PRAMANA
— journal of
physics

Vol. 69, No. 2
August 2007
pp. 241–253

Propagation of ion-acoustic waves in a dusty plasma with non-isothermal electrons

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MS received 30 April 2005; revised 18 April 2007; accepted 1 May 2007

Abstract. For an unmagnetised collisionless plasma consisting of warm ions, non-isothermal electrons and cold, massive and charged dust grains, the Sagdeev potential equation, considering both ion dynamics and dust dynamics has been derived. It has been observed that the Sagdeev potential $V(\phi)$ exists only for $\phi > 0$ up to an upper limit ($\phi \simeq 1.2$). This implies the possibility of existence of compressive solitary wave in the plasma. Exhaustive numerics done for both the large-amplitude and small-amplitude ion-acoustic waves have revealed that various parameters, namely, ion temperature, non-isothermality of electrons, Mach numbers etc. have considerable impact on the amplitude as well as the width of the solitary waves. Dependence of soliton profiles on the ion temperature and the Mach number has also been graphically displayed. Moreover, incorporating dust-charge fluctuation and non-isothermality of electrons, a non-linear equation relating the grain surface potential to the electrostatic potential has been derived. It has been solved numerically and interdependence of the two potentials for various ion temperatures and orders of non-isothermality has been shown graphically.

Keywords. Non-isothermal electrons; Sagdeev potential; dust-charge fluctuation.

PACS Nos 52.25.Vy; 52.27.Lw; 52.35.Fp

1. Introduction

The reductive perturbation method was firstly employed by Washimi and Taniuti [1] to investigate the various salient features of solitary waves in a cold, collisionless and unmagnetised plasma and they derived K-dV equation to study plasma dynamics. Moreover, the K-dV theory was applied to ion-acoustic waves by Zakharov [2] and Gardner *et al* [3]. The main point in this theory is that the non-linear steepening is balanced by dispersion and not by dissipation as in ordinary gas dynamics. Gardner and Morikawa [4,5] and Berezin and Karpman [6] also made use of this theory for the magneto-sonic waves excited in a magnetised plasma. In addition to these, many authors incorporated various parameters or ambient conditions e.g., ion temperature [7], non-isothermality, i.e. resonant electrons (both trapped and free) [8], ion-beam [9], two-temperature electrons [10], density gradient [11], temperature gradient [12], Landau damping [13], magnetic field [14] etc. for theoretical analysis of solitary waves in plasmas. In recent years, there has been a great deal of interest in numerous collective processes in a plasma contaminated by the charged dust grains encountered very often in space and laboratory plasmas because of their vital role in understanding various interesting and novel aspects of cosmic environments (such as planetary rings, cometary tails, intersteller clouds etc.) low-temperature radio-frequency plasma discharges and so on.

The presence of massive and charged dust particles in a plasma can drastically influence its dispersion and nonlinear properties. Several authors [15,16] have studied the low-frequency ion and dust-acoustic wave modes in dusty plasmas. Dust grains are usually charged negatively because of the ion current and electron current on their surface. In the presence of disturbances, the dust charge needs to be treated as a dynamical variable in studying dynamics of a dusty plasma. It should be noted here that this charge fluctuation plays an important role in distinguishing dusty plasma from negative-ion multicomponent plasma. In many laboratory and industrial plasmas, the ion-acoustic waves are more relevant than the dust-acoustic waves. However, not many theoretical works on large-amplitude ion-acoustic waves in dusty plasma have been done. Nejoh [17] has investigated the effect of ion temperature along with dust-charge fluctuation on large-amplitude ion-acoustic wave in a dusty plasma. But this analysis does not involve non-isothermality of electrons. Das *et al* [18] have made detailed study on the dynamical aspects of various solitary waves (K-dV solitons, spiky solitary waves, explosive solitary waves etc.) in dusty plasma, incorporating non-isothermality of electrons. In their work, only dust dynamics has been considered.

In our paper, we have followed the pseudopotential approach to study the effect of ion temperature, Mach number, non-isothermality of electrons etc. on the large-amplitude waves excited in the dusty plasma, considering the ion dynamics. We have also studied the mode of dependence of the grain surface potential on the electrostatic potential. Furthermore, in the small-amplitude limit, sech^4 type of soliton solution has been obtained. Amplitude and width of the ion-acoustic soliton have been calculated for different ion temperatures and Mach numbers and their variations have been studied. In addition to this, a relation connecting the grain surface potential and the electrostatic potential in the limit $\psi \to 0$ has been derived and then it has been presented graphically.

2. Formulation of the problem

(A) In the present paper, we assume the plasma to be unmagnetised, collisionless and to consist of warm ions, hot and non-isothermal electrons and cold dust grains because the dust grains are massive compared to the ions. We restrict ourselves to one-dimensional propagation. The non-isothermality of the plasma is revealed through the modified Boltzmann distribution of electron density of the form [18]

$$n_{\mathrm{e}} = [\exp(\phi) - G(\phi)], \tag{1}$$

where

$$G(\phi) = \frac{4}{3}b_1\phi^{3/2} + \frac{8}{15}b_2\phi^{5/2} + \frac{16}{105}b_3\phi^{7/2} + \cdots$$
$$b_1 = \frac{1-\beta}{\sqrt{\pi}}, \quad b_2 = \frac{1-\beta^2}{\sqrt{\pi}}, \quad b_3 = \frac{1-\beta^3}{\sqrt{\pi}}, \ldots; \quad \beta = T_{\rm ef}/T_{\rm et}; \tag{2}$$

$T_{\rm ef}$ and $T_{\rm et}$ being the temperatures of the free and trapped electrons respectively. Moreover, $G = 0$ represents the isothermal plasma in which electrons are Boltzmannian.

The non-dimensional equations which do govern the dynamics of the ion and the dust particles, following Nejoh [17] are

For the ions:

$$\frac{\partial n_{\rm i}}{\partial t} + \frac{\partial(n_{\rm i}v_{\rm i})}{\partial x} = 0, \tag{3}$$

$$\frac{\partial v_{\rm i}}{\partial t} + v_{\rm i}\frac{\partial v_{\rm i}}{\partial x} + \frac{3\sigma}{(1+az_{\rm d})^2}n_{\rm i}\frac{\partial n_{\rm i}}{\partial x} + \frac{\partial\phi}{\partial x} = 0. \tag{4}$$

Here, $\sigma = T_{\rm i}/T_{\rm e}$ and $a = n_{\rm d0}/n_0$ where $T_{\rm i}, T_{\rm e}, n_0, n_{\rm d0}$ denote the ion temperature, electron temperature, the equilibrium values of the background electron density and the number density of the dust particles respectively. $z_{\rm d}$ is the charge number of the dust grains.

For the dust grains:

$$\frac{\partial n_{\rm d}}{\partial t} + \frac{\partial(n_{\rm d}v_{\rm d})}{\partial x} = 0, \tag{5}$$

$$\left(\frac{\partial}{\partial t} + v_{\rm d}\frac{\partial}{\partial x}\right)v_{\rm d} - \left(\frac{z_{\rm d}}{\mu_{\rm d}}\right)\frac{\partial\phi}{\partial x} = 0, \tag{6}$$

where the dust charge variable is given by $Q_{\rm d} = -z_{\rm d}e$; e and $z_{\rm d}$ being the magnitudes of the electron charge and the dust charge number respectively.

The above equations are supplemented by Poisson's equation which is

$$\frac{\partial^2\phi}{\partial x} = n_{\rm e} - n_{\rm i} + z_{\rm d}n_{\rm d}. \tag{7}$$

In eq. (6), $\mu_{\rm d} = m_{\rm d}/m_{\rm i}$ where $m_{\rm d}$ and $m_{\rm i}$ are the dust grain mass and the ion mass respectively. The plasma variables $n_{\rm e}, n_{\rm i}, n_{\rm d}, v_{\rm d}, v$ and ϕ represent the electron density, the ion density, the dust grain density, dust particle velocity, the ion velocity and the electrostatic potential respectively. Here, the densities have been normalised by the background electron density n_0. The space coordinate x, time coordinate t, velocities and electrostatic potential have been normalised by the electron Debye length $\lambda_{\rm De} = (\epsilon_{\rm o}K_{\rm B}T_{\rm e}/n_{\rm o}e^2)^{1/2}$, the inverse ion plasma frequency $\omega_{\rm i}^{-1} = (\epsilon_o m_{\rm i}/n_{\rm o}{\rm e}^2)^{1/2}$, the ion sound velocity $C_{\rm s} = (K_{\rm B}T_{\rm e}/m_{\rm i})^{1/2}$ and $K_{\rm B}T_{\rm e}/e$, respectively, where $\epsilon_{\rm o}$ is the permittivity of free space and $K_{\rm B}$ is the Boltzmann constant. We further assume that the phase velocity of the ion-acoustic wave is low compared to the electron thermal velocity. The charge neutrality in the state of equilibrium is given by the condition $n_{\mathrm{i}0} = n_0 + z_{\mathrm{d}} n_{\mathrm{d}0}$ where $n_{\mathrm{i}0}$ denotes the unperturbed value of the ion density.

The variable dust grain charge Q_{d} defined earlier satisfies the following differential equation

$$\left(\frac{\partial}{\partial t} + v_{\mathrm{d}} \frac{\partial}{\partial x}\right) Q_{\mathrm{d}} = I_{\mathrm{i}} + I_{\mathrm{e}}, \tag{8}$$

where I_{i} and I_{e} are the ion current and the electron current respectively, given by [19]

$$I_{\mathrm{e}} = -\pi r^2 e \left(\frac{8T_{\mathrm{e}}}{\pi m_{\mathrm{e}}}\right)^{1/2} [\exp(\phi) - G(\phi)] \exp\left(\frac{e\psi}{T_{\mathrm{e}}}\right), \tag{9}$$

$$I_{\mathrm{i}} = \pi r^2 e \left(\frac{8T_{\mathrm{i}}}{\pi m_{\mathrm{i}}}\right)^{1/2} n_{\mathrm{i}} \left(1 - \frac{e\psi}{T_{\mathrm{i}}}\right), \tag{10}$$

where $\psi = Q_{\mathrm{d}}/r$, stands for the dust grain surface potential relative to the plasma electrostatic potential ϕ, r being the radius of the dust grain and other symbols are carrying usual meanings.

Now eqs (1)–(7) have to be transformed, on introducing the Galilean transformation involving a new variable ξ given by $\xi = x - Mt$ where M is the velocity of the moving frame. Following Nejoh [17], we obtain the ion density and the dust density given by

$$n_{\mathrm{i}} = (1 + az_{\mathrm{d}}) / \left[1 - \frac{2\phi}{M^2 - 3\sigma}\right]^{1/2}, \tag{11}$$

$$n_{\mathrm{d}} = a / \left[1 + \frac{z_{\mathrm{d}}}{\mu_{\mathrm{d}}} \cdot \frac{2\phi}{M^2}\right]^{1/2}, \tag{12}$$

where the boundary conditions $\phi \to 0$, $n_{\mathrm{d}} \to a$, $n_{\mathrm{i}} \to (1 + az_{\mathrm{d}})$, $v_{\mathrm{i}} \to 0$, $v_{\mathrm{d}} \to 0$ as $\xi \to \infty$ are used.

Using (1), (11) and (12) in the transformed Poisson's equation we readily have

$$\begin{aligned}\frac{\partial^2 \phi}{\partial \xi^2} &= \exp(\phi) - G(\phi) - (1 + az_{\mathrm{d}}) / \left[1 - \frac{2\phi}{M^2 - 3\sigma}\right]^{1/2} \\ &\quad + z_{\mathrm{d}} a / \left[1 + \frac{z_{\mathrm{d}}}{\mu_{\mathrm{d}}} \cdot \frac{2\phi}{M^2}\right]^{1/2} = -\frac{\partial V(\phi)}{\partial \phi},\end{aligned} \tag{13}$$

where $G(\phi)$ has been defined earlier and $V(\phi)$ denotes the pseudopotential.

Integration of (13) yields

$$\frac{1}{2}\left(\frac{\partial\phi}{\partial\xi}\right)^2 + V(\phi) = 0, \tag{14}$$

where

$$\begin{aligned} V(\phi) &= 1 - \exp(\phi) + \frac{8}{15}\, b_1\, \phi^{5/2} + \frac{16}{105}\, b_2\, \phi^{7/2} + \frac{32}{945}\, b_3\, \phi^{9/2} \\ &\quad -(1 + az_{\mathrm{d}})(M^2 - 3\sigma)\left[\sqrt{1 - \frac{2\phi}{M^2 - 3\sigma}} - 1\right] \\ &\quad -a\mu_{\mathrm{d}} M^2 \left[\sqrt{1 + \frac{z_{\mathrm{d}}}{\mu_{\mathrm{d}}}\frac{2\phi}{M^2}} - 1\right]. \end{aligned} \tag{15}$$

At equilibrium,

$$I_{\mathrm{e}} + I_{\mathrm{i}} = 0, \tag{16}$$

where I_{e} and I_{i} are respectively given by (9) and (10). We are led by this equation to have on normalisation

$$\exp(\phi + \psi) - G(\phi)\, \exp(\psi) = \frac{\sigma - \psi}{\sqrt{\sigma\mu_{\mathrm{i}}}}(1 + az_{\mathrm{d}}) \cdot \frac{1}{\sqrt{1 - \frac{2\phi}{M^2 - 3\sigma}}}. \tag{17}$$

Equation (17) may be regarded as a new equation for the dust-charge fluctuation. This equation has to be solved numerically.

(B) Small-amplitude approximation

Equation (15), under small-amplitude approximation reduces to the form

$$V(\phi) = \left[-\frac{1}{2} + \frac{(1 + az_{\mathrm{d}})}{2(M^2 - 3\sigma)} + \frac{az_{\mathrm{d}}^2}{2\mu_{\mathrm{d}} M^2}\right]\phi^2 + \frac{8}{15} b_1 \phi^{5/2}. \tag{18}$$

With the use of the energy-integral equation (14) one obtains

$$\begin{aligned} \left(\frac{\mathrm{d}\phi}{\mathrm{d}\xi}\right)^2 &= \left[1 - \frac{(1 + az_{\mathrm{d}})}{(M^2 - 3\sigma)} - \frac{az_{\mathrm{d}}^2}{\mu_{\mathrm{d}} M^2}\right]\phi^2 - \frac{16}{15} b_1 \phi^{5/2} \\ &= A\phi^2 - B\phi^{5/2}, \end{aligned} \tag{19}$$

where

$$A = \left[1 - \frac{(1 + az_{\mathrm{d}})}{M^2 - 3\sigma} - \frac{az_{\mathrm{d}}^2}{\mu_{\mathrm{d}} M^2}\right] \quad \text{and} \quad B = \frac{16}{15} b_1.$$

Now, in order to solve the non-linear equation (19), following the newly proposed simple wave solution technique of Das *et al* [20] we make use of a transformation

$$\eta = \alpha(x - \lambda t) \tag{20}$$

relative to a frame moving with the velocity λ where α is such a parameter that its reciprocal represents the characteristic width of the soliton and consequently eq. (19) becomes

$$\alpha^2 \frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = A\phi - \frac{5}{4}B\phi^{3/2}. \tag{21}$$

Based on the knowledge about the solitary wave solution of the K-dV equation which, in general, comes out to be hyperbolic function, we, following Das *et al* [18] make an additional substitution $z = \tanh\eta$ and $W(z) = \phi(\eta)$. As a result, eq. (21) is transformed into another differential equation of the form (Fuchyan type)

$$\alpha^2(1-z^2)\frac{\mathrm{d}^2W}{\mathrm{d}z^2} - 2\alpha^2 z(1-z^2)\frac{\mathrm{d}W}{\mathrm{d}z} - AW(z) + \frac{5}{4}BW^{3/2} = 0. \tag{22}$$

The power series solution of (22) may be taken to be of the form

$$W(z) = \sum_{r=0}^{\infty} a_r z^{m+r}. \tag{23}$$

But this method due to Frobenius is too elaborate and tedious. It is to be mentioned here that though eq. (21) can be solved by direct integration to have the soliton solution, the tanh method is better because it is more general and can be applied to an equation involving term like $\phi^{5/2}$ where the ordinary integration method fails to be applied. So, following the same method [20] in which the series is truncated to a finite series with $(N+1)$ terms where N comes out to be 4 in our case and the series (23) with $m = 0$ reduces to

$$W(z) = \sum_{r=0}^{4} a_r z^r. \tag{24}$$

For clarity, it needs to be mentioned that by equating the power of z associated with the highest order derivative to the maximum power of z one gets $N = 4$ ($\frac{3N}{2} = N - 2 + 4$, i.e., $N = 4$). The nature of the differential equation dictates that there should be no odd terms in the series solution (24), i.e., $a_1 = a_3 = 0$ and thus finally $W(z)$ comes out to be

$$W(z) = a_\mathrm{o} + a_2 z^2 + a_4 z^4 = a_\mathrm{o}(1-z^2)^2 \tag{25}$$

with $a_\mathrm{o} = -a_2/2 = a_4$. Using the expression (25) for $W(z)$ in eq. (22) one readily gets

$$2\alpha^2(5z^2 - 1) - A + \frac{5}{4}B\sqrt{a_\mathrm{o}}(1-z^2) = 0. \tag{26}$$

Equating the coefficients of z^0 and z^2 in eq. (26) the unknown parameters a_o and α turn out to be

$$a_\mathrm{o} = \left(\frac{A}{B}\right)^2, \quad \alpha = \sqrt{\frac{A}{8}}$$

and thus we have the soliton solution as

$$\phi(\eta) = \phi_0 \operatorname{sech}^4 \eta = \phi_0 \operatorname{sech}^4 \left(\frac{x - \lambda t}{1/\alpha}\right) = \phi_0 \operatorname{sech}^4 \left(\frac{x - \lambda t}{\delta}\right), \tag{27}$$

where

$$a_{\mathrm{o}} = \phi_0 = \left(\frac{A}{B}\right)^2 \quad \text{and} \quad \delta = \sqrt{\frac{8}{A}} \tag{28}$$

are the amplitude and the width of the soliton respectively. In §2, eq. (17) can be rewritten as

$$\exp(\phi + \psi) - G(\phi)\exp(\psi) = \frac{\sigma - \psi}{\sqrt{\sigma\mu_{\mathrm{i}}}}(1 + \gamma\psi)\frac{1}{\sqrt{1 - \frac{2\phi}{M^2 - 3\phi}}} \tag{29}$$

in which $\gamma\psi$ has been substituted for az_{d}, because $\gamma = ar/e$, $\psi = Q_{\mathrm{d}}/r$, $Q_{\mathrm{d}} = ez_{\mathrm{d}}$ (magnitude only).

For vanishingly small value of ψ, the grain surface potential, eq. (29) leads us to have

$$\psi = \frac{g(\phi)}{f(\phi)} \tag{30}$$

where

$$g(\phi) = \sqrt{\frac{\sigma}{\mu_{\mathrm{i}}}} \cdot \frac{1}{\sqrt{1 - \frac{2\phi}{M^2 - 3\sigma}}} + G(\phi) - \exp(\phi)$$

and

$$f(\phi) = \exp(\phi) - G(\phi) + (1 - \gamma\sigma) \cdot \frac{1}{\sqrt{\sigma\mu_{\mathrm{i}}}} \cdot \frac{1}{\sqrt{1 - \frac{2\phi}{M^2 - 3\sigma}}}. \tag{31}$$

Numerical computation of (30) has to be done to study the mode of dependence of ψ on ϕ for the plasma containing isothermal electrons and non-isothermal electrons.

3. Graphical illustration of the results

(a) The pseudopotential profiles as depicted in figures 1a and 1b reveal the possibility of excitation of compressive solitons in the plasma. Figure 1a vividly shows contrast between two types of plasmas in which electrons are isothermal and non-isothermal. For both the plasmas, solitons are likely to exist but the potential wells represented by the $V(\phi)$ profiles are of different depths. Because of greater depth, in isothermal plasma soliton formation is more likely compared to that for non-isothermal plasma. Moreover, for the same set of parameter values, amplitude of the soliton is maximum for the isothermal plasma and the soliton amplitude gradually decreases as higher orders of non-isothermality are considered.

Figure 2 also exhibits trapping of particles, thereby implying the formation of soliton. It is very interesting to note that the soliton amplitude increases remarkably as the ion temperature decreases for a dusty plasma with non-isothermal electrons and this is in tune with what have been displayed in figures 1a and 1b; 5a and 5b.

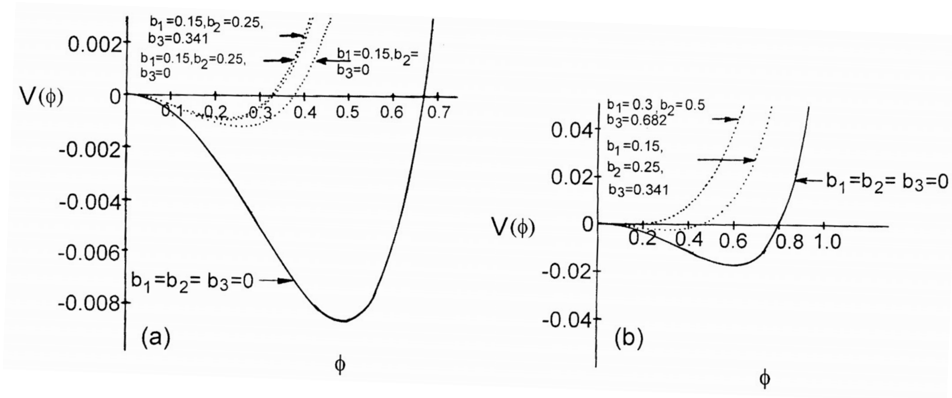

Figure 1. (**a**) Contrast between isothermality and non-isothermality relative to the profile of $V(\phi)$ when $M = 1.6$, $\sigma = 0.08$, $a = 10^{-3}$, $\mu_{\rm d} = 10^{6}$, $z_{\rm d} = 10^{3}$. (**b**) Effect of non-isothermality on the pseudopotential profile when the plasma parameters are: $M = 1.6$, $\sigma = 0.05$, $a = 10^{-3}$, $\mu_{\rm d} = 10^{6}$, $z_{\rm d} = 10^{3}$.

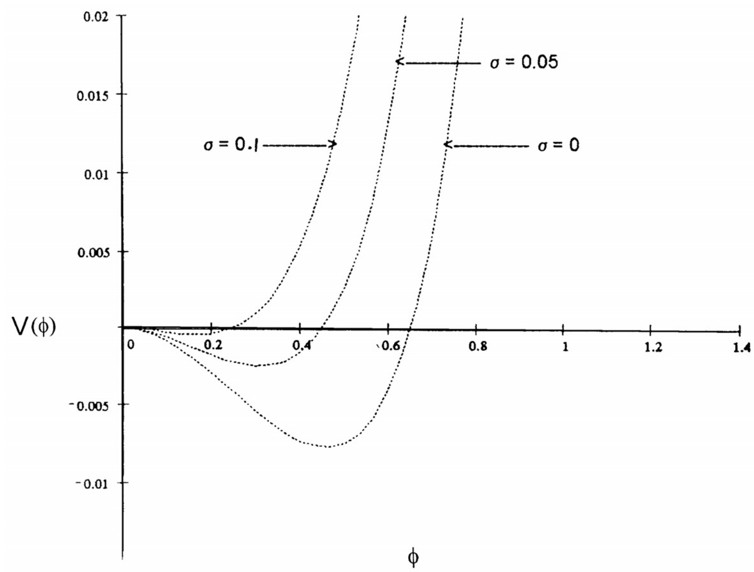

Figure 2. Effect of ion temperature on the profile of $V(\phi)$ for the following plasma parameter values: $a = 10^{-3}$, $\mu_{\rm d} = 10^{6}$, $z_{\rm d} = 10^{3}$, $M = 1.6$, $b_1 = 0.15$, $b_2 = 0.25$, $b_3 = 0.341$.

In figure 3, effect of Mach number on the Sagdeev potential profile has been clearly displayed. Here also $V(\phi) < 0$ for $\phi > 0$ which means that compressive soliton is excited. Moreover, it is observed that for the same non-isothermal plasma, higher the value of the Mach number, greater is the soliton amplitude and this is consistent with what figure 5b shows.

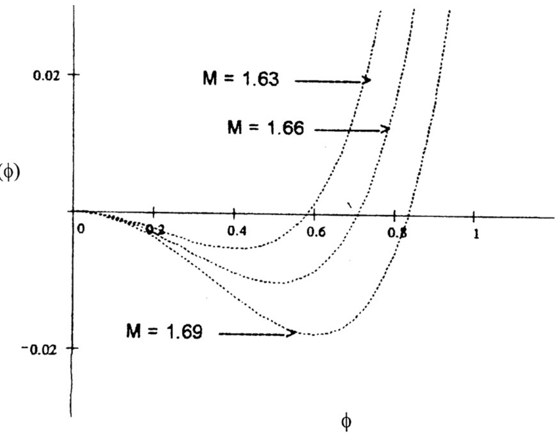

Figure 3. Effect of Mach number on the pseudopotential curves for non-isothermal plasma when the parameter values are: $a = 10^{-3}$, $\mu_{\rm d} = 10^6$, $z_{\rm d} = 10^3$, $\sigma = 0.05$, $b_1 = 0.15$, $b_2 = 0.25$, $b_3 = 0.341$.

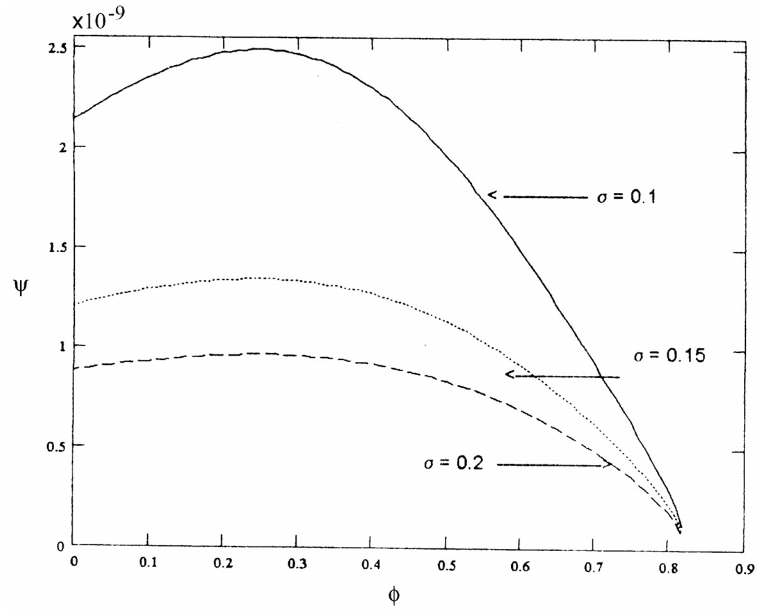

Figure 4. Variation of the grain surface potential with the electrostatic potential when σ is a parameter and $b_1 = 0.15$, $b_2 = 0.25$, $b_3 = 0.314$, $M = 1.6$, $\mu_{\rm d} = 10^6$, $a = 10^{-3}$ and $\mu_{\rm i} = 1836$.

From figure 4, one can extract various important informations concerning mode of dependence of the grain surface potential (ψ) on the electrostatic plasma potential (ϕ) and the ion temperatures. For $\sigma = 0.1$ which is the lowest, the grain surface potential is found to increase first with the increase of the plasma potential (ϕ)

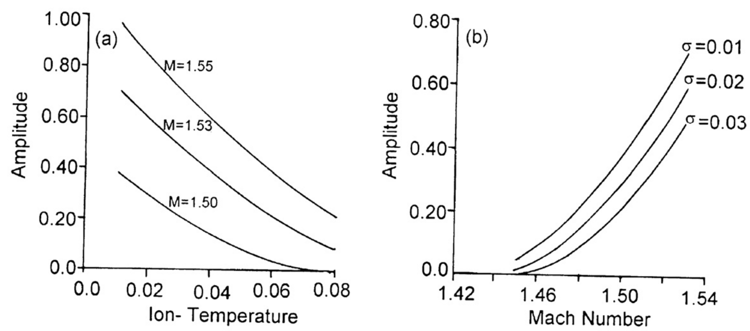

Figure 5. (**a**) Variation of amplitude (ϕ_0) with the ion temperature with Mach number as parameter when $a = 10^{-3}$, $z_\mathrm{d} = 10^3$, $\mu_\mathrm{d} = 10^6$, $b_1 = 0.15$. (**b**) Change of amplitude with the Mach number with ion temperature as parameter when $a = 10^{-3}$, $z_\mathrm{d} = 10^3$, $\mu_\mathrm{d} = 10^6$, $b_1 = 0.15$.

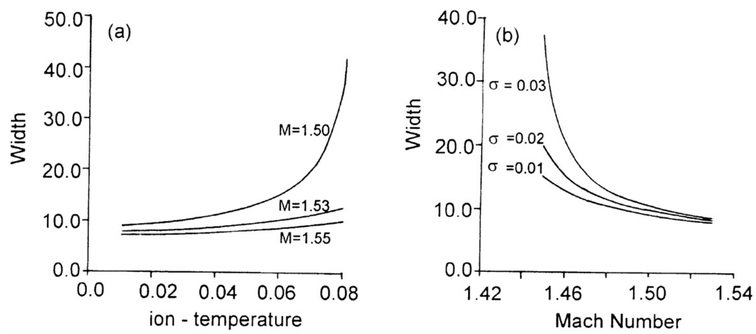

Figure 6. (**a**) Plot of variation of width (δ) with the ion temperature with Mach number as parameter when $a = 10^{-3}$, $z_\mathrm{d} = 10^3$, $\mu_\mathrm{d} = 10^6$, $b_1 = 0.15$. (**b**) Plot of dependence of width on the Mach number for various values of ion temperature when $a = 10^{-3}$, $z_\mathrm{d} = 10^3$, $\mu_\mathrm{d} = 10^6$, $b_1 = 0.15$.

till the peak is reached and then starts decreasing with further increase in ϕ. The hump in the curves ceases to appear as the ion temperature is increased further. The striking point to note here is that all the curves appear to meet at a point signifying that the grain surface potential is independent of σ where ϕ is slightly greater than 0.8. Now it is to be mentioned that all the curves presented in figures 1–4 have been drawn for the values of ϕ satisfying the inequality $0 \leq \phi \leq 1.2$.

(b) For the ion-acoustic soliton obtained in the small-amplitude approximation, variation of amplitude and width with ion temperature and Mach number when Mach number and ion temperature are parameters, have been depicted in figures 5 and 6 respectively. In figure 5a we notice that the amplitude decreases with the ion temperature when Mach number is held constant whereas in figure 5b it is found to increase with the Mach number when the ion temperature is maintained constant. If figures 5a and 5b are analysed carefully and simultaneously then it will be observed that Mach number plays a crucial role in connection with the existence,

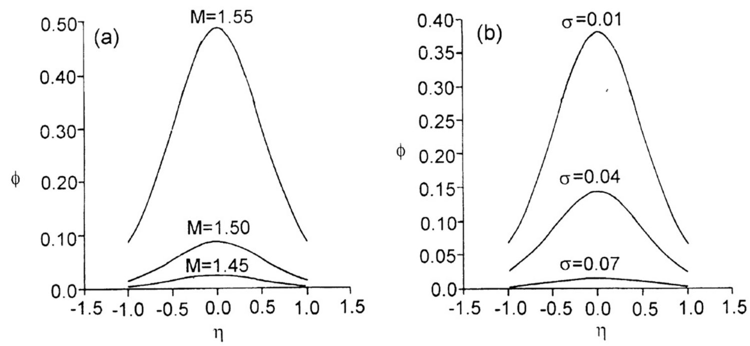

Figure 7. (**a**) Solitary wave profiles for various values of Mach number when $a = 10^{-3}$, $z_{\rm d} = 10^3$, $\sigma = 0.05$, $\mu_{\rm d} = 10^6$, $b_1 = 0.15$. (**b**) Solitary wave profiles for different values of ion temperatures when $a = 10^{-3}$, $z_{\rm d} = 10^3$, $M = 1.5$, $\mu_{\rm d} = 10^6$, $b_1 = 0.15$.

rather excitation of solitary wave in a plasma. In our problem, when $\sigma = 0.03$ for any $M < 1.45$, no soliton can be excited (figure 5b) and when $\sigma = 0.08$, for any $M < 1.5$, no solitary wave is likely to be excited. That means for a particular ion temperature (normalised), there exists a threshold value of M (Mach number) below which solitary wave cannot exist in a plasma. In view of the dependence of M (the Mach number) on σ (the ion temperature) and for a set of values of the parameters $a, \mu_{\rm d}$ and $z_{\rm d}$, values of M covering the range 1.45–1.6 have been used in doing the numerics. In figure 6a, in contrast to amplitude variation, it is observed that the width increases with the ion temperature which is expected to happen usually. Moreover, in this case our attention is drawn to the fact that the rate of increase of the width is slower for greater Mach number but the rate is faster for smaller value of Mach number particularly when the ion temperature is higher. It is evident from figure 6b that the width of the solitary wave decreases as the Mach number increases for certain ion-temperature. Furthermore, it is seen that the rate of fall in width is more rapid when ion temperature is higher.

The profiles of the ion-acoustic solitary waves have been displayed in figures 7a and 7b. It is obvious from figure 7a that for a particular ion temperature the amplitude of the wave decreases whereas width increases along with the decrease in the Mach number and this is consistent with what are found in figures 5b and 6b respectively. Again what happens to the amplitude and the width of the ion-acoustic soliton due to change in the ion temperature and the Mach number as shown in figure 7b are in agreement with figures 5a and 6a respectively.

Figure 8 reveals the nature of change of the grain surface potential with the electrostatic potential for both isothermal and non-isothermal cases. When the plasma contains isothermal electrons, i.e. for $b_1 = 0$, $b_2 = 0$, the grain surface potential, at first, is found to increase with ϕ though not uniformly, then to remain almost unchanged for a narrow range of values of ϕ and finally to fall sharply. For the non-isothermal case, the plot shows similar type of variation of ψ with ϕ. The most striking point to note here is that the maximum value of ψ diminishes with the increase in the order of non-isothermality. Also, there is no region of constancy of

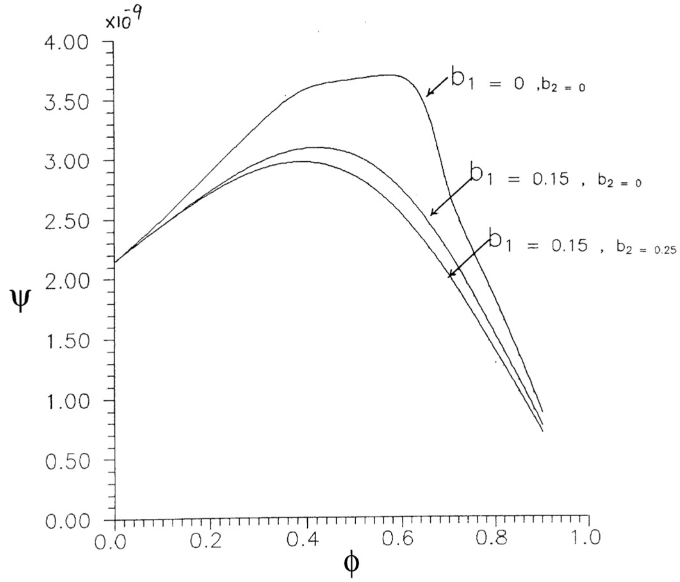

Figure 8. Dependence of the grain surface potential (ψ) on the ambient plasma potential (ϕ) when $a = 10^{-3}$, $z_{\mathrm{d}} = 10^3$, $M = 1.5$, $\sigma = 0.1$, $\mu_{\mathrm{d}} = 10^6$.

the grain-surface potential. It is worth mentioning that the parameter values used in drawing the graphs are relevant to either space plasma laboratory plasma [17].

Acknowledgement

The author likes to thank the University Grants Commission for financial support vide No. F. PSW-025/01-02(ERO) dated 14 August 2001. The author is also thankful to S K Bhattacharya for his help in doing part of the numerical computation of the manuscript and to IUCAA (Inter-University Centre for Astronomy and Astrophysics), Ganeshkhind, Pune, India for their hospitality while doing the Associateship programme there during which the author has prepared this paper.

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