PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 70, No. 1
January 2008
pp. 113–120

π^--^{12}C elastic scattering above the Δ resonance using diffraction model

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MS received 31 January 2007; revised 28 June 2007; accepted 7 August 2007

Abstract. Phenomenological analysis of the π^--^{12}C elastic scattering differential cross-section at 400, 486, 500, 584, 663, 672 and 766 MeV is presented. The analysis is made in the diffraction model framework using the recently proposed parametrization of the phase-shift function. Good description of the experimental data is achieved at all energies. Microscopic interpretation of the parameters of the phase-shift function is provided in terms of Helm's model density parameters.

Keywords. Pion–nucleus scattering; diffraction model; Helm's model.

PACS Nos 24.10.Ht; 25.80.Dj

1. Introduction

Over the past two decades or so, several microscopic studies of pion–nucleus elastic scattering at incident pion energies above the Δ resonance have appeared in the literature [1–6]. These studies show that the first-order multiple scattering optical potentials do not provide a satisfactory description of the empirical data and that the conventional second-order corrections to the first-order theory are not helpful in significantly improving the theoretical situation. Some phenomenological analyses based on the optical model and the strong absorption model have also appeared in the literature. For example, Hong and Kim [7] have used the conventional six-parameter Wood–Saxon optical potential to obtain a satisfactory description of the π^--^{12}C elastic scattering data over a wide range of energies. On the other hand, Choudhury and Scura [8] have applied McIntyre parametrization of the elastic S-matrix element S_ℓ , to derive a closed expression for the elastic scattering amplitude under the strong absorption approximation which they used to fit the π^--^{12}C and π^--^{40}Ca elastic scattering data at 800 MeV/c.

In a recent article, Ahmad and Arafah [9] (hereafter to be referred to as I) have examined Choudhury and Scura's S_ℓ. They have found that this S_ℓ when substituted in the exact partial wave expression of the scattering amplitude does not provide a satisfactory fit to the experimental data. This situation shows that the strong absorption approximation as used by Choudhary and Scura [8] does not work well in this case. Also, it suggests that some other parametrization for S_ℓ should be used for the diffraction model analysis of the pion data. Following the suggestion by McEvan, Cooper and Mackintosh (MCM) [10] that in the diffraction model analysis of the elastic scattering data, the parametrization of S_ℓ should better be based on some theory, the authors of ref. [9] proposed a parametrization of S_ℓ which is based on the optical limit approximation of the Glauber theory [11]. The proposed parametrization which involves four adjustable parameters has been shown to work exceedingly well for π-^{12}C system at 800 MeV/c. In I, a microscopic interpretation of the geometrical parameters of S_ℓ in terms of the modified harmonic oscillator density parameters has also been provided.

This work is an extension of the study reported in I. Here we show that the diffraction model phenomenology as proposed in I provides a very good discretion of the π^--^{12}C elastic scattering data over a wide range of energies from 400 to 766 MeV. Hence, it could be applied for obtaining analytic optical potential by inversion in this energy range. This optical potential involves lesser number of adjustable parameters than the conventional Wood–Saxon potential [7]. Further, in I the geometrical parameters of the phase-shift function were related to the parameters of the modified harmonics oscillator model density using the optical limit approximation of the Glauber model and the zero-range approximation for the πN interaction. This density model is of limited applicability as it is generally used for lighter nuclei. Some other density models have also been used for ^{12}C which belong to the leptoderous class and are of wider applicability. One of them is the Helms model in which the nuclear density is obtained by convoluting the uniform spherical density of the radius R with the Gaussian density of variance σ. This model which describes the two main features of the nuclear density namely, the extension and the diffuseness in terms of the parameters R and σ respectively has been used by Friedrich and Voegler [12] successfully to describe the nuclear charge distribution over almost the whole mass spectrum.

Therefore, it would be interesting to establish some relationship between the parameters of the phase-shift function and Helms model density parameters. This is another modification of the present work.

2. Theoretical considerations

In this work the elastic scattering differential cross-section for π^--^{12}C system is calculated using the following partial wave expression for the elastic scattering amplitude:

$$F_{\mathrm{el}}(\theta) = F_{\mathrm{c}}(\theta) + \frac{i}{2k}\sum_{\ell=0}^{\infty}(2\ell+1)\mathrm{e}^{2i\sigma_\ell}[1-S_\ell]P_\ell(\cos\theta), \tag{1}$$

where $F_{\mathrm{c}}(\theta)$ is the point Coulomb scattering amplitude, σ_ℓ is the point Coulomb phase-shift, k is the c.m. momentum, P_ℓ (cos θ) is the Legendre polynomial and S_ℓ is the elastic S-matrix element. Following I, we write S_ℓ as

$$S_\ell = S(b)\mathrm{e}^{i\delta_{\mathrm{c}}(b)}|_{kb=\ell+\frac{1}{2}}, \tag{2}$$

where b is the impact parameter, S is the nuclear part of the S-matrix element, and $\delta_{\mathrm{c}}(b)$ is the difference between the phase-shift function of the Coulomb potential due to the extended charge distribution of the target nucleus and the corresponding point charge [9].

Next, we express $S(b)$ in terms of the phase-shift function $\chi(b)$ as

$$S(b) = \mathrm{e}^{i\chi(b)}, \tag{3}$$

and assume that

$$\chi(b) = (c_r + ic_i)(1 + c_1 b^2)\mathrm{e}^{-c_2 b^2}, \tag{4}$$

where c_r, c_i, c_1 and c_2 are parameters. In I, a microscopic interpretation for the above parametrization has been given using the zero-range Glauber optical limit approximation [11] and the modified harmonic oscillator model target density distribution. Below, we give another interpretation using the Helm's model for the target density distribution. This alternative interpretation enables us to determine the radius or extension parameter of the density distribution from the parameters c_1 and c_2 of the parametrization (4).

In the optical limit approximation, the Glauber model phase-shift function for the π-nucleus system may be expressed as [13]

$$\chi_{\mathrm{op}}(b) = \frac{A}{k}\int_0^\infty \mathrm{d}q\, q J_0(qb) F(q) f(q), \tag{5}$$

where A is the target mass number, $f(q)$ is the isospin averaged πN scattering amplitude, $F(q)$ is the target form factor and J_0 is the Bessel function of zero order. In writing eq. (5) it has been assumed that the neutron and proton density distributions in the target are the same.

In the Helm's model, the nuclear density is obtained by convoluting a uniform density distribution of radius R with the normalized Gaussian density distribution of variance σ (also called diffusion parameter). Accordingly, the Helm's model form factor is of the form [12]:

$$F(q) = \frac{3}{qR} j_1(qR)\, \mathrm{e}^{-\sigma^2 q^2/2}, \tag{6}$$

where $j_1(qR)$ is the spherical Bessel function of order 1. The Helm's model rms radius r_m is given by [12]

$$r_m^2 = \frac{3}{5}R^2\left[1 + 5\left(\frac{\sigma}{R}\right)^2\right]. \tag{7}$$

With regard to the amplitude $f(q)$, one may take the generally used Gaussian parametrization [13]:

$$f(q) = \frac{ik\sigma_{\mathrm{t}}(1 - i\alpha)}{4\pi}\mathrm{e}^{-\beta q^2/2}, \tag{8}$$

where $\sigma_{\rm t}$ is the πN total cross-section, α is the ratio of real to imaginary parts of the πN amplitude and β is the slope parameter.

With the Helm's model $F(q)$ as given by eq. (6) the optical phase-shift function $\chi_{\rm op}(b)$ can be evaluated only numerically. Therefore, we invoke some approximation to obtain a closed expression for $\chi_{\rm op}(b)$. Using the product representation of the Bessel function [14], the form factor $F(q)$ may be written as

$$F(q) = \left[\prod_{i=1}^{\infty}\left(1 - \frac{q^2R^2}{x_i^2}\right)\right] {\rm e}^{\frac{-\sigma^2}{2}q^2}, \tag{9}$$

where x_is are the zeros of $j_1(x)$. Now, we approximate the above form factor as

$$F(q) \approx \left(1 - \frac{q^2R^2}{x_1^2}\right) {\rm e}^{-(\nu R^2 + \frac{\sigma^2}{2})q^2}, \tag{10}$$

where $x_1(= 4.4934)$ is the first zero of the Bessel function and

$$\nu = (0.1 - 1/x_1^2) = 0.0505. \tag{11}$$

Expression (10) ensures that the approximate form factor has the same rms radius and the first zero as the exact one. Thus the approximate form factor given by eq. (10) correctly describes the low $-q$ behavior of the Helm's model form factor. This is satisfying, since in expression (5) for $\chi_{\rm op}(b)$, the main contribution to the integral, because of the oscillatory nature of the Bessel function, comes from the small q region. Therefore, it is hoped that the approximation (10) would give reasonably good results for $\chi_{\rm op}(b)$. Substitution of eq. (10) in eq. (5) gives

$$\chi_{\rm op}(b) = \frac{A\sigma_{\rm t}(i+\alpha)}{8\pi}\,\frac{(D-Y)}{D^2}\left[1 + \frac{Y\,b^2}{4D(D-Y)}\right]{\rm e}^{-\frac{b2}{4D}}, \tag{12}$$

where

$$D = (\nu\, R^2 + \sigma^2/2) + \beta/2, \tag{13}$$

$$Y = \frac{R^2}{x_1^2}. \tag{14}$$

Comparing eqs (4) and (12), we find that the geometrical parameters c_1 and c_2 of the diffraction model phenomenology are related to the density parameters as

$$c_2 = \frac{1}{4D} \tag{15}$$

and

$$c_1 = \frac{Y}{4D(D-Y)}. \tag{16}$$

Using eqs (13)–(16) the following relationships between the parameters c_1 and c_2 and the density parameters may be easily derived:

$$R^2 = \frac{c_1 x_1^2}{4c_2(c_1+c_2)}, \tag{17}$$

$$\sigma^2 + \beta = 2\left(\frac{1}{4c_2} - 0.0505R^2\right). \tag{18}$$

Thus from the phenomenological parameters c_1 and c_2 one can determine the radius parameter R using eq. (17). However, because of the appearance of β in eq. (18), these parameters can be used to determine σ if either the zero-range approximation holds good or β is precisely known.

3. Results and discussion

Using the parametrization of the phase-shift function as given by eq. (4) and the expression (1) for the elastic scattering amplitude we fit the π^--^{12}C elastic scattering differential cross-section data at 400, 486, 500, 584, 663, 673 and 766 MeV by varying the parameters c_r, c_i, c_1 and c_2. The quantity $S_\ell(b)$ as given by eq. (2) is evaluated using the same expression for $S_\ell(b)$ as given in I. The results of the fit are shown in figures 1a and 1b. The corresponding χ^2 and parameter values are given in table 1. It is seen that in general, reasonably satisfactory fits to the experimental data are achieved. However, at 400 MeV although there is fairly good qualitative agreement with the data, yet the χ^2 value is fairly large. This is mainly due to wiggles in the experimental values in the region of the minimum and large disagreement between the calculated and experimental values in the extreme forward angle region. In the last two columns of table 1 we give the Helm's model radius parameter R and the effective diffuseness $\sigma^2 + \beta$ as calculated from eqs (17) and (18). The R values in the table compare reasonably well with the value 2.44 fm as extracted from the study of the charge density distribution of ^{12}C nucleus [12]. Here, it may be added that in multiparameter fitting of the elastic scattering angular distribution the value of a parameter depends upon several factors such as the quality of the experimental data, momentum transfer covered by the experimental values, correlation with other parameters, χ^2-value achieved, etc. From table 1, it

Table 1. Parameter values of the phase-shift function for π^--^{12}C system.

Energy (MeV)	c_r	c_i	c_1 (fm^{-2})	c_2 (fm^{-2})	χ^2	$\sigma^2+\beta$ (fm^2)	R (fm)
400	1.1209	0.8807	0.3853	0.3719	29.15	0.65	2.63
486	0.8383	0.6101	0.5291	0.4587	2.58	0.50	2.43
500	0.6967	0.9322	0.3656	0.3884	10.20	0.65	2.51
584	0.6925	0.8273	0.4171	0.4320	1.48	0.58	2.40
663	0.2832	1.0961	0.4351	0.4210	1.52	0.57	2.47
672.5	0.2683	1.6459	0.2469	0.3960	0.8	0.80	1.78
766.2	0.144	1.257	0.307	0.374	1.00	0.72	2.47

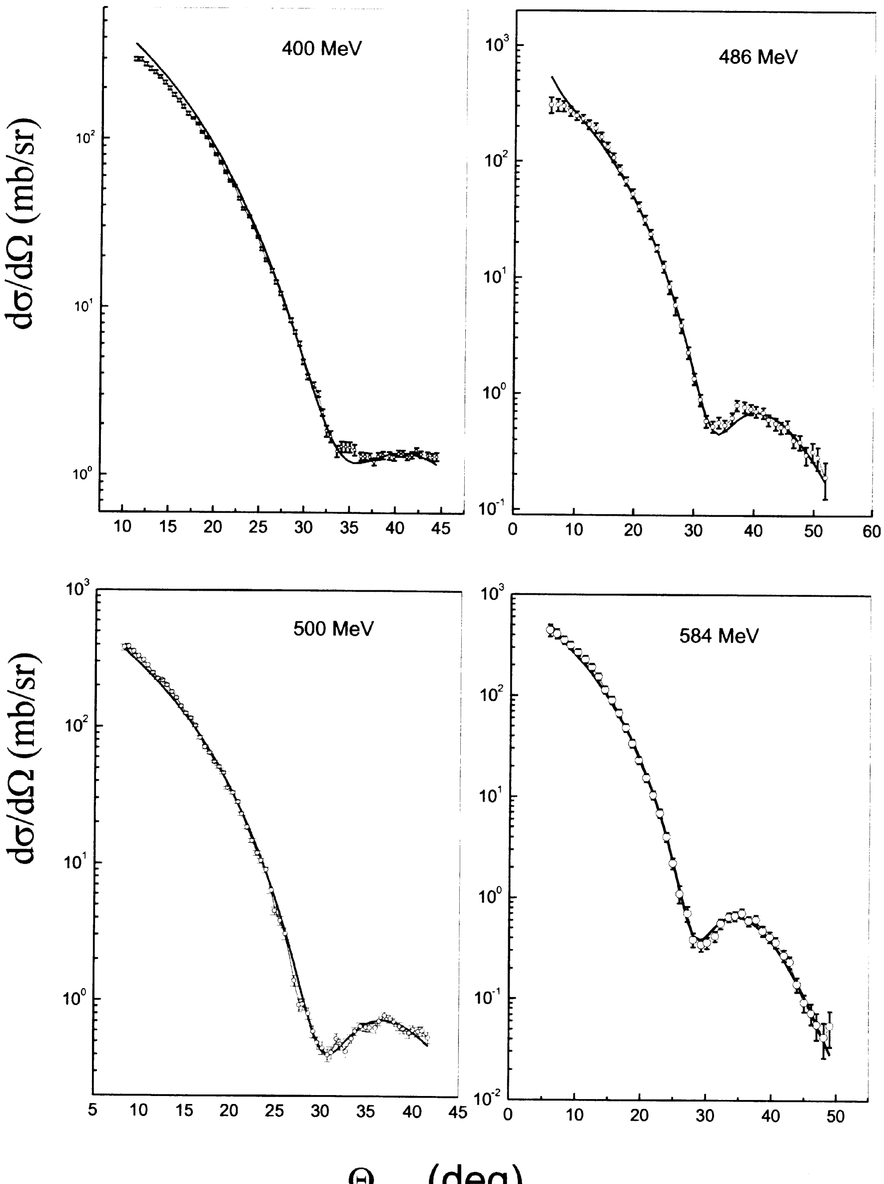

Figure 1. (**a**) π^--^{12}C elastic scattering differential cross-sections. The solid curves show the results of our fit with the parameter values given in table 1. The experimental data have been taken from refs [15] and [16].

is seen that at 400 and 500 MeV the χ^2-value are relatively large which perhaps are due to the large wiggles in the experimental data around the region of the minimum as mentioned earlier (figure 1a). Therefore, the extracted values of R at these two

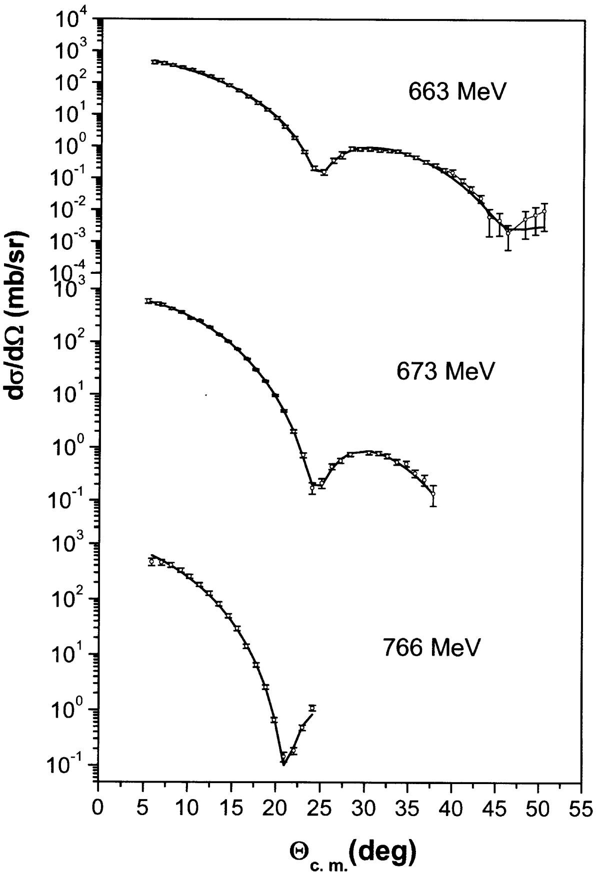

Figure 1. (**b**) Same as in figure 1a.

energies are expected to be correct only qualitatively. At the incident pion energies 486, 584, 663 and 766.2 MeV where the fits are relatively much better the values of R as obtained from the fittings are in better agreement with the electron scattering value. However, at 672.5 MeV the value of R is not in good agreement with the electron scattering value, though the χ^2-value in this case is the smallest. This is because the χ^2 minimum in this case is reached with a fairly large value of the effective diffuseness $(\sigma^2 + \beta)$ as may be seen from the table. Such a correlation between the radius parameter and the surface diffuseness is commonly observed in the optical model fittings.

Coming to the effective diffuseness $\sigma^2 + \beta$ in table 1, it is seen that the values are in reasonably fair agreement with the charge density diffuseness parameter $\sigma^2 \approx 0.65$ fm^2 as obtained from the analysis of the ^{12}C charge density distribution [12]. Here, it may be mentioned that the diffuseness σ^2 for the point nucleon density distribution is related with $\sigma^2_{\rm ch}$ as $\sigma^2 = \sigma^2_{\rm ch} - r^2_{\rm ch}/3$ where $r_{\rm ch}$ is the charge rms radius of proton.

Thus within the diffraction model framework, there are two main advantages of the presently used parametrization of the elastic S-matrix element. First, its geometrical parameters c_1 and c_2 are simply related to the extension and diffuseness parameters R and σ of the nucleon density distribution. This is not the case with the generally used McIntyre parametrization of the S-matrix. For example, the diffraction model radius and diffuseness parameters of ref. [8] have no explicit relationship with the radius and the diffuseness parameters of the nucleon density distribution. Second, as discussed at length in I, our parametrization of S-matrix element may be used to obtain a closed expression for the optical potential by the method of inversion. It is reasonable to hope that this inversion optical potential which involves only four adjustable parameters is more reliable than the conventional six-parameter phenomenological Woods–Saxon potential to be used for DWBA type calculation.

Acknowledgements

The author is grateful to Professor I Ahmad for suggesting application of Helm's model for the microscopic interpretation of the phenomenology and for reading the manuscript.

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