\langle 3| + \lambda \mathrm{e}^{i\phi}(a_1|1\rangle\langle 2| + a_2|2\rangle\langle 3|) + \lambda \mathrm{e}^{-i\phi}(a_1^{+}|2\rangle\langle 1| + a_2^{+}|3\rangle\langle 2|) \tag{3.16}$$

with $U(\phi) = \exp[-i\phi(a_1^{+}a_1 + a_2^{+}a_2)]$. The eigenvalues satisfy (3.6) with

$$q = \Delta_1\Delta_2 + \lambda^2(n_1 + n_2 + 2),\ p = \lambda^2(n_2 + 1)\Delta_1 - \lambda^2(n_1 + 1)\Delta_2. \tag{3.17}$$

The corresponding eigenfunctions are found to be

$$|\psi_{\varepsilon}\rangle = \frac{1}{\sqrt{N}}\left[\frac{\lambda\sqrt{n_1+1}}{\varepsilon - \Delta_1}\mathrm{e}^{in_1\phi}|1, n_1, n_2\rangle + |2, n_1 + 1, n_2\rangle + \frac{\lambda\sqrt{n_2+1}}{\Delta_2 + \varepsilon}\mathrm{e}^{-in_2\phi}|3, n_1 + 1, n_2 + 1\rangle\right]$$
$$N = 1 + \frac{\lambda^2(n_1+1)}{(\Delta_1 - \varepsilon)^2} + \frac{\lambda^2(n_2+1)}{(\Delta_2 + \varepsilon)^2}. \tag{3.18}$$

For the state (3.8), the Berry phase is found to be

$$\gamma = \frac{2\pi}{N}\left[\frac{\lambda^2(n_2+1)}{(\Delta_2 + \varepsilon)^2} - \frac{\lambda^2(n_1+1)}{(\Delta_1 - \varepsilon)^2}\right]. \tag{3.19}$$

By substituting $n_1 \to n$ and $n_2 \to n + 1$, the eigenvalues and the Berry phases become equal to that of the one-mode case. When $n_1 = 0$ and $n_2 = 0$, the Berry phase is

$$\gamma_{\varepsilon} = \frac{2\pi}{N}\left[\frac{\lambda^2}{(\Delta_2 + \varepsilon)^2} - \frac{\lambda^2}{(\Delta_1 - \varepsilon)^2}\right] \tag{3.20}$$

which is the vacuum-induced Berry phase for the three-level atom interacting with a two-mode quantum field. If $n_1 = 0$and $n_2 \neq 0$, the Berry phase will be

$$\gamma_{\varepsilon} = \frac{2\pi}{N}\left[\frac{\lambda^2(n_2+1)}{(\Delta_2 + \varepsilon)^2} - \frac{\lambda^2}{(\Delta_1 - \varepsilon)^2}\right]. \tag{3.21}$$

The detuning Δ_1 also enters the expression, which means the mode a_1 still affects the Berry phase even if its photon number is chosen zero.

4. Berry phases for V-type atom

For the V-type atom, the allowed dipole transitions are $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$, but not $1 \leftrightarrow 2$. The Hamiltonian is then

$$H = \omega_{\mathrm{f}}a^{+}a + E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2| + E_3|3\rangle\langle 3| + \lambda(a|1\rangle\langle 3| + a|2\rangle\langle 3| + a^{+}|3\rangle\langle 1| + a^{+}|3\rangle\langle 2|), \tag{4.1}$$

where $\Delta_1 = E_1 - E_3 - \omega_{\rm f}$ and $\Delta_2 = E_2 - E_3 - \omega_{\rm f}$ are the detunings. Through some computations, we have the eigenfunctions

$$|\psi_\varepsilon(\phi)\rangle = \frac{1}{\sqrt{N}}\left[\frac{\lambda\sqrt{n_1+1}}{\varepsilon-\Delta_1}|1,n\rangle + \frac{\lambda\sqrt{n_2+1}}{\varepsilon-\Delta_2}|2,n\rangle + {\rm e}^{-i\phi}|3,n+1\rangle\right],$$
$$N = 1 + \frac{\lambda^2(n+1)}{(\Delta_1-\varepsilon)^2} + \frac{\lambda^2(n+1)}{(\varepsilon-\Delta_2)^2}, \tag{4.2}$$

where the eigenvalue satisfies

$$\varepsilon^3 - (\Delta_2+\Delta_1)\varepsilon^2 - [-\Delta_1\Delta_2 + 2\lambda^2(n+1)]\varepsilon + \lambda^2(n+1)(\Delta_2+\Delta_1) = 0. \tag{4.3}$$

Starting from (4.2), the Berry phase can be calculated as

$$\gamma_\varepsilon = 2\pi/N. \tag{4.4}$$

Considering a special case $\Delta_1 = -\Delta_2 = \Delta$, the eigenvalue equation (4.3) has the simple solutions

$$\varepsilon_0 = 0,$$
$$\varepsilon_+ = -\varepsilon_- = \sqrt{\Delta^2 + 2\lambda^2(n+1)}. \tag{4.5}$$

For $\varepsilon_0 = 0$ and in the classical limit, the Berry phase is found to be

$$\gamma_0 = 2\pi\frac{\Delta^2}{\Delta^2 + 2\lambda^2\alpha} \tag{4.6}$$

which is non-zero. Such a property is different from that of the Ξ-type atom, where $\gamma_0 = 0$ in the classical limit.

The Berry phase can also be derived for the two-mode case. By substituting $n_1 \to n$ and $n_2 \to n$, the eigenvalues and the Berry phases become equal to that of the one-mode case. For the Λ-type atom, behaviors of the Berry phase are similar to that of the V-type atom.

We may understand different behaviors for different types of atoms from the following point of view. For the Ξ-type atom, by defining the operators

$$S_+ = |1\rangle\langle 2| + |2\rangle\langle 3|$$
$$S_- = |2\rangle\langle 1| + |3\rangle\langle 2|$$
$$S_z = |1\rangle\langle 1| - |3\rangle\langle 3| \tag{4.7}$$

and $S_x = (S_+ + S_-)/2,\ S_y = (S_+ - S_-)/(2i)$, the Hamiltonian (3.3) can be written as

$$H_{\rm int} = S_x\lambda(a^+ + a)/2 + S_y\lambda(a^+ - a)/(2i) + S_z\Delta. \tag{4.8}$$

It is not difficult to show that operators S_x, S_y, S_z obey the commutation relations of the angular momentum operator. The eigenvalues of the third component S_z in (4.7) is 0 and ±1. So, the Hamiltonian (4.8) describes a particle with pseudospin-1 in a quantized field $(\lambda(a^+ + a)/2, \lambda(a^+ - a)/(2i), \Delta)$. However, for the Λ-type or V-type atom, there is no result similar to (4.8).

5. Conclusions

The Berry phase of a three-level atom interacting with quantized light fields is discussed. Though the Ξ-type, Λ-type and V-type atoms are all three-state systems, properties of their Berry phases are different. For the Ξ-type atom, there is a Berry phase that does not exist classically. However, for the Λ-type or V-type atom, there is no such phase. In the classical case, the Berry phase of an arbitrary spin rotating in an external field can be generally written as Ωm [25], where Ω is the solid angle subtended by the classical field and m is the quantum number of the third component of the spin operator. If the field is quantized, there is no such general expression, which can be seen from the results for the Ξ-type atom in this paper and that in [17] for the two-level atom, which is a particle with pseudospin-1/2.

Berry phases for the atom interacting with the two-mode field are also studied. Berry's phases for the one-mode field and the two-mode field can be equal so long as the photon numbers of the two-mode field are properly chosen. In the two-mode case, Berry's phase for the atom interacting with one mode is affected by the other mode even if the photon number of the other mode is zero, which is another vacuum effect.

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