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Analytical prediction of forced convective heat transfer of fluids embedded with nanostructured materials (nanofluids)

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Abstract. Nanofluids are a new class of heat transfer fluids developed by suspending nanosized solid particles in liquids. Larger thermal conductivity of solid particles compared to the base fluid such as water, ethylene glycol, engine oil etc. significantly enhances their thermal properties. Several phenomenological models have been proposed to explain the anomalous heat transfer enhancement in nanofluids. This paper presents a systematic literature survey to exploit the characteristics of nanofluids, viz., thermal conductivity, specific heat and other thermal properties. An empirical correlation for the thermal conductivity of Al_2O_3 + water and Cu + water nanofluids, considering the effects of temperature, volume fraction and size of the nanoparticle is developed and presented. A correlation for the evaluation of Nusselt number is also developed and presented and compared in graphical form. This enhanced thermophysical and heat transfer characteristics make fluids embedded with nanomaterials as excellent candidates for future applications.

Keywords. Nanofluids; thermal properties; conductivity models; empirical relation.

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1. Introduction

Nanotechnology has been widely used in traditional industry because materials with grain size of nanometers posses unique optical, electrical and chemical properties. An innovative utility of this emerging technology is that nanoparticles can be dispersed in conventional heat transfer fluids such as water, glycol or oil to produce a new class of high efficiency heat exchange media [1]. The key idea is to exploit the very high thermal conductivities of solid particles which can be hundreds or even thousands of times greater than those of fluids. The superior properties of nanoparticle fluid mixtures relative to those of fluids without particles or with large-sized

particles include high thermal conductivities, stability and prevention of clogging in microchannels. It is well-known that metals in solid form have thermal conductivities that are much higher than those of fluids. For example, the thermal conductivity of copper at room temperature is about 700 times and Al is 400 times greater than that of water. Therefore, fluids containing suspended solid metallic particles are expected to display significantly enhanced thermal conductivities relative to those of conventional heat transfer fluids. Despite several attempts a satisfactory explanation for increase in thermal conductivity of nanofluid is yet unclear [2]. However, the ballistic nature of heat transfer in nanoparticles combined with direct or fluid-mediated clustering effects provide the path for rapid heat transfer and are key factors for promoting the thermal properties of nanofluids. The Brownian motion of particles is found to be too slow to transport significant amount of heat through a nanofluid. The study of heat transport in solid dispersion is relatively recent. Ahuja [3] showed that sub-micron polystyrene suspensions in aqueous glycerin increased the heat transfer by two times under laminar flow conditions. Further, the difference in pressure drop is found negligible even at 9% volume fraction.

With the advent of nanotechnology in the early nineties, it become possible to manufacture nanosized particles. These particles, due to their extremely small sizes, can form very stable colloidal systems which are currently known as nanofluids. The first heat transfer enhancement with nanosized particle was reported by Masuda *et al* [4] in Japan. They demonstrated that ultrafine suspensions of alumina, silica and other oxides in water increase the thermal conductivity by a substantial amount for a particle volume fraction of 4.3%. Choi *et al* [1] at the Argonne National Laboratory proposed to construct a new class of engineered fluids with superior heat transfer capabilities in 1995. Wang *et al* [5] reported enhanced thermal conductivity for alumina and cupric oxide with a variety of base fluids including water and ethylene glycol. With alumina particles they observed a maximum of 12% increase in the conductivity at a volume fraction of 3%. The viscosity on the other hand showed an increase of 20–30% for the same volume fraction. Das *et al* [6] measured the conductivities of alumina and cupric oxide for different temperatures ranging from 20 to 50°C and found linear increase in the conductivity ratio with temperature. However, for the same loading fraction the ratio of increase was higher for cupric oxide than for alumina. Eastman *et al* [7] showed that 10 nm copper particles in ethylene glycol could enhance the conductivity by 40% with very low particle loading fraction. With cupric oxide (35 nm) the enhancement was 20% for a volume fraction of 4%. Keblinski *et al* [8] reported that the nanoparticle range of interest lies between 1 and 50 nm. Practically it was demonstrated that solid nanoparticle colloids with grains of diameter less than 50 nm are extremely stable and exhibit no significant settling under static conditions even after a few weeks. When the particle size is less than 1 nm the diffusive heat transport ballistic phonon effects become invalid which consequently lead to insignificant increase in thermal conductivity.

2. Thermal properties of nanofluids

The thermal conductivity measurement of nanofluids was the main focus in the early stages of nanofluid research. Recently, studies have been carried out on the heat

transfer coefficient of nanofluids in natural [9–11] and forced flows. Most studies carried out to date are limited to the thermal characterization of nanofluids without phase change (boiling, evaporation, or condensation). However, nanoparticles in nanofluids can play a vital role in two-phase heat transfer systems, and there is a great need to characterize nanofluids in boiling and condensation heat transfer. Das *et al* [9] initiated experiments on the boiling characteristics of nanofluids.

In convection heat transfer, the heat transfer coefficient depends not only on the thermal conductivity but also on other properties such as the specific heat, density, and dynamic viscosity of a nanofluid. The various thermal properties of nanofluids can be evaluated as follows.

2.1 *Density*

The density of a nanofluid can be calculated by

$$\rho_{\mathrm{nf}} = (1-\phi)\rho_{\mathrm{f}} + \phi\rho_{\mathrm{p}}. \tag{1}$$

For typical nanofluids with nanoparticles at a value of volume fraction less than 1%, a change of less than 5% in the fluid density is expected.

2.2 *Specific heat*

The specific heat C_{nf} of a nanofluid can be calculated by

$$C_{\mathrm{nf}} = \frac{(1-\phi)\rho_{\mathrm{f}}C_{\mathrm{f}} + \phi\rho_{\mathrm{p}}C_{\mathrm{p}}}{\rho_{\mathrm{nf}}}. \tag{2}$$

Using these equations, one can predict that small decreases in specific heat will typically result when solid particles are dispersed in liquids. For example, adding 3% Al_2O_3 by volume to water leads to decrease the specific heat by approximately 7–8% compared with that of water alone. The simple equations obtained by using energy balance as above may need to be modified if nanoparticles are found to exhibit a size-dependent specific heat.

2.3 *Viscosity*

Wang *et al* [5] measured the viscosity of water-based nanofluids containing Al_2O_3 nanoparticles dispersed by different dispersion techniques and showed that nanofluids have lower viscosities when the particles are better dispersed. They also showed an increase of about 30% in viscosity at 3 vol.% Al_2O_3, compared with that of water alone. However, the viscosity of the Al_2O_3/water nanofluids prepared by Pak and Cho [12] was three times higher than that of water. For metallic nanofluids containing a low volume fraction of nanoparticles (usually <0.01), Einstein proposed a model that would predict the viscosity as

$$\mu_{\rm nf} = (1 + 2.5\phi)\mu_{\rm f}. \tag{3}$$

Das *et al* [6] measured the viscosity of Al_2O_3 in water and CuO in water nanofluids as a function of shear rate and showed Newtonian behavior of the nanofluids for a range of volume fractions between 1% and 4%.

Brinkman [13] has modified eq. (3) into more generalized form as

$$\mu_{\rm nf} = \frac{\mu_{\rm f}}{(1-\phi)^{2.5}}. \tag{4}$$

Wang *et al* [5] gave a correlation as follows and compared it with various other models and presented in graphical form as shown in figure 1.

For water–Al_2O_3 nanofluid,

$$\mu_{\rm nf} = 123\phi^2 + 7.3\phi + 1. \tag{5}$$

Pak and Cho [12] gave correlation for the viscosity of nanofluids as follows:

For water–Al_2O_3,

$$\mu_{\rm nf} = \mu_{\rm f}(1 + 39.11\phi + 533.9\phi^2). \tag{6}$$

2.4 *Thermal conductivity models*

Many theoretical and empirical models have been proposed to predict the effective thermal conductivity of nanofluids. Using potential theory, Maxwell [14] obtained a simple relationship for the conductivity of randomly distributed and non-interacting homogeneous spheres in a homogeneous medium as

$$\frac{k_{\rm nf}}{k_{\rm f}} = 1 + \frac{3(\alpha-1)\phi}{(\alpha+2) - (\alpha-1)\phi}, \tag{7}$$

where

$$\alpha = \frac{k_{\rm p}}{k_{\rm f}}.$$

Hamilton and Crosser [15] modified the Maxwell correlation as

$$\frac{k_{\rm nf}}{k_{\rm f}} = \frac{[k_{\rm p} + (n-1)k_{\rm f} - (n-1)\phi(k_{\rm f} - k_{\rm p})]}{[k_{\rm p} + (n-1)k_{\rm f} + \phi(k_{\rm f} - k_{\rm p})]}, \tag{8}$$

where the parameter n is the 'shape factor' defined as

$$n = \frac{3}{\psi}.$$

where ψ, called the 'sphericity', is defined as the ratio of the surface area of the sphere to that of the particle for the same volume. For spherical particles $\psi = 1$, and for the cylinders $\psi = 0.5$. Jang and Choi [16] found that the Brownian motion of nanoparticles at the molecular and nanoscale level is a key mechanism governing the thermal behavior of nanofluids. They derived a model, which considers

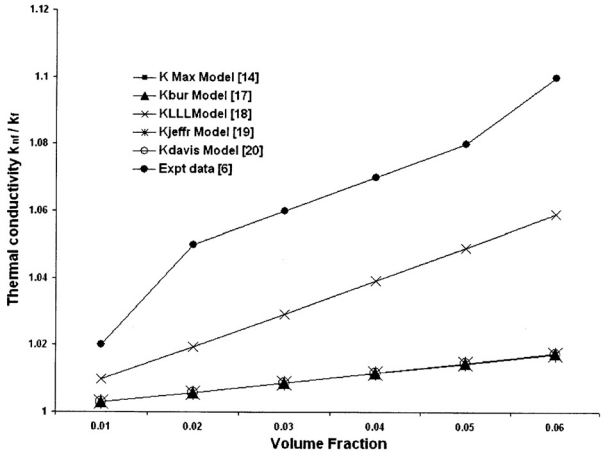

Figure 1. Comparison of the thermal conductivity of nanofluid models with the experimental data [6].

the concentration, temperature and, size. They proposed four modes of energy transport, viz., (1) collision between base fluid molecules, (2) thermal diffusion in nanoparticles suspended in fluids, (3) collision between nanoparticles and (4) thermal interactions of dynamic or dancing nanoparticles with base fluid molecules and derived the thermal conductivity of nanofluid k_{nf} by neglecting the third mode.

The above thermal conductivity models are compared with the experimental data [6] and are presented in graphical form as shown in figure 1. It is found that the predicted values of various models are far less than experimental observations. Hence the present work is aimed at developing a suitable correlation to predict the thermal conductivity of nanofluids.

3. Present model for thermal conductivity

Thus from the above models it is clear that the thermal conductivity of a nanofluid, k_{nf}, is given by

$$k_{\mathrm{nf}} = f\left[v, d_{\mathrm{p}}, \rho_{\mathrm{p}}, T, k_{\mathrm{p}}, k_{\mathrm{f}}, \phi\right]. \tag{9}$$

These variables can be grouped and can be expressed in non-dimensional terms as

$$k_{\mathrm{nf}} = f\left[\mathrm{Re}_{\mathrm{m}}, \phi, \frac{k_{\mathrm{p}}}{k_{\mathrm{f}}}\right]. \tag{10}$$

Therefore

$$\frac{k_{\mathrm{nf}}}{k_{\mathrm{f}}} = c \cdot \mathrm{Re}_{\mathrm{m}}^{\mathrm{p}} \phi^{\mathrm{q}} \left(\frac{k_{\mathrm{p}}}{k_{\mathrm{f}}}\right)^{r}, \tag{11}$$

where Re_{m} the modified Reynolds number [21] is given as $\frac{1}{\nu_{\mathrm{p}}}\sqrt{\frac{18k_{\mathrm{b}}T}{\Pi\rho_{\mathrm{p}}d_{\mathrm{p}}}}$.

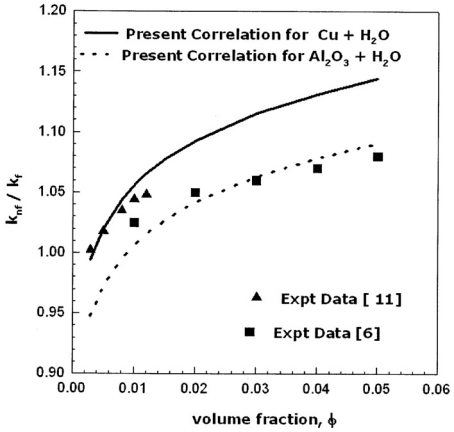

Figure 2. Comparison of the present correlation with the experimental data [6,11].

The constants in the above equation (11) are obtained by using experimental data. The data available in [6–10] for Al_2O_3 + H_2O and Cu + H_2O nanofluids for different nanoparticle sizes at different volume fractions and at different temperatures are used. Using nonlinear regression analysis an empirical correlation to predict the k of Al_2O_3 + H_2O and Cu + H_2O nanofluids is developed as

$$\frac{k_{\mathrm{nf}}}{k_{\mathrm{f}}} = \mathrm{Re}_{\mathrm{m}}\phi^{0.05}\left(\frac{k_{\mathrm{p}}}{k_{\mathrm{f}}}\right)^{0.2324} \tag{12}$$

for Al_2O_3 + H_2O and

$$\frac{k_{\mathrm{nf}}}{k_{\mathrm{f}}} = 0.74\ \mathrm{Re}_{\mathrm{m}}\phi^{0.05}\left(\frac{k_{\mathrm{p}}}{k_{\mathrm{f}}}\right)^{0.2324} \tag{13}$$

for Cu + H_2O.

Equations (12) and (13) are applicable only when volume fraction is greater than zero and valid with an average deviation of 0.8% and standard deviation of 1%. The above equations take care of the diameter of the nanoparticle, concentration and temperature effects. The correlations give good agreement with the experimental results as shown in figure 2.

Using the empirical correlation obtained, figures 3a–3c are drawn. Figure 3a shows the effect of particle diameter on the thermal conductivity of nanofluid at various volume fractions. It indicates that with increasing particle diameter the thermal conductivity enhancement decreases. Further, it shows that with increasing volume fraction the effective thermal conductivity of a nanofluid increases. The rate of increase of the k value is found to be less at higher volume fractions compared to lower fractions. Moreover, beyond 12% of volume fraction it appears that the improvement in k value may be insignificant.

Figure 3b shows the effect of temperature on the thermal conductivity of a nanofluid. It indicates that with increasing temperature the k value of the nanofluid

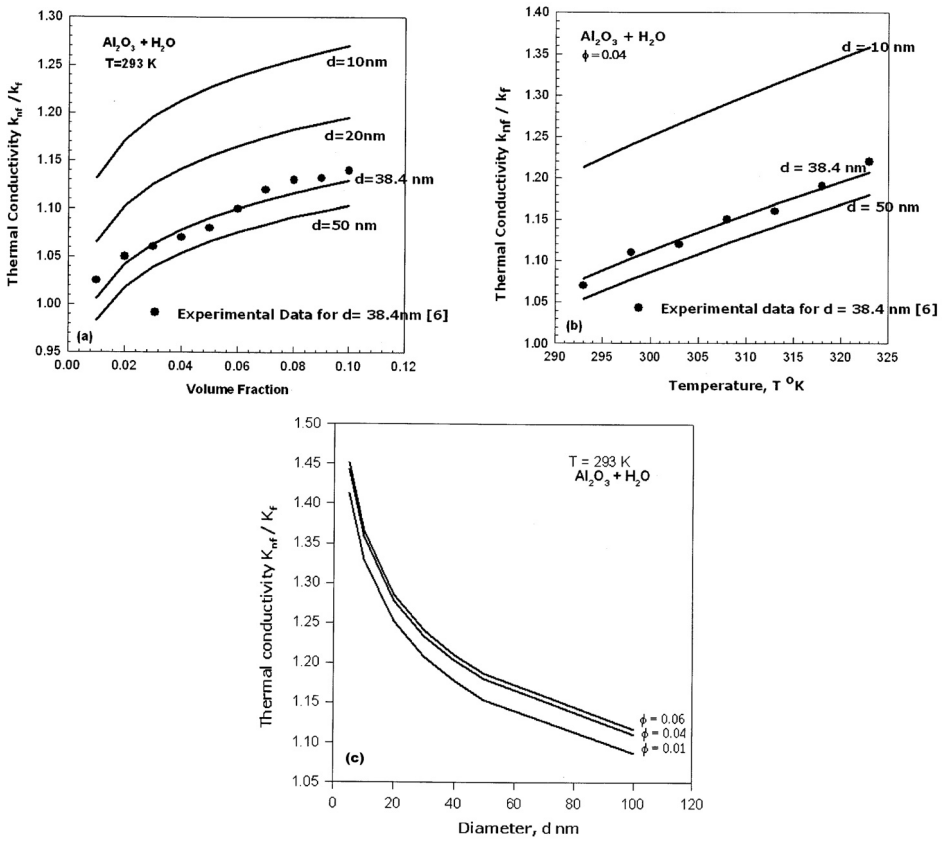

Figure 3. The effect of (**a**) volume fraction, (**b**) temperature and (**c**) diameter of nanoparticle on thermal conductivity of nanofluid.

increases. Further, we can conclude from the graph that the effect is more dominant in the small-sized particles than with large-sized ones. Figure 3c shows the effect of particle diameter on the k value of nanofluid. It indicates that with increase in particle size the thermal conductivity effect decreases. Further, it is evident that the effect of volume fraction on low-sized particle is almost nil, and hence it is advisable to go for small-sized low concentration of particles to have a better k value.

4. Convection of nanofluids

The present study is aimed at developing a correlation to predict heat transfer coefficient for steady, forced turbulent convection flow of nanofluid inside a tube. It is evident from literature survey that there exists no formulated theory till date that could predict the heat transfer characteristics of nanofluid by considering it as multi-component model. In general, most nanofluids used in practical application

contain oxide particles and can be easily fluidized. Consequently, it can be treated as single-phase fluid which possesses thermophysical properties as explained earlier. Thus a direct extension of conventional fluid to nanofluid is feasible and classical theory developed for single-phase fluid is also applicable to nanofluid. The conservation of mass, momentum, energy are also applicable. Applicability of the above assumptions is difficult to assess due to lack of data. However, under negligible slip condition, assuming thermal equilibrium between the phases and assuming the particle spatial distribution as uniform the single-phase assumption is valid. From the practical point of view, mixtures with relatively low particle concentration ensure a perfect mixing of particles inside the liquid phase. The stability of particle suspension is also crucial for experimental verification. The experimental results by Pak and Cho [12] and Xuan and Li [11] in which correlations of a form similar to that of the well-known Dittus–Boelter formula were proposed to characterize heat transfer of nanofluids. Pak and Cho [12] and Xuan and Li [11] proposed correlation for the calculation of Nusselt number as follows:

$$\mathrm{Nu} = 0.021(\mathrm{Re_f})^{0.8}(\mathrm{Pr_f})^{0.5} \tag{14}$$

$$\mathrm{Nu} = 0.059\left[1 + 7.6286\phi^{0.6886}\left(\mathrm{Re_f Pr_f}\frac{d_\mathrm{p}}{D}\right)^{0.001}\right]\mathrm{Re_f^{0.9238}\ Pr_f^{0.4}}. \tag{15}$$

In the above correlations the Reynolds and Prandtl numbers are evaluated by considering the base fluid only. The Nu values obtained by the above two equations are found either overestimating or underestimating compared to experimental data. Thus in the present work, we adopted the above 'single-phase fluid' approach in order to study the thermal behavior of nanofluids in which the thermophysical properties of the nanofluid are considered instead of base fluid properties. The convective heat transfer coefficient and Nusselt number are related as

$$\mathrm{Nu_{nf}} = \frac{h_\mathrm{nf} D}{k_\mathrm{nf}}. \tag{16}$$

The heat transfer coefficient of turbulent flow through circular tube can be calculated from Dittus–Boelter equation in the following form, where c, m and n represent the coefficients suitable to nanofluid experimental data.

$$\mathrm{Nu_{nf}} = c(\mathrm{Re_{nf}})^m(\mathrm{Pr_{nf}})^n. \tag{17}$$

$\mathrm{Re_{nf}}$ and $\mathrm{Pr_{nf}}$ are defined as follows:

$$\mathrm{Re_{nf}} = \frac{\rho_\mathrm{nf} U D}{k_\mathrm{nf}}, \tag{18}$$

$$\mathrm{Pr_{nf}} = \frac{C_\mathrm{nf}\mu_\mathrm{nf}}{k_\mathrm{nf}}. \tag{19}$$

The properties of nanofluid in eqs (16)–(19) are calculated using eqs (1)–(6), (12), (13).

The Nusselt number for the Al_2O_3 + H_2O and Cu + H_2O nanofluids is obtained from the regression analysis using the experimental data obtained from [10–12]. The values of m and n are fixed as 0.8 and 0.4 to simulate Dittus–Boelter correlation. The constant c is obtained as 0.0256 for Al_2O_3 + H_2O and 0.027 for Cu + H_2O nanofluids. Thus the correlations for Nusselt number are developed with an average deviation of 5% and standard deviation of 6.4% as follows:

$$\mathrm{Nu} = 0.0256(\mathrm{Re_{nf}})^{0.8}(\mathrm{Pr_{nf}})^{0.4} \quad \text{for } Al_2O_3 + H_2O, \tag{20}$$

$$\mathrm{Nu} = 0.027(\mathrm{Re_{nf}})^{0.8}(\mathrm{Pr_{nf}})^{0.4} \quad \text{for Cu} + H_2O. \tag{21}$$

The above equations take care of the diameter of the nanoparticle, concentration and temperature effects. The correlations give good agreement with the experimental results as shown in figures 4 and 5.

5. Conclusion

- Various models to predict thermal conductivity of nanofluid is presented in a chronological order. The comparison of models with experimental data is shown in graphical form.
- A simple analytical correlation to predict k of Al_2O_3 + H_2O and Cu + H_2O nanofluids taken into effect the temperature, volume fraction and particle size is presented.
- A simple analytical correlation to predict Nu of Al_2O_3 + H_2O and Cu + H_2O nanofluids is presented and comparison of Nu with experimental and existing correlations is shown in graphical form.
- The correlations obtained for k and Nu are in good agreement with experimental data.

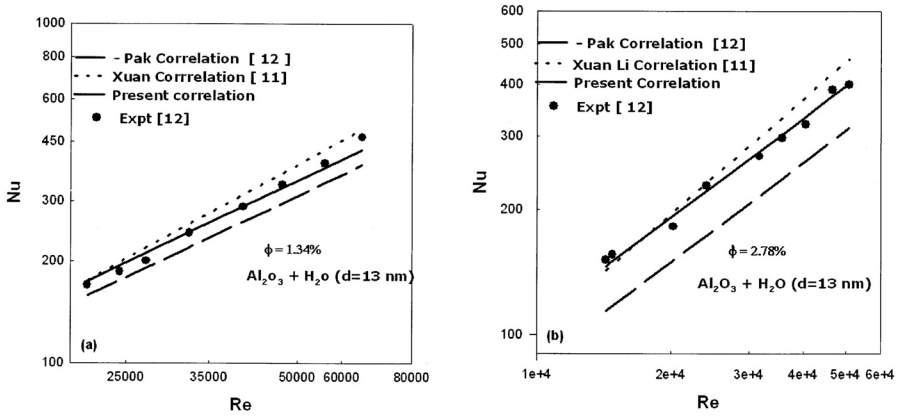

Figure 4. Nusselt number comparison for alumina + water nanofluid correlation with the experimental data. (**a**) $\phi = 1.34\%$, (**b**) $\phi = 2.78\%$.

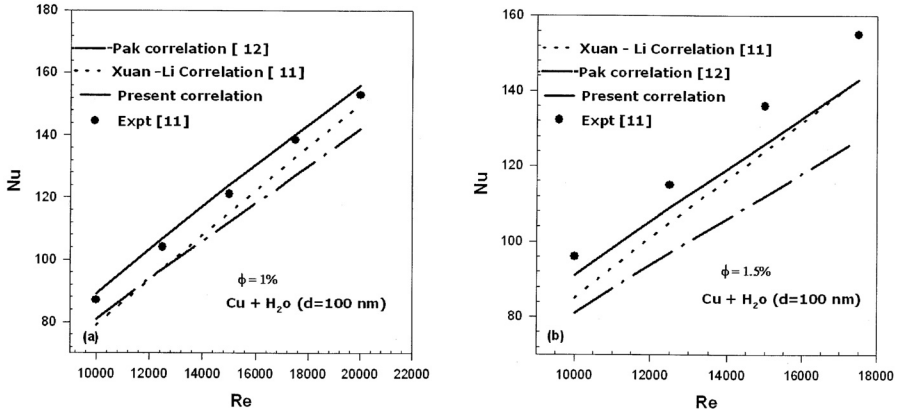

Figure 5. Nusselt number comparison for Cu + water nanofluid correlation with the experimental data. (**a**) $\phi = 1\%$, (**b**) $\phi = 1.5\%$.

Hence it is concluded that adding nanosized materials to base fluids enhances thermal properties and makes them more suitable to heat exchanger applications.

Nomenclature

C	specific heat (kJ/kg K)
D	diameter of tube (m)
d	diameter (nm)
k	thermal conductivity (W/m K)
k_b	Boltzmann constant ($1.3807 \cdot 10^{-23}$) (J/K)
Nu	Nusselt number
Pr	Prandtl number
Re	Reynolds number
v	velocity (m/s)
n	shape factor
$\mathrm{Re_m}$	Reynolds number $\frac{1}{\nu_\mathrm{p}}\sqrt{\frac{18k_\mathrm{b}T}{\Pi\rho_\mathrm{p}d_\mathrm{p}}}$
T	temperature (K)

Greek

ρ	density (kg/m^3)
μ	viscosity (Pa·s)
ϕ	volume fraction
ψ	sphericity
ν	viscosity (m^2/s)

Subscripts

nf nanofluids
p nanoparticle
f base fluid

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