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Effect of field quantization on Rabi oscillation of equidistant cascade four-level system

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Abstract. We have exactly solved a model of equidistant cascade four-level system interacting with a single-mode radiation field both semiclassically and quantum mechanically by exploiting its similarity with Jaynes–Cummings model. For the classical field, it is shown that the Rabi oscillation of the system initially in the first level (second level) is similar to that of the system when it is initially in the fourth level (third level). We then proceed to solve the quantized version of the model where the dressed state is constructed using a six-parameter four-dimensional matrix and show that the symmetry exhibited in the Rabi oscillation of the system for the semiclassical model is completely destroyed on the quantization of the cavity field. Finally, we have studied the collapse and revival of the system for the cavity field-mode in a coherent state to discuss the restoration of symmetry and its implication is discussed.

Keywords. Rabi oscillation; four-level system; collapse and revival.

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1. Introduction

Over the decades, the theory of electron spin resonance (ESR) has been regarded as the key model to understand various fundamental aspects of the semiclassical two-level system [1]. Its fully quantized version, namely, the two-level Jaynes–Cummings model (JCM), has also been proven to be a useful theoretical laboratory to address many subtle issues of the light–atom interaction which eventually gives birth to the cavity electrodynamics [1,2]. A natural but non-trivial extension of the JCM is the three-level system and it exhibits a wide variety of quantum-optical phenomena such as two-photon coherence [3], resonance Raman scattering [4], double resonance process [5], population trapping [6], three-level super radiance [7], three-level echoes [8], STIRAP [9], quantum jump [10], quantum zeno effect [11] etc. As a straightforward generalization of the three-level system, the multi-level system interacting with monochromatic laser is also extensively investigated [12–17]. Thus, it is understood that the increase of the number of level leads to the emergence of a plethora of phenomena and the upsurge of ongoing investigations of the four-level system is undoubtedly to predict more phenomena. For example, out of different configurations of the four-level system, the tripod configuration has come into the purview of recent studies particularly because it exhibits the phenomenon of the electromagnetically induced transparency (EIT) [18–25] which also received experimental confirmation [26–29]. Such a system is proposed to generate the non-Abelian phases [30], qubit rotation [31], coherent quantum switching [32], coherent controlling of nonlinear optical properties [33], embedding two qubits [34] etc. These developments lead to the careful scrutiny of all possible configurations of the four-level system including the cascade four-level system which we shall discuss here.

In the recent past, the equidistant cascade four-level system interacting with the semiclassical and quantized field was discussed mainly within the framework of generalized N-level system [35–40]. The other variant of this configuration, often referred to as Tavis–Cummings model, is studied to construct possible controlled unitary gates relevant for the quantum computation [41,42]. However, these treatments are not only devoid of the explicit calculation of the probabilities for all possible initial conditions [43,44], but they also bypass the comparison between the semiclassical and the quantized models which is crucial to discern the exact role of the field quantization on the population oscillation. In this work we have developed a dressed atom approach of calculating the probabilities with all possible initial conditions especially in the spirit of the basic theory of the ESR model and JCM taking the field to be either monochromatic classical or quantized field [1]. This work is the natural extension of our previous works on the equidistant cascade three-level model [44] where it is explicitly shown that the symmetric pattern observed in the population dynamics for the classical field is completely spoilt on the quantization of the cavity mode.

The remaining sections of the paper are organized as follows. In §2 we discuss the equidistant cascade four-level system modeled by the generators of the spin-$\frac{3}{2}$ representation of $SU(2)$ group and then study its Rabi oscillation with different initial conditions taking interacting field to be the classical field. Section 3 deals with the solution of the four-level system taking the cavity field mode to be the quantized mode. In §4 we compare the Rabi oscillation of the system of the semiclassical model with that of the quantized field and discuss the collapse and revival phenomenon and its implications. Finally, in §5 we summarize our results and discuss the outlook of our investigation.

2. The semiclassical cascade four-level system

The Hamiltonian of the equidistant cascade four-level system is given by

$$H(t) = \hbar\omega_0 J_3 + \hbar\kappa(J_+ \exp(-i\Omega t) + J_- \exp(i\Omega t)), \qquad (1)$$

where J_+, J_- and J_3 are the generators of the spin-$\frac{3}{2}$ representation of $SU(2)$ group given by

$$J_+ = \begin{bmatrix} 0 & \sqrt{3} & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad J_- = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \sqrt{3} & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \sqrt{3} & 0 \end{bmatrix},$$
$$J_3 = \begin{bmatrix} \frac{3}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{bmatrix}. \tag{2}$$

In eq. (1), $\hbar\omega_0$ is the equidistant energy gap between the levels, Ω is the frequency of the classical mode and κ is the coupling constant of the light–atom interaction respectively. The time evolution of the system is described by the Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = H(t)\Psi, \tag{3}$$

where the above time-dependent Hamiltonian in the matrix form is given by

$$H(t) = \begin{bmatrix} \frac{3}{2}\hbar\omega_0 & \sqrt{3}\hbar\kappa\exp[-i\Omega t] & 0 & 0 \\ \sqrt{3}\hbar\kappa\exp[i\Omega t] & \frac{1}{2}\hbar\omega_0 & 2\hbar\kappa\exp[-i\Omega t] & 0 \\ 0 & 2\hbar\kappa\exp[i\Omega t] & -\frac{1}{2}\hbar\omega_0 & \sqrt{3}\hbar\kappa\exp[-i\Omega t] \\ 0 & 0 & \sqrt{3}\hbar\kappa\exp[i\Omega t] & -\frac{3}{2}\hbar\omega_0 \end{bmatrix}. \tag{4}$$

To find the amplitudes, let the solution of the Schrödinger equation corresponding to this Hamiltonian is given by

$$\Psi(t) = C_1(t)\,|1\rangle + C_2(t)\,|2\rangle + C_3(t)\,|3\rangle + C_4(t)\,|4\rangle, \tag{5}$$

where $C_1(t)$, $C_2(t)$, $C_3(t)$ and $C_4(t)$ are the time-dependent normalized amplitudes with basis states

$$|1\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad |2\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad |3\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad |4\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{6}$$

respectively. The Schrödinger equation in eq. (3) can be written as

$$i\hbar\frac{\partial\tilde{\Psi}}{\partial t} = \tilde{H}\tilde{\Psi}, \tag{7}$$

where the time-independent Hamiltonian is given by

$$\widetilde{H} = -i\hbar U^\dagger\dot{U} + U^\dagger H(t)U, \tag{8}$$

with the unitary operator $U(t) = \mathrm{e}^{-i\Omega J_3 t}$. The rotated wave function appearing in eq. (7) is obtained by the unitary transformation

$$\begin{aligned}\widetilde{\Psi}(t) &= U(t)^{\dagger}\Psi(t) \\ &= \mathrm{e}^{-i\frac{3}{2}\Omega t}C_1(t)\,|1\rangle + \mathrm{e}^{-i\frac{1}{2}\Omega t}C_2(t)\,|2\rangle \\ &\quad + \mathrm{e}^{i\frac{1}{2}\Omega t}C_3(t)\,|3\rangle + \mathrm{e}^{i\frac{3}{2}\Omega t}C_4(t)\,|4\rangle\,. \end{aligned} \tag{9}$$

We thus note that the amplitudes are simply modified by a phase term and hence do not contribute to the probabilities. The time-independent Hamiltonian in eq. (8) is given by

$$\widetilde{H} = \hbar \begin{bmatrix} \frac{3}{2}\Delta & \sqrt{3}\kappa & 0 & 0 \\ \sqrt{3}\kappa & \frac{1}{2}\Delta & 2\kappa & 0 \\ 0 & 2\kappa & -\frac{1}{2}\Delta & \sqrt{3}\kappa \\ 0 & 0 & \sqrt{3}\kappa & -\frac{3}{2}\Delta \end{bmatrix}, \tag{10}$$

where $\Delta = \omega_0 - \Omega$. At resonance ($\Delta = 0$), the eigenvalues of the Hamiltonian are given by $\lambda_1 = -\lambda_4 = -3\hbar\kappa$ and $\lambda_2 = -\lambda_3 = -\hbar\kappa$ respectively which can also be generated by the transformation

$$\mathrm{diag}(\lambda_1, \lambda_2, \lambda_3, \lambda_4) = T_{\alpha}\widetilde{H}T_{\alpha}^{-1}, \tag{11}$$

where T_{α} is the transformation matrix given by

$$T_{\alpha} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{bmatrix}. \tag{12}$$

The different elements of matrix, which preserves the orthogonality, are given by [45]

$$\begin{aligned}
\alpha_{11} &= c_1c_5 + s_1s_3s_4s_5, \\
\alpha_{12} &= -c_1s_5s_6 + s_1c_3c_6 + s_1s_3s_4c_5s_6, \\
\alpha_{13} &= s_1s_3c_4, \\
\alpha_{14} &= -c_1s_5c_6 - s_1c_3s_6 + s_1s_3s_4c_5c_6, \\
\alpha_{21} &= -s_1c_2c_5 + (c_1c_2s_3 - s_2c_3)s_4s_5, \\
\alpha_{22} &= s_1c_2s_5s_6 + (c_1c_2c_3 + s_2s_3)c_6 + (c_1c_2s_3 - s_2c_3)s_4c_5s_6, \\
\alpha_{23} &= (c_1c_2s_3 - s_2c_3)c_4, \\
\alpha_{24} &= s_1c_2s_5c_6 - (c_1c_2c_3 + s_2s_3)s_6 + (c_1c_2s_3 - s_2c_3)s_4c_5c_6, \\
\alpha_{31} &= -s_1s_2c_5 + (c_1s_2s_3 + c_2c_3)s_4s_5, \\
\alpha_{32} &= s_1s_2s_5s_6 + (c_1s_2c_3 - c_2s_3)c_6 + (c_1s_2s_3 + c_2c_3)s_4c_5s_6, \\
\alpha_{33} &= (c_1s_2s_3 + c_2c_3)c_4, \\
\alpha_{34} &= s_1s_2s_5c_6 - (c_1s_2c_3 - c_2s_3)s_6 + (c_1s_2s_3 + c_2c_3)s_4c_5c_6, \\
\alpha_{41} &= c_4s_5,
\end{aligned}$$

$$\alpha_{42} = c_4 c_5 s_6,$$
$$\alpha_{43} = -s_4,$$
$$\alpha_{44} = c_4 c_5 c_6, \tag{13}$$

where $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$ $(i = 1, 2, 3, 4, 5, 6)$. A straightforward calculation gives various angles to be

$$\theta_1 = \arccos\left[-\sqrt{\frac{2}{5}}\right], \quad \theta_2 = \frac{3\pi}{4}, \quad \theta_3 = -\frac{\pi}{2},$$
$$\theta_4 = -\arcsin\left[\sqrt{\frac{3}{8}}\right], \quad \theta_5 = \arcsin\left[\sqrt{\frac{1}{5}}\right], \quad \theta_6 = \frac{\pi}{3}. \tag{14}$$

The time-dependent probability amplitudes of the four-levels are given by

$$\begin{bmatrix} C_1(t) \\ C_2(t) \\ C_3(t) \\ C_4(t) \end{bmatrix} = T_\alpha^{-1} \begin{bmatrix} \mathrm{e}^{-i\lambda_1 t} & 0 & 0 & 0 \\ 0 & \mathrm{e}^{-i\lambda_2 t} & 0 & 0 \\ 0 & 0 & \mathrm{e}^{-i\lambda_3 t} & 0 \\ 0 & 0 & 0 & \mathrm{e}^{-i\lambda_4 t} \end{bmatrix} T_\alpha \begin{bmatrix} C_1(0) \\ C_2(0) \\ C_3(0) \\ C_4(0) \end{bmatrix}. \tag{15}$$

Later in §4, we proceed to analyze the probabilities of the four levels numerically for four distinct initial conditions, namely, Case I: $C_1(0) = 1$, $C_2(0) = 0$, $C_3(0) = 0$, $C_4(0) = 0$, Case II: $C_1(0) = 0$, $C_2(0) = 1$, $C_3(0) = 0$, $C_4(0) = 0$, Case III: $C_1(0) = 0$, $C_2(0) = 0$, $C_3(0) = 1$, $C_4(0) = 0$ and Case IV: $C_1(0) = 0$, $C_2(0) = 0$, $C_3(0) = 0$, $C_4(0) = 1$, respectively.

3. The Jaynes–Cummings model of cascade four-level system

We now consider the equidistant cascade four-level system interacting with a monochromatic quantized cavity field. The Hamiltonian of such a system in the rotating wave approximation (RWA) is given by

$$H = \hbar(\omega_0 J_3 + \Omega a^\dagger a) + \hbar(\Delta J_3 + g(J_+ a + J_- a^\dagger)). \tag{16}$$

This is an archetype JCM where the Pauli matrices are replaced by the spin-$\frac{3}{2}$ representation of $SU(2)$ group. Using the algebra of the $SU(2)$ group and that of the field mode it is easy to see that the two parts of the Hamiltonian shown in the parenthesis of eq. (16) commute with each other indicating that they have the simultaneous wave function. Let the eigenfunction corresponding to this Hamiltonian is given by

$$|\Psi_q(t)\rangle = \sum_{n=0}^{\infty}[C_1^{n+2}(t)|n+2,1\rangle + C_2^{n+1}(t)|n+1,2\rangle$$
$$+C_3^{n}(t)|n,3\rangle + C_4^{n-1}(t)|n-1,4\rangle], \tag{17}$$

where n represents the number of photons in the cavity field. The Hamiltonian couples the atom-field states $|n+2,1\rangle$, $|n+1,2\rangle$, $|n,3\rangle$, and $|n-1,4\rangle$ respectively.

At resonance $\Delta = 0$, the interaction part of the Hamiltonian in the matrix form is given by

$$H_{\rm int} = g\hbar \begin{bmatrix} 0 & \sqrt{3(n+2)} & 0 & 0 \\ \sqrt{3(n+2)} & 0 & 2\sqrt{n+1} & 0 \\ 0 & 2\sqrt{n+1} & 0 & \sqrt{3n} \\ 0 & 0 & \sqrt{3n} & 0 \end{bmatrix}, \tag{18}$$

with the eigenvalues

$$\lambda_{1q} = -\lambda_{4q} = -g\hbar\sqrt{5(1+n)+b}, \tag{19a}$$

$$\lambda_{2q} = -\lambda_{3q} = -g\hbar\sqrt{5(1+n)-b}, \tag{19b}$$

respectively where $b = \sqrt{25+16n(2+n)}$. The dressed eigenstates are constructed by rotating the bare states as

$$\begin{bmatrix} |n,1\rangle \\ |n,2\rangle \\ |n,3\rangle \\ |n,4\rangle \end{bmatrix} = T_n \begin{bmatrix} |n+2,1\rangle \\ |n+1,2\rangle \\ |n,3\rangle \\ |n-1,4\rangle \end{bmatrix}, \tag{20}$$

where T_n is similar to the aforementioned orthogonal transformation matrix whose different elements are given by

$$\begin{aligned}
\alpha_{11} &= -\alpha_{41} = -\frac{(1+b-2n)\sqrt{5+b+5n}}{2\sqrt{3(2+n)\{5(5+b)+2n(16+b+8n)\}}}, \\
\alpha_{21} &= -\alpha_{31} = \frac{(b-1+2n)\sqrt{(5+5n-b)(5+2n+b)}}{12\sqrt{n(n+1)(n+2)b}} \\
\alpha_{12} &= \alpha_{42} = \frac{5+2n+b}{2\sqrt{5(5+b)+2n(16+b+8n)}}, \\
\alpha_{13} &= -\alpha_{43} = -\frac{\sqrt{(1+n)(5+b+5n)}}{\sqrt{5(5+b)+2n(16+b+8n)}}, \\
\alpha_{22} &= \alpha_{32} = -\frac{\sqrt{3n(1+n)}}{\sqrt{b(5+b+2n)}}, \quad \alpha_{14} = \alpha_{44} = \frac{\sqrt{(b-5-2n)}}{2\sqrt{b}}, \\
\alpha_{23} &= -\alpha_{33} = -\frac{\sqrt{(5+5n-b)(5+2n+b)}}{2\sqrt{3nb}}, \\
\alpha_{24} &= \alpha_{34} = \frac{\sqrt{5+2n+b}}{2\sqrt{b}}.
\end{aligned} \tag{21}$$

A straightforward but rigorous calculation gives the explicit expressions of the angle of rotation for the quantized model

$$\theta_1 = \arccos\left[\frac{-\alpha_{11}}{\sqrt{(1-\alpha_{13}^2)(1-\alpha_{11}^2-\alpha_{13}^2)}}\right],$$
$$\theta_2 = \arccos\left[\frac{\alpha_{11}\alpha_{13}\alpha_{23}+(1-\alpha_{13}^2)\sqrt{(1-2\alpha_{11}^2-2\alpha_{13}^2)(1-2\alpha_{13}^2-\alpha_{23}^2)}}{(2\alpha_{13}^2-1)\sqrt{(\alpha_{13}^2-1)^2+\alpha_{11}^2(\alpha_{13}^2-2)}}\right],$$
$$\theta_3 = \arcsin\left[\frac{\alpha_{13}\sqrt{\alpha_{11}^2+\alpha_{13}^2-1}}{\sqrt{\alpha_{11}^2(2-\alpha_{13}^2)+(1-\alpha_{13}^2)^2}}\right],$$
$$\theta_4 = \arcsin[\alpha_{13}],$$
$$\theta_5 = -\arcsin\left[\frac{\alpha_{11}}{\sqrt{1-\alpha_{13}^2}}\right],$$
$$\theta_6 = \arcsin\left[\frac{\alpha_{12}}{\sqrt{1-\alpha_{11}^2-\alpha_{13}^2}}\right], \tag{22}$$

where different elements α_{ij} appearing in the rotation matrix are defined in eq. (21). It is easy to see that in the limit $n \to \infty$, these angles precisely yield those of the semiclassical model given in eq. (14). This clearly shows that our treatment of the quantized model is in conformity with the Bohr correspondence principle and indicates the consistency of our treatment.

The time-dependent probability amplitudes of the four levels are given by

$$\begin{bmatrix} C_1^{n+2}(t) \\ C_2^{n+1}(t) \\ C_3^{n}(t) \\ C_4^{n-1}(t) \end{bmatrix} = T_n^{-1} \begin{bmatrix} e^{-i\lambda_{1q}t} & 0 & 0 & 0 \\ 0 & e^{-i\lambda_{2q}t} & 0 & 0 \\ 0 & 0 & e^{-i\lambda_{3q}t} & 0 \\ 0 & 0 & 0 & e^{-i\lambda_{4q}t} \end{bmatrix} \times T_n \begin{bmatrix} C_1^{n+2}(0) \\ C_2^{n+1}(0) \\ C_3^{n}(0) \\ C_4^{n-1}(0) \end{bmatrix}. \tag{23}$$

In the next section we proceed to analyze the probabilities of the four levels for aforesaid initial conditions, namely, Case V: $C_1^{n+2}(0) = 1$, $C_2^{n+1}(0) = 0$, $C_3^n(0) = 0$, $C_4^{n-1}(0) = 0$, Case VI: $C_1^{n+2}(0) = 0$, $C_2^{n+1}(0) = 1$, $C_3^n(0) = 0$, $C_4^{n-1}(0) = 0$, Case VII: $C_1^{n+2}(0) = 0$, $C_2^{n+1}(0) = 0$, $C_3^n(0) = 1$, $C_4^{n-1}(0) = 0$ and Case VIII: $C_1^{n+2}(0) = 0$, $C_2^{n+1}(0) = 0$, $C_3^n(0) = 0$, $C_4^{n-1}(0) = 1$, respectively and then compare the results with those of the semiclassical model.

4. Numerical results

We are now in position to explore the physical content of our treatment by comparing the probabilities of the semiclassical and quantized cascade four-level system respectively. Figures 1a–d show the plots of the probabilities $|C_1^i(t)|^2$ (level 1, dot–dashed line), $|C_2^i(t)|^2$ (level 2, dotted line), $|C_3^i(t)|^2$ (level 3, dashed line) and $|C_4^i(t)|^2$ (level 4, solid line) for the semiclassical model corresponding to Cases I,

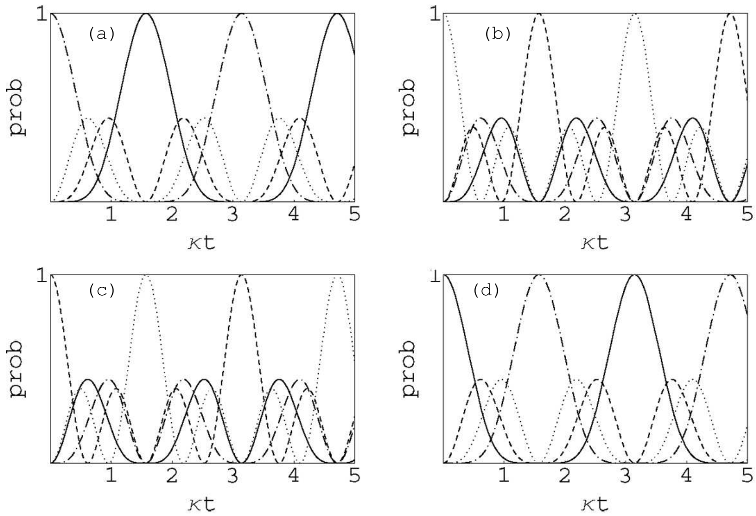

Figure 1. For the semiclassical model, the time evolution (scaled with κ) of the probabilities for Cases I, II, III and IV are shown in figures 1a, 1b, 1c and 1d, respectively. The Rabi oscillation of figures 1a and 1d and that of figures 1b and 1c are found to be similar. The probabilities of level 1 (dot–dashed line) and level 4 (solid line) and those of level 2 (dotted line) and level 3 (dashed line) are interchanged.

II, III and IV respectively. The comparison of figure 1a (figure 1b) and figure 1d (figure 1c) shows that the pattern of the probability oscillation of Case I (Case II) is similar to that of Case IV (Case III) except the probabilities of level 1 and level 4 and also that of level 2 and level 3 are interchanged. Thus, a regular pattern of the probability oscillation reveals the symmetric behavior of Rabi oscillation for the semiclassical four-level cascade system.

Following ref. [44], in the case of the quantized field, we consider the time evolution of the probabilities for two distinct situations, first, when the field is in a number state representation and then, when the field is in the coherent state representation.

For the number state representation, the Rabi oscillation for Cases V, VI, VII and VIII of the quantized system are shown in figures 2a–d. Here we note that for Case V (Case VI), the oscillation pattern of the system is completely different from that of Case VIII (Case VI). Thus the symmetry observed in the pattern of the population dynamics of the semiclassical model between Case I and Case IV and also between Case II and Case III no longer exists. In other words, for the quantized field, in contrast to the semiclassical case, whether the system initially stays in any one of the four levels, the symmetry of the Rabi oscillation in all cases is completely spoilt. As pointed out earlier, the disappearance of the symmetry is essentially due to the vacuum fluctuation of the quantized cavity mode which survives even at $n = 0$. Recently, we have reported similar breaking pattern in the Rabi oscillation for the equidistant cascade [44] and also for lambda and vee three-level systems [46]. Such breaking is not observed in the case of two-level

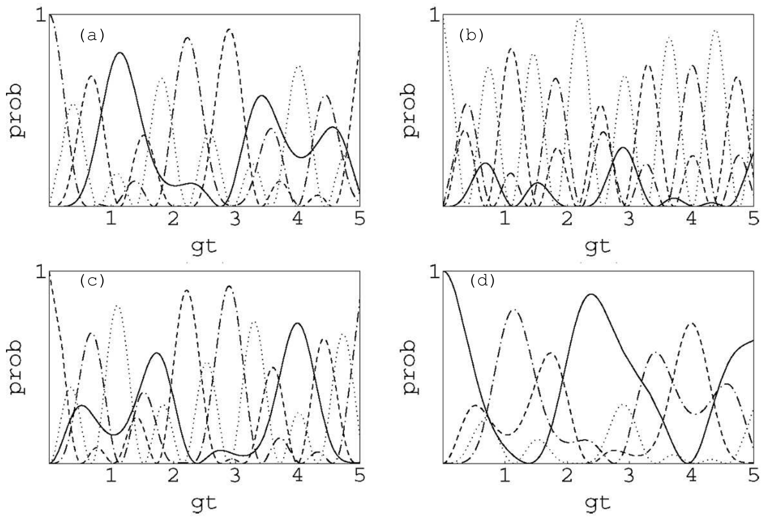

Figure 2. The Rabi oscillation (scaled with g) for Cases V, VI, VII and VIII with quantized cavity mode shows the breaking of the aforesaid symmetry between Case I and Case IV and between Case II and Case III, respectively.

Jaynes–Cumming model and hence is essentially a nontrivial feature of multi-level systems when the number of levels exceeds two.

Finally, we consider the model interacting with the monochromatic quantized field which is in the coherent state. The coherently averaged probabilities of the system for level-1, level-2, level-3 and level-4 are given by

$$\langle P_1(t)\rangle = \sum_n w_n \left|C_1^{n+2}(t)\right|^2 \tag{24a}$$

$$\langle P_2(t)\rangle = \sum_n w_n \left|C_2^{n+1}(t)\right|^2 \tag{24b}$$

$$\langle P_3(t)\rangle = \sum_n w_n \left|C_3^{n}(t)\right|^2 \tag{24c}$$

$$\langle P_4(t)\rangle = \sum_n w_n \left|C_4^{n-1}(t)\right|^2, \tag{24d}$$

respectively, where $w_n = \exp[-\bar{n}]\frac{\bar{n}^n}{n!}$ is the coherent distribution with $\bar{n}$ being the mean photon number of the quantized field mode. Figures 3 and 4 display the numerical plots of eq. (24) with $\bar{n} = 48$ for Case V, VI, VII and VIII, respectively where the collapse and revival of the Rabi oscillation is clearly evident. The collapse and revival for Case V depicted in figures 3a–d is compared with that of Case VIII shown in figures 3e–h. We note that figures 3a, 3b, 3c and 3d are precisely identical to figures 3h, 3g, 3f and 3e respectively. Similarly, figure 4 compares the collapse and revival of the system for Case VI with that of Case VII, where, similar to the semiclassical model, we note that figures 4a, 4b, 4c and 4d are similar to figures 4h, 4g, 4f and 4e, respectively. Note that a distinct collapse and revival

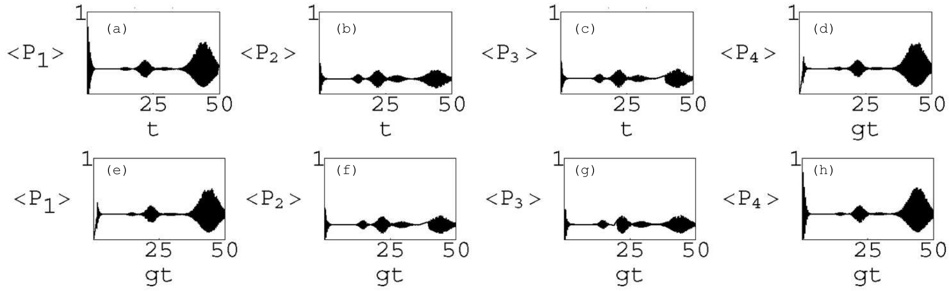

Figure 3. Figures 3a–d and figures 3e–h depict the time-dependent collapse and revival phenomenon for Case V and Case VIII respectively. We note that the oscillation pattern of levels 1, 2, 3 and 4 in Case V is similar to that of levels 4, 3, 2 and 1 for Case VIII, respectively.

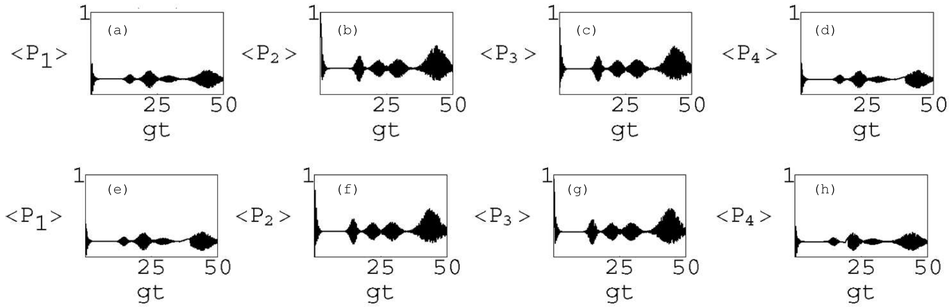

Figure 4. Figures 4a–d and figures 4e–h depict the collapse and revival phenomenon for Case VI and Case VII, respectively. Here we find that the oscillation pattern of levels 1, 2, 3 and 4 for Case VI is similar to that of levels 4, 3, 2 and 1 for Case VII, respectively.

pattern appears for a coherent cavity field only for a large average photon number. This clearly recovers the symmetric pattern exhibited by the semiclassical four-level system with the classical field mode.

5. Conclusion

This paper examines the behaviour of the oscillation of probability of a cascade four-level system taking the field to be either classical or quantized. The Hamiltonian of the system is constructed from the generators of the spin-$\frac{3}{2}$ representation of the $SU(2)$ group and the probabilities of the four levels are computed for different initial conditions using a generalized Euler angle representation. We argue that the symmetry exhibited in the Rabi oscillation with the classical field is completely destroyed due to the quantum fluctuation of the cavity mode. This symmetry is, however, restored by taking the cavity mode as a coherent state with large average photon number. It is interesting to look for the effect of field quantization on the

Rabi oscillation with other configurations of the four-level system and to scrutinize its nontrivial effect on the various coherent phenomena involving multilevel systems.

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