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Deviation from tri-bimaximal mixings through flavour twisters in inverted and normal hierarchical neutrino mass models

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Abstract. We explore a novel possibility for lowering the solar mixing angle (θ_{12}) from tri-bimaximal mixings, without sacrificing the predictions of maximal atmospheric mixing angle ($\theta_{23} = 45°$) and zero reactor angle ($\theta_{13} = 0°$) in the inverted and normal hierarchical neutrino mass models having 2-3 symmetry. This can be done through the identification of a flavour twister term in the texture of neutrino mass matrix and the variation of such term leads to lowering of solar mixing angle. For the observed ranges of Δm^2_{21} and Δm^2_{23}, we calculate the predictions on $\tan^2\theta_{12} = 0.5, 0.45, 0.35$ for different input values of the parameters in the neutrino mass matrix. We also observe a possible transition from inverted hierarchical model having even CP parity (Type-IHA) to inverted hierarchical model having odd CP parity (Type-IHB) in the first two mass eigenvalues, when there is a change in input values of parameters in the same mass matrix. The present work differs from the conventional approaches for the deviations from tri-bimaximal mixing, where the 2-3 symmetry is broken, leading to $\theta_{23} \neq 45°$ and $\theta_{13} \neq 0°$.

Keywords. Inverted hierarchical mass matrix; normal hierarchical mass matrix; tri-bimaximal mixings, solar angle.

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1. Introduction

Current observational data [1] on neutrino oscillations indicate a clear departure from tri-bimaximal mixings (TBM) or Harrison–Perkins–Scott (HPS) mixing pattern [2]. The most recent SNO experimental determination [3] of solar angle gives $\tan^2\theta_{12} = 0.45^{+0.09}_{-0.08}$ compared with $\tan^2\theta_{12} = 0.50$ in HPS scheme. There is no strong claim for substantial departure from the maximal atmospheric mixing ($\tan^2\theta_{23} = 1$), and zero reactor angle ($\sin\theta_{13} = 0$). Only upper bound for $\sin\theta_{13}$ is known at the moment and future measurements may possibly give a very small value which can be approximated by zero [4]. This does not yet contradict with the non-observation of Dirac CP phase angle. There are several discussions [5] on the experimental requirements for mass hierarchy measurements at $\sin\theta_{13} = 0$.

Conditions of maximal atmospheric mixing ($\theta_{23} = 45°$) and exact zero reactor angle ($\theta_{13} = 0°$) are, in fact, necessary and sufficient condition [4–9] for the leptonic mixing matrix obtained from the diagonalisation of the left-handed Majorana neutrino mass matrix having 2-3 symmetry (or μ-τ symmetry) which implies an invariance under the simultaneous permutation of the second and third rows as well as the second and third columns [4]. On the basis where charged lepton mass matrix is diagonal, a 2-3 symmetry can be generally realised in all neutrino mass models [10,11]. Further constraints such as zero determinant [4,12,13] or zero trace [14] of the neutrino mass matrix, which lead to other interesting properties, are not considered in the present analysis. This freedom allows us to consider larger values of non-zero mass eigenvalue m_3 within the framework of inverted hierarchical models.

A general form of inverted hierarchical mass matrix $m_{\rm LL}$ having 2-3 symmetry, can be written as [4],

$$m_{\rm LL} = \begin{pmatrix} m_{11} & m_{12} & m_{12} \\ m_{12} & m_{22} & m_{23} \\ m_{12} & m_{23} & m_{22} \end{pmatrix} \tag{1}$$

which is diagonalised by the relation $m_{\rm LL} = UDU^\dagger$ where U is given by

$$U = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ \frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{2}$$

Here $c_{12} = \cos\theta_{12}$ and $s_{12} = \sin\theta_{12}$; and we have $\theta_{23} = \pi/4$ and $\theta_{13} = 0°$. On the basis where charged lepton mass matrix is diagonal, U in eq. (2) is identified as the MNS mixing matrix $U_{\rm MNS}$ [15] where the solar mixing angle θ_{12} is arbitrary, atmospheric mixing angle θ_{23} is maximal and reactor angle θ_{13} is exactly zero. For bimaximal mixings, we choose $c_{12} = 1/\sqrt{2}$ and $s_{12} = 1/\sqrt{2}$, leading to $\tan^2\theta_{12} = 1.0$, whereas for tri-bimaximal mixings (TBM) [2], we have $c_{12} = \sqrt{\frac{2}{3}}$ and $s_{12} = 1/\sqrt{3}$, leading to $\tan^2\theta_{12} = 0.5$. Then the MNS mixing matrix, $U_{\rm MNS} = O_{23}O_{12}$, assumes the following form:

$$U_{\rm TBM} = O_{23}O_{12} = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{3}$$

where

$$O_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{4}$$

and

$$O_{12} = \begin{pmatrix} \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

For diagonal neutrino mass matrix $D = \text{diag}(m_1, m_2, m_3)$, the mass matrix $m_{\text{LL}} = U_{\text{TMB}} D U^{\dagger}_{\text{TBM}}$ generally takes a simple form

$$m_{\text{LL}} = \begin{pmatrix} A & B & B \\ B & A-C & B+C \\ B & B+C & A-C \end{pmatrix} m_0, \tag{6}$$

where the elements are expressible in terms of linear combinations of three masses: $A = (m_2 + 2m_1)/3$, $B = (m_2 - m_1)/3$, and $C = (m_1 - m_3)/2$ respectively. This form of mass matrix which is a consequence of tri-bimaximal mixings, can be derived from the general $S3$ symmetry [8]. Another equivalent form due to Harrison *et al* [2] in terms of m^2_{LL} for tri-bimaximal mixings derived from $S3$ symmetry, is given by

$$m^2_{\text{LL}} = \begin{pmatrix} s+t+u & u & u \\ u & s+u & t+u \\ u & t+u & s+u \end{pmatrix}, \tag{7}$$

where $m_1^2 = s + t, m_2^2 = s + t + 3u$ and $m_3^2 = s - t$ respectively.

We are now interested in investigating the condition for fixing the arbitrary solar mixing angle to its tri-bimaximal value [2] in eq. (6) and for lowering this value without sacrificing maximal atmospheric mixing and exact zero value of reactor angle. This amounts to minimal deviation from $S3$ symmetry while preserving 2-3 symmetry (1). We confine the present analysis in inverted as well as normal hierarchical neutrino mass models. At the end we attempt to give some simple realisations of the models, pending a complete derivation at the Lagrangian level using some symmetry.

2. Analysis of inverted hierarchical models

We have in general two types [10,11,16] of inverted hierarchical models based on the relative sign of the first two mass eigenvalues m_1 and m_2: Type-IHA for same CP parity $(m_1, m_2, \pm m_3)$ and Type-IHB for opposite CP parity $(m_1, -m_2, m_3)$. Type-IHB is found to be more stable under radiative corrections in MSSM [17–19], whereas Type-IHA is more stable under the presence of left-handed Higgs triplet term in Type-II see-saw mechanism [20]. For our present analysis, we will not address the issue of stability of neutrino mass model. Instead, we explore the properties of these two types of inverted hierarchical mass matrices and their connections.

2.1 *Inverted hierarchy of Type-IHA*

We start with a specific choice of the parametrisation of mass matrix which can lead to inverted hierarchy of Type-IHA having 2-3 symmetry (1) as

$$M_{\rm IHA} = \begin{pmatrix} 1-2\epsilon & -\epsilon & -\epsilon \\ -\epsilon & \frac{1}{2} & \frac{1}{2}-\eta \\ -\epsilon & \frac{1}{2}-\eta & \frac{1}{2} \end{pmatrix} m_0, \tag{8}$$

where the symmetry breaking parameters are $\epsilon, \eta < 1$ and $m_0 = 0.05$ eV as imput value [10]. Such left-handed Majorana mass matrix can be realised in the canonical see-saw formula using a generalised diagonal form of Dirac mass matrix $m_{\rm LR}$ and non-diagonal form of right-handed Majorana mass matrix $M_{\rm RR}$ [10]. $M_{\rm IHA}$ in eq. (8) can be reduced to the zeroth order texture [16] when $\epsilon = \eta = 0$, and the resulting zeroth-order mass matrix has a degeneracy in the first two mass eigenvalues, $(1, 1, 0)m_0$, and such degeneracy makes the solar mixing angle θ_{12} arbitrary, and may have infinite values lying between 0° and $\pi/4$. Once the degeneracy is removed as in eq. (8), the solar angle is then fixed at a particular value. Such freedom in fixing the solar angle does not destroy 2-3 symmetry of the mass matrix, and it depends absolutely on the choice of input values of η and ϵ, without disturbing the predictions on atmospheric angle $\theta_{23} = \pi/4$ and reactor angle $\theta_{13} = 0^\circ$. Diagonalising (6) we obtain the three mass eigenvalues:

$$m_1 = (2 - 2\epsilon - \eta - y)\frac{m_0}{2}, \quad m_2 = (2 - 2\epsilon - \eta + y)\frac{m_0}{2}, \quad m_3 = \eta m_0, \tag{9}$$

where

$$y^2 = 12\epsilon^2 - 4\epsilon\eta + \eta^2. \tag{10}$$

The solar mixing is now fixed by the relation [7]

$$\tan 2\theta_{12} = \frac{2\sqrt{2}}{2 - (\eta/\epsilon)}, \tag{11}$$

where the flavour twister term η/ϵ in eq. (11), plays an important role. For tri-bimaximal mixing, we find two solutions of η/ϵ at 1 and 3 respectively. Similarly, for deviation from tri-bimaximal solar mixing to lower values, we have the corresponding values of flavour twister: $\eta/\epsilon < 1$ and $\eta/\epsilon > 3$. Once the solar angle is fixed, then the range of η or ϵ can be solved through a search programme. Table 1 represents a summary of our results.

2.2 *Inverted hierarchy of Type-IHB*

For the solution of eq. (8) in the range $\epsilon > 0.5$, a possible transition from inverted hierarchy with even CP parity (Type-IHA) to inverted hierarchy with odd parity (Type-IHB) in the first two mass eigenvalues, can be observed through the following parametrisation:

Table 1. Prediction of the solar mixing angle $\tan^2\theta_{12}$ and its deviation from tri-bimaximal mixings, along with other predictions on Δm^2_{21} and Δm^2_{23} in inverted hierarchical model. Within a given value of $\tan^2\theta_{12}$, the three cases in order represent A, B, and C as shown in numerical demonstrations of the text.

$\tan^2\theta_{12}$	η/ϵ	Range of η	Δm^2_{21} (10^{-5} eV^2)	Δm^2_{23} (10^{-3} eV^2)
0.5	1.0	0.0048 − 0.0064	7.15 − 9.51	2.50 − 2.50
0.5	1.0	0.6607 − 0.6618	9.50 − 7.20	1.41 − 1.41
0.5	3.0	−0.0187 − −0.0142	9.41 − 7.20	2.63 − 2.60
0.45	0.8405	0.0040 − 0.0053	7.29 − 9.54	2.5 − 2.5
0.45	0.8405	0.5865 − 0.5878	9.52 − 7.27	2.03 − 2.03
0.45	3.16	−0.0193 − −0.0147	9.47 − 7.24	2.63 − 2.60
0.35	0.4462	0.0020 − 0.0026	7.22 − 9.30	2.51 − 2.51
0.35	0.4462	0.3622 − 0.3628	9.43 − 7.22	4.01 − 4.01
0.35	3.55	−0.0206 − −0.0157	9.50 − 7.20	2.63 − 2.60

$$\delta_1 = 2\left(1-\frac{1}{2\epsilon}\right),\ \delta_2 = -\frac{1}{2\epsilon},\ \delta_3 = \left(\frac{\eta}{\epsilon}-\frac{1}{2\epsilon}\right),\quad m_0' = m_0(-\epsilon). \tag{12}$$

Thus the mass matrix in eq. (8) becomes Type-IHB [8],

$$M_{\rm IHB} = \begin{pmatrix} \delta_1 & 1 & 1 \\ 1 & \delta_2 & \delta_3 \\ 1 & \delta_3 & \delta_2 \end{pmatrix} m_0', \tag{13}$$

where $\delta_{1,2,3}$ are smaller than unity. The zeroth order mass matrix of eq. (11) has the form [16]

$$M^0_{\rm IHB} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} m_0', \tag{14}$$

which has non-degenerate eigenvalues $(1,-1,0)m_0'$ compared to that of eq. (8). In addition, the solar mixing is now fixed at the maximal value ($\theta_{12}=\pi/4$) unlike eq. (8) where it takes an arbitrary value. The diagonalisation of (13) leads to the following mass eigenvalues:

$$m_{1,2} = (\delta_1+\delta_2+\delta_3 \pm x)\frac{m_0'}{2},\quad m_3 = (\delta_2-\delta_3)m_0', \tag{15}$$

where

$$x^2 = 8 + (\delta_1^2+\delta_2^2+\delta_3^2) - 2\delta_1\delta_2 - 2\delta_1\delta_3 + 2\delta_2\delta_3; \tag{16}$$

and the solar mixing,

$$\tan 2\theta_{12} = \frac{2\sqrt{2}}{(\delta_1-\delta_2-\delta_3)} \tag{17}$$

which leads to the same expression in eq. (11).

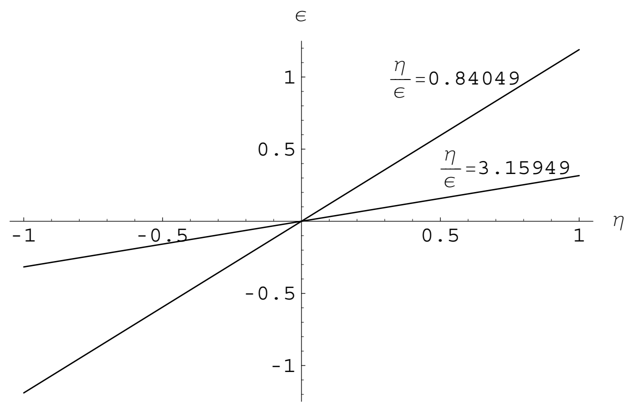

Figure 1. Two solutions of $\tan^2\theta_{12} = 0.45$ for inverted hierarchical model corresponding to the flavour twister $(\eta/\epsilon) = 0.8405$ and 3.16 respectively.

Though the inverted hierarchical model with odd CP (Type-IHB) generally predicts nearly maximal solar mixing which requires corrections from charged lepton mass matrix [21] to tone down its value, the present form in (13) has the ability to predict lower values of solar angle without such correction from charged lepton sector. This possibility is due to our parametrisation in eq. (12) and proper choice of input values of parameters. Such novel procedures do not sacrifice the predictions of maximal atmospheric mixing angle and zero reactor angle.

2.3 *Numerical analysis*

We follow two important steps for carrying out numerical estimations. In the first step, we choose a specific value of solar mixing $\tan^2\theta_{12}$ via eq. (11) or (17), and then solve for possible values of the ratio η/ϵ. In the second step we take up a particular value of this ratio η/ϵ, and find out the ranges of either η or ϵ for the given ranges of Δm^2_{21} and Δm^2_{23} which are consistent with observational data [1].

For a demonstration, we present here the numerical estimations for the value of solar mixing $\tan^2\theta_{12} = 0.45$ which in turn corresponds to two values of $r = \eta/\epsilon$ at $r = 0.8405$ and $r = 3.16$ derived from eq. (11). The first value leads to two ranges of η as (A) $0.0040 \le \eta \le 0.0053$ and (B) $0.5865 \le \eta \le 0.5878$, whereas the second one has only one range of η as (C) $-0.0193 \le \eta \le -0.0147$. As discussed before, case (B) belongs to inverted hierarchy (Type-IHB) and cases (A,C) belong to Type-IHA. However, the expressions for eigenvalues and solar mixing angle are the same. We use standard procedure to estimate neutrino masses and mixings with mid-values of η and ϵ of the corresponding range [10,11].

Case (A): *Type-IHA where mass eigenvalues are of the form* (m_1, m_2, m_3). Using the value $\eta = 0.0046$ and $\epsilon = 0.0055$, we have

$$m_{\rm LL} = M_{\rm IHA} = \begin{pmatrix} 0.0495 & -0.0003 & -0.0003 \\ -0.0003 & 0.025 & 0.0248 \\ -0.0003 & 0.0248 & 0.025 \end{pmatrix}. \tag{18}$$

Diagonalising the above mass matrix we have three mass eigenvalues:

$$m_i = (0.0492, 0.0500, 0002) \text{ eV}, \quad i = 1, 2, 3$$

leading to $\Delta m^2_{21} = 8.35 \times 10^{-5}$ eV^2 and $\Delta m^2_{23} = 2.50 \times 10^{-3}$ eV^2. The MNS mixing matrix is extracted as

$$U_{\text{MNS}} = \begin{pmatrix} 0.8308 & 0.5566 & 0.0 \\ 0.3936 & -0.5875 & -0.7071 \\ 0.3936 & -0.5875 & 0.7072 \end{pmatrix} \tag{19}$$

which gives $\tan^2\theta_{12} = 0.45$, $\tan^2\theta_{23} = 1$ and $\sin\theta_{13} = 0$.

Case (B): *Type-IHB where the mass eigenvalues are of the form* $(-m_1, m_2, m_3)$. For the input value $\eta = 0.5872$ and $\epsilon = 0.6986$ we have

$$m_{\text{LL}} = M_{\text{IHB}} = \begin{pmatrix} -0.0199 & -0.0349 & -0.0349 \\ -0.0349 & 0.025 & -0.0044 \\ -0.0349 & -0.0044 & 0.025 \end{pmatrix}. \tag{20}$$

Diagonalising the above mass matrix we have three mass eigenvalues:

$$m_i = (-0.0530, 0.0538, 0.0294) \text{ eV}, \quad i = 1, 2, 3$$

leading to $\Delta m^2_{21} = 8.33 \times 10^{-5}$ eV^2 and $\Delta m^2_{23} = 2.03 \times 10^{-3}$ eV^2. The model is quite different from degenerate model where the overall magnitude of neutrino masses is of the order of 0.4 eV. The MNS mixing matrix is extracted as

$$U_{\text{MNS}} = \begin{pmatrix} -0.8305 & 0.5571 & 0.0 \\ -0.3939 & -0.5872 & -0.7071 \\ -0.3939 & -0.5872 & 0.7072 \end{pmatrix}, \tag{21}$$

which gives $\tan^2\theta_{12} = 0.45$, $\tan^2\theta_{23} = 1$ and $\sin\theta_{13} = 0$.

Case (C): *Type-IHA where the mass eigenvalues are of the form* $(m_1, m_2, -m_3)$. For the input value $\eta = -0.0170$ and $\epsilon = -0.0054$ we have

$$m_{\text{LL}} = M_{\text{IHA}} = \begin{pmatrix} 0.0505 & 0.0003 & 0.0003 \\ 0.0003 & 0.025 & 0.0259 \\ 0.0003 & 0.0259 & 0.025 \end{pmatrix}. \tag{22}$$

Diagonalising the above mass matrix we have three mass eigenvalues:

$$m_i = (0.0503, 0.0511, -0.0009) \text{ eV}, \quad i = 1, 2, 3$$

leading to $\Delta m^2_{21} = 8.35 \times 10^{-5}$ eV^2 and $\Delta m^2_{23} = 2.61 \times 10^{-3}$ eV^2. The MNS mixing matrix is extracted as

$$U_{\text{MNS}} = \begin{pmatrix} 0.8305 & -0.5571 & 0.0 \\ -0.3939 & -0.5872 & -0.7071 \\ -0.3939 & -0.5872 & 0.7072 \end{pmatrix} \tag{23}$$

which gives $\tan^2\theta_{12} = 0.45$, $\tan^2\theta_{23} = 1$ and $\sin\theta_{13} = 0$.

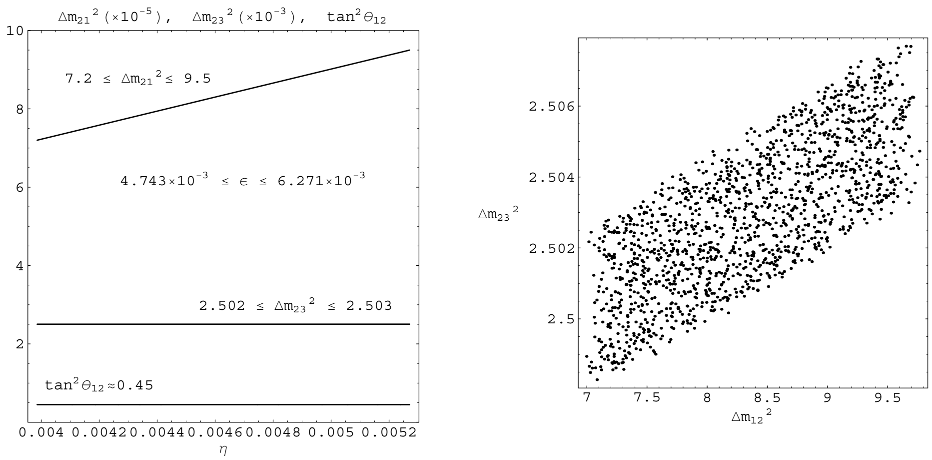

Figure 2. Predictions on Δm^2_{21} in the unit (10^{-5} eV^2) and Δm^2_{23} in the unit (10^{-3} eV^2) for the value $\tan^2\theta_{12} = 0.45$ in the range $0.0040 \le \eta \le 0.0053$ and the corresponding correlation graph for inverted hierarchical model.

We present our calculations in table 1 for all possible ranges of η and ϵ leading to different values of solar mixing $\tan^2\theta_{12}$ at 0.5 for tri-bimaximal mixing, and then 0.45 and 0.35 as possible deviations from tri-bimaximal mixing. The present analysis shows a wide scope for lowering the solar mixing angle without sacrificing predictions $\tan^2\theta_{23} = 1$ and $\sin\theta_{13} = 0$. It is interesting to note that only two parameters η and ϵ play the key roles in the whole analysis. For each value of $\tan^2\theta_{12}$ we have three solutions corresponding to two values of η/ϵ, and every solution has a particular range of η. These values satisfy observed ranges of Δm^2_{21} and Δm^2_{23}.

As a representative example, we present in figure 1 the graphical solution of the ratio η/ϵ corresponding to $\tan^2\theta_{12} = 0.45$. In figures 2–4 we summarise all the results of the calculation corresponding to $\tan^2\theta_{12} = 0.45$ case. In particular, figure 2 presents a graphical summary of the predictions on Δm^2_{21} and Δm^2_{23}, and a corresponding correlation graph between them for the valid range $0.0040 \le \eta \le 0.0053$. Similarly, figures 3 and 4 present corresponding correlation graphs for $0.5865 \le \eta \le 0.5878$ and $-0.0193 \le \eta \le -0.0147$ ranges, respectively.

3. Mass matrix for normal hierarchical mass model

We propose a form of the normal hierarchical neutrino mass matrix having the 2-3 symmetry,

$$m_{\mathrm{LL}} = \begin{pmatrix} -\eta & -\epsilon & -\epsilon \\ -\epsilon & 1-\epsilon & -1 \\ -\epsilon & -1 & 1-\epsilon \end{pmatrix} m_0 \tag{24}$$

and the neutrino mass eigenvalues are given by

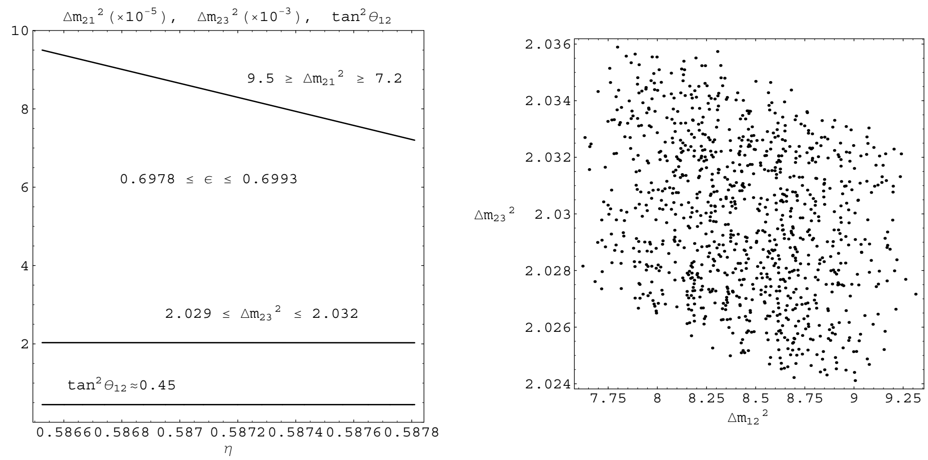

Figure 3. Predictions on Δm_{21}^2 in the unit (10^{-5} eV^2) and Δm_{23}^2 in the unit (10^{-3} eV^2) for the value $\tan^2\theta_{12} = 0.45$ in the valid range $0.5865 \leq \eta \leq 0.5878$ and the corresponding correlation graph for inverted hierarchical model.

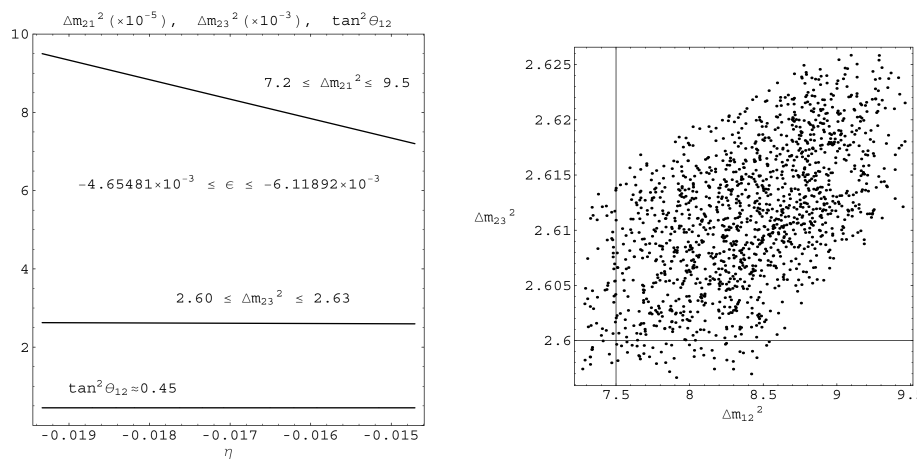

Figure 4. Predictions on Δm_{21}^2 in the unit (10^{-5} eV^2) and Δm_{23}^2 in the unit (10^{-3} eV^2) for the value $\tan^2\theta_{12} = 0.45$ in the valid range $-0.0193 \leq \eta \leq -0.0147$ and the corresponding correlation graph for inverted hierarchical model.

$$m_1 = \frac{1}{2}m_0(-\epsilon - \eta + \sqrt{9\epsilon^2 - 2\epsilon\eta + \eta^2}),$$
$$m_2 = \frac{1}{2}m_0(-\epsilon - \eta - \sqrt{9\epsilon^2 - 2\epsilon\eta + \eta^2}), \quad m_3 = (2-\epsilon)m_0, \tag{25}$$

where $m_0 = 0.03$ eV. The solar mixing angle is given by

$$\tan 2\theta_{12} = \frac{2\sqrt{2}}{(\eta/\epsilon) - 1}. \tag{26}$$

Table 2. Choice of η and ϵ for $\tan^2\theta_{12} \simeq 0.50$ along with other predictions on Δm^2_{21} and Δm^2_{23} in normal hierarchical model.

$\tan^2\theta_{12}$	η/ϵ	ϵ	Δm^2_{21} (10^{-5} eV^2)	Δm^2_{23} (10^{-3} eV^2)
0.4900	−0.0303	0.165	7.15	2.93
0.4913	−0.0265	0.189	9.42	2.82
0.4960	−0.0122	0.164	7.18	2.94
0.4965	−0.0106	0.188	9.45	2.83
0.4980	−0.007	0.164	7.22	2.94
0.4982	−0.0053	0.188	9.49	2.93
0.4990	−0.0030	0.164	7.24	2.94
0.4991	−0.0027	0.188	9.52	2.83

Table 3. Lowering of solar angle from tri-bimaximal mixings in normal hierarchical model.

$\tan^2\theta_{12}$	η/ϵ	ϵ	Δm^2_{21} (10^{-5} eV^2)	Δm^2_{23} (10^{-3} eV^2)
0.45	−0.1595	0.1762 − 0.2025	7.18 − 9.48	2.89 − 2.77
0.40	−0.3416	0.197 − 0.2263	7.19 − 9.49	2.80 − 2.67
0.35	−0.5538	0.2356 − 0.2707	7.19 − 9.49	2.63 − 2.47

For tri-bimaximal mixing $\tan^2\theta_{12} = 0.5$, we have $\tan 2\theta_{12} = -2\sqrt{2}$ leading to $\eta = 0$ [2,8]. For such a case, eq. (22) reduces to the mass matrix in eq. (6). Deviation from the tri-bimaximal mixing can be realised for $(\eta/\epsilon) \leq 0$ where (η/ϵ) is the flavour twister term. In table 2 we present the neutrino mass parameters for nearly tri-bimaximal mixings, and in table 3 for lowering solar mixing angles through flavour twister term.

4. Realisation of the neutrino mass models

It is important to examine how the neutrino mass matrices in eqs (8) and (24) are realised in practice, and how the deviations from tri-bimaximal mixings can be achieved without destroying 2-3 symmetry.

4.1 *Inverted hierarchical model*

In order to realise the mass matrix in eq. (8), we start with two parts of neutrino mass matrix, $m_{\mathrm{LL}} = m^o_{\mathrm{LL}} + \Delta m_{\mathrm{LL}}$, which can be diagonalised by tri-bimaximal mixing matrix (3). For the inverted hierarchy the structure of the dominant term m^o_{LL} having 2-3 symmetry is given by

$$m_{\mathrm{LL}}^{o} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m_0 \tag{27}$$

which is diagonalised as

$$O_{23}^{\mathrm{T}} m_{\mathrm{LL}}^{o} O_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} m_0. \tag{28}$$

The second perturbative term Δm_{LL} can also be diagonalised by $(O_{23}O_{12})$,

$$\Delta m_{\mathrm{LL}} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} m_0(-\eta), \tag{29}$$

where η is a very small parameter. The diagonalisation with tri-bimaximal mixing matrix (3),

$$(O_{23}O_{12})^{\mathrm{T}} \Delta m_{\mathrm{LL}} (O_{23}O_{12}) = O_{12}^{\mathrm{T}} (O_{23}^{\mathrm{T}} \Delta m_{\mathrm{LL}} O_{23}) O_{12} \tag{30}$$

gives

$$O_{12}^{\mathrm{T}} O_{23}^{\mathrm{T}} \begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} O_{23} O_{12} m_0(-\eta)$$

$$= O_{12}^{\mathrm{T}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ \sqrt{2} & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} O_{12} m_0(-\eta)$$

$$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} m_0(-\eta). \tag{31}$$

Thus, from eqs (28) and (31), the diagonalisation of the total mass matrix, $U_{\mathrm{TBM}}^{\mathrm{T}} m_{\mathrm{LL}} U_{\mathrm{TBM}} = O_{23}^{\mathrm{T}} m_{\mathrm{LL}}^{o} O_{23} + (O_{23}O_{12})^{\mathrm{T}} \Delta m_{\mathrm{LL}} (O_{23}O_{12})$ leads to

$$\begin{pmatrix} 1-3\eta & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \eta \end{pmatrix} m_0. \tag{32}$$

The deviation of solar angle from tri-bimaximal mixings can be introduced through the replacement Δm_{LL} by $\Delta m'_{\mathrm{LL}}$ using a flavour twister $x = \epsilon/\eta$ where

$$\Delta m'_{\mathrm{LL}} = \begin{pmatrix} 2x & x & x \\ x & 0 & 1 \\ x & 1 & 0 \end{pmatrix} m_0(-\eta) \tag{33}$$

which still has 2-3 symmetry. This can be diagonalised by O_{23} but O_{12} is no longer valid and it is now replaced by a new matrix O'_{12}. Thus $O'^{\mathrm{T}}_{12}(O^{\mathrm{T}}_{23}\Delta m'_{\mathrm{LL}}O_{23})O'_{12}$ leads to

$$O'^{\mathrm{T}}_{12}O^{\mathrm{T}}_{23}\begin{pmatrix} 2x & x & x \\ x & 0 & 1 \\ x & 1 & 0 \end{pmatrix} O_{23}O'_{12}m_0(-\eta)$$

$$= O'^{T}_{12}\begin{pmatrix} 2x & 2x/\sqrt{2} & 0 \\ 2x/\sqrt{2} & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} O'_{12}m_0(-\eta)$$

$$= \begin{pmatrix} (1+2x+y)/2 & 0 & 0 \\ 0 & (1+2x-y)/2 & 0 \\ 0 & 0 & -1 \end{pmatrix} m_0(-\eta), \tag{34}$$

where $y = \sqrt{1-4x+12x^2}$ and the new O'_{12} is obtained as

$$O'_{12} = \begin{pmatrix} \frac{-1+2x+y}{\sqrt{(-1+2x+y)^2+(2\sqrt{2}x)^2}} & \frac{-1+2x-y}{\sqrt{(-1+2x-y)^2+(2\sqrt{2}x)^2}} & 0 \\ \frac{2\sqrt{2}x}{\sqrt{(-1+2x+y)^2+(2\sqrt{2}x)^2}} & \frac{2\sqrt{2}x}{\sqrt{(-1+2x-y)^2+(2\sqrt{2}x)^2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{35}$$

The solar angle $\tan^2\theta_{12}$ is extracted as

$$\tan^2\theta_{12} = \left(\frac{(-1+2x-y)^2}{(-1+2x+y)^2}\right)\left(\frac{(-1+2x+y)^2+(2\sqrt{2}x)^2}{(-1+2x-y)^2+(2\sqrt{2}x)^2}\right). \tag{36}$$

The corresponding new mass eigenvalues for m'_{LL} are calculated as

$$m_{1,2} = \frac{m_o}{2}[2-\eta(1+2x\pm y)]; \quad m_3 = \eta m_o. \tag{37}$$

After substitution of $x = \epsilon/\eta$ in eqs (36) and (37), we find that these relations are exactly same as those given in eqs (9) and (11), and the mass matrix (8) can be recovered. Further, for $x = 1$, we get $O_{12} = O'_{12}$ leading to $\tan^2\theta_{12} = 0.5$. These equations are consistent with the data in table 1 for the case of inverted hierarchy Type-IHB.

4.2 *Normal hierarchical model*

In the case of normal hierarchy (24), we start with two parts of neutrino mass matrix, $m_{\mathrm{LL}} = m^o_{\mathrm{LL}} + \Delta m_{\mathrm{LL}}$, which can be diagonalised by tri-bimaximal mixing matrix (3). The structure of the dominant term m^o_{LL} having 2-3 symmetry, can be taken as

$$m^o_{\mathrm{LL}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} m_0 \tag{38}$$

which can be diagonalised by

$$O_{23}^{\mathrm{T}} m_{\mathrm{LL}}^{o} O_{23} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} m_0. \tag{39}$$

However, the second term Δm_{LL} which can be diagonalised by $(O_{23}O_{12})$, can be taken as

$$\Delta m_{\mathrm{LL}} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} m_0(-\epsilon), \tag{40}$$

where ϵ is a very small real parameter. The diagonalisation of eq. (40) with tri-bimaximal mixing matrix, $(O_{23}O_{12})^{\mathrm{T}} \Delta m_{\mathrm{LL}} (O_{23}O_{12}) = O_{12}^{\mathrm{T}} (O_{23}^{\mathrm{T}} \Delta m_{\mathrm{LL}} O_{23}) O_{12}$ leads to

$$O_{12}^{\mathrm{T}} O_{23}^{\mathrm{T}} \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} O_{23} O_{12} m_0(-\epsilon)$$

$$= O_{12}^{\mathrm{T}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ \sqrt{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} O_{12} m_0(-\epsilon)$$

$$= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_0(-\epsilon). \tag{41}$$

From eqs (39) and (41) the diagonalisation, $U_{\mathrm{TBM}}^{\mathrm{T}} m_{\mathrm{LL}} U_{\mathrm{TBM}} = O_{23}^{\mathrm{T}} m_{\mathrm{LL}}^{o} O_{23} + (O_{23}O_{12})^{\mathrm{T}} \Delta m_{\mathrm{LL}} (O_{23}O_{12})$ leads to

$$\begin{pmatrix} -3\epsilon & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (2-\epsilon) \end{pmatrix} m_0. \tag{42}$$

The deviation of solar angle from tri-bimaximal mixings can be done through the replacement Δm_{LL} by $\Delta m'_{\mathrm{LL}}$ using a flavour twister $x = \eta/2\epsilon$,

$$\Delta m'_{\mathrm{LL}} = \begin{pmatrix} 2x & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} m_0(-\epsilon) \tag{43}$$

which still has 2-3 symmetry and can be diagonalised by O_{23}. Thus $O_{12}'^{\mathrm{T}} (O_{23}^{\mathrm{T}} \Delta m'_{\mathrm{LL}} O_{23}) O'_{12}$ leads to

$$O_{12}'^{\mathrm{T}} O_{23}^{\mathrm{T}} \begin{pmatrix} 2x & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} O_{23} O'_{12} m_0(-\epsilon)$$

$$= O_{12}'^{\mathrm{T}} \begin{pmatrix} 2x & \sqrt{2} & 0 \\ \sqrt{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} O_{12}' m_0(-\epsilon)$$

$$= \begin{pmatrix} (1+2x-y)/2 & 0 & 0 \\ 0 & (1+2x+y)/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_0(-\epsilon), \tag{44}$$

where $y = \sqrt{9-4x+4x^2}$ and the new O_{12}' is obtained as

$$O_{12}' = \begin{pmatrix} \frac{1-2x+y}{\sqrt{(1-2x+y)^2+(2\sqrt{2})^2}} & \frac{-1+2x+y}{\sqrt{(1-2x-y)^2+(2\sqrt{2})^2}} & 0 \\ \frac{2\sqrt{2}}{\sqrt{(1-2x+y)^2+(2\sqrt{2})^2}} & \frac{2\sqrt{2}}{\sqrt{(1-2x-y)^2+(2\sqrt{2})^2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{45}$$

The solar angle $\tan^2\theta_{12}$ is extracted as

$$\tan^2\theta_{12} = \left(\frac{(-1+2x+y)^2}{(1-2x+y)^2}\right)\left(\frac{(1-2x+y)^2+(2\sqrt{2})^2}{(1-2x-y)^2+(2\sqrt{2})^2}\right). \tag{46}$$

The corresponding new mass eigenvalues for m_{LL}' are

$$m_{1,2} = \frac{m_o}{2}[-\epsilon \pm \eta \pm \epsilon y]; \quad m_3 = (2-\epsilon)m_o. \tag{47}$$

After substitution of $x = \eta/(2\epsilon)$ in eqs (46) and (47) we recover the earlier results in eqs (25) and (26), and the mass matrix in eq. (24). For the value $x = 0$, we have the tri-bimaximal condition $O_{12} = O_{12}'$ leading to $\tan^2\theta_{12} = 0.5$. These new relations are numerically consistent with the results in tables 2 and 3.

Though the above analysis demonstrates how the choices of perturbation terms are made for some viable variations of the tri-bimaximal mixing pattern for allowed range of x, it will be more important if such mass matrices are derived from the Lagrangian level based on some appropriate symmetry considerations. Such extension will be an ambitious project and is beyond the scope of the present work. Other similar models with either $m_1 = 0$ or $m_3 = 0$, have also been reported in [22].

5. Summary and discussions

We summarise the main points in this work. The proposed inverted and normal hierarchical neutrino mass matrices having 2-3 symmetry has the potential to predict tri-bimaximal mixings. It also imparts a possible mechanism to lower the solar mixing angle from its tri-bimaximal value, without sacrificing predictions on maximal atmospheric mixing angle and zero reactor angle. Since no correction is taken from charged lepton mass matrix, the desired 2-3 symmetry is always maintained in the neutrino mass matrix. We have shown that two types of inverted hierarchical models (Types IHA and IHB) are found to have same origin of mass matrix but the range of input values are different.

A few discussions are in order. We once again emphasise here the basic differences between the present work and other contemporary attempts on deviations from tri-bimaximal neutrino mixings available in the literature. In refs [23,24] the breaking of 2-3 symmetry generally leads to $\theta_{13} \neq 0°$, $\theta_{23} \neq 45°$ and a non-trivial CP-violating phase δ. In special cases, if the 2-3 symmetry is softly broken, either $\theta_{13} = 0°$ or $\theta_{23} = 0°$ might survive [24] but not both at all. However, in ref. [22], deviation from tri-bimaximal mixing has been achieved along with the conditions $\theta_{13} = 0°$ and $\theta_{23} = 45°$. In such approach either $m_1 = 0$ or $m_3 = 0$ condition has been used, and is entirely different from the present work. Deviations from tri-bimaximal mixings using radiative corrections with Planck scale effects [25] or threshold corrections [26], and also from diagonalisation of charged lepton sector [21], are not discussed here. In short the present attempt is unique in its approach in the sense that it does not destroy 2-3 symmetry. Extension to degenerate neutrino mass models, stability analysis under radiativie corrections, and application to baryon asymmetry, are in progress.

The present analysis has relevance in the context of quark-lepton complementarity scenario which would be true at low energy scale. It is known that the renormalisation group running of solar angle always increases its value from high scale to the low scale in MSSM [27]. The present work has enough scope to predict solar angle lower than tri-bimaximal value at high scale and this can accommodate the RG running effects at lower energy compared to data. The present analysis though phenomenological, may have important implications in model buildings [28] on tri-bimaximal mixings and its possible deviations, based on various discrete symmetries as well as non-Abelian gauge groups.

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