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$q\bar{q}$ Pair production in non-Abelian gauge fields

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Abstract. We calculate the $q\bar{q}$ pair production probability in the colour-flux tube model by considering the effect of non-Abelian interactions in the theory. Non-Abelian interactions in the colour field are time-dependent and hence should oscillate with a characteristic frequency ω_0, which depends on the amplitude of the field strength. Using the WKB approximation in complex time, we calculated the pair production probability. When the strength of the field is comparable to the quark masses, the corresponding pair creation probability is maximum, and for the static field $w_0 \to 0$, we recovered the well-known Schwinger result.

Keywords. Quark-gluon plasma; pair production; quarks; relative heavy-ion collisions; particles and resonance production.

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1. Introduction

The quark–anti-quark creation probability in hadron–hadron or nucleus–nucleus collisions has been a topic of great interest in high-energy physics. Moreover, it has been the subject of extensive research since the early days of strong interactions. In ultra-relativistic heavy-ion collisions (URHIC), it is believed that a large amount of energy is deposited in a small space-time region. It is this energy stored in the form of colour electric field energy that is thought to be responsible for the production of quark–anti-quark ($q\bar{q}$) pairs and eventually quark-gluon plasma (QGP). The gluon-induced pair creation, in the presence of a strong colour electric field mechanism is again discussed here with a new non-Abelian model solution, which reveals a surprising result. The advantages of this model solution are several. One is that it is an actual non-Abelian model compared to models that reflect only the non-Abelian nature as previously used by several authors [1–3]. The second is that calculation is purely analytical, without resorting to any approximation. Thirdly, we obtained a consistent result using real physics that is better compared to previous results by other authors [2–5].

2. Basic concept of the problem

When two nucleons collide at high energy, they almost just pass through each other. In the process, nucleons are excited, but in addition they leave a flux tube of deposited energy in the region of space they pass through. This energy is rapidly transformed into hadrons, the secondary particles usually produced in such collisions. By analysing proton–proton collision data, it is known that the deposition of energy at a higher rate is approximately $\frac{1}{3}$ GeV fm^{-3}. It does not increase much more with a further increase in collision energy, which only makes the flux tube longer.

If, instead of nucleons, two heavy nuclei collide (for simplicity, both with mass number A), a multiple superposition of the phenomenon just described is obtained, so that a given region of the flux tube now receives a much greater deposition of energy [6]. Simple geometric arguments reveal an energy density of at least $\varepsilon N A^{1/3}$ GeV fm^{-3}. This means that an energetic collision of two ^{238}U nuclei provides an average deposition of approximately 6 GeV fm^{-3}, which is well above the level needed for plasma formation [7].

Many authors have considered $q\bar{q}$ pair production in the above flux tube model [4,5,8–14]. The basic idea is that when two nuclei collide, they pass each other and become colour-charged, and are thus connected by colour flux tubes. In these flux tubes, a strong colour electric field is set up, which makes the vacuum unstable to pair production via the Schwinger mechanism [1]. As a result, the colour field energy in the flux tube is transformed into the energy of $q\bar{q}$ pairs. Thus, in the collision process, a large number of $q\bar{q}$ pairs, together with gluons produced through interactions, ultimately leads to the formation of a QGP. The self-interaction of gluons in the flux tube is likely to polarise the medium between the two receding nuclei. This leads to a characteristic normal mode of oscillations. Therefore, the colour field in the flux tube should be time-dependent (colour particles are coupled to each other via the gauge fields). These collective oscillations are non-Abelian. Hence, we assume a non-Abelian colour field model in which the characteristic frequency of collective oscillation depends on the amplitude of the oscillation.

During a nucleus–nucleus collision, a QGP is formed at the centre of the tube, which is of parallel plates of colour sources. Here we take an approximation whereby in the region of QGP formation, i.e. at the centre point between the receding nuclei, there is no colour source, only colour fields. This approximation is justifiable, because when compared to the size of the region of QGP formation, the receding nuclei are farther away at approximately infinity. Hence, the colour source effect can be neglected. The fields left behind at the centre of QGP formation by the receding nuclei are nothing but the solution of the Yang–Mills equation in vacuum. Furthermore, it is suggested that the colour field should be time-dependent owing to the non-Abelian interactions of gluons.

3. Model solution

According to our present level of understanding, all fundamental interactions except gravity are described by non-Abelian gauge theories. The gauge bosons of various non-Abelian symmetries mediate different interactions. All known fundamental fermion fields are divided into two broad classes. One class, called leptons, such as the electron, muon and neutron, does not have any strong interactions. In gauge theory terms, we can state that these are singlets under strong interaction gauge groups. The other class, called quarks, can be described by $SU(3)$ symmetry, and therefore has strong interactions. Under global $SU(3)$ symmetry, the three colours of the quarks transform like a triplet. If we try to gauge this colour symmetry, we obtain a theory called quantum chromodynamics (QCD). This contains eight gauge fields that are called gluons. To understand features due to non-Abelian effects, we have to solve the Yang–Mills equation. For simplicity, we consider the time-dependent vacuum solution of the $SU(2)$ Yang–Mills equation, which satisfies

$$\partial_\mu G_a^{\mu\nu} + g\varepsilon_{abc}A_{\mu b}G_c^{\mu\nu} = 0,$$

where the field tensor is

$$G_a^{\mu\nu} = \partial^\mu A_a^\mu - \partial^\nu A_a^\mu + g\varepsilon_{abc}A_b^\mu A_c^\nu,$$

and a, b, c are colour indices that take values $1, 2, 3$ and Lorentz indices $\mu, \nu = 0, 1, 2, 3$, with metric $(1, -1, -1, -1)$. ε_{abc} is an anti-symmetric Levicivita tensor.

Taking the temporal gauge $A^0 = 0$, the space homogeneity and the colour potential A_a are assumed to be the same in all colour directions, and the Yang–Mills equation is solved to obtain the colour potential $A(t)$. The general solution of this is a Jacobian elliptical function. As a special case, this becomes $\vec{A}(t) = -\sqrt{2}(E_\alpha/\omega_0)\tanh\omega_0 t$ where $\omega_0 = \sqrt{2gE}$

$$\vec{\varepsilon} = -\frac{\partial\vec{A}}{\partial t} = \sqrt{2}E_0\,\mathrm{sech}^2\,\omega_0 t. \tag{1}$$

This form of electric field is quite physical, where the electric field dies off smoothly as a function of time. It is due to the nonlinearity of the gauge theory. Further, the qualitative feature of eq. (1) is obvious and easy to understand. The colour field has an impulse profile. The amplitude and frequency of the plasma oscillation increase with the increase in the strength of the external impulse field. The impulse profile is roughly symmetric about $t = 0$, where the field assumes its maximum value $\sqrt{2}E_0$. It produces charged particles and accelerates them, producing a positive current and an associated field that continues to oppose the external field until the net field vanishes. At that time the particle production ceases and current reaches a maximum value. However, the external field is dying away. Since its lifetime is small, the electric field due to the particle's own motion quickly finds itself too strong. This excess of field strength begins to produce particles. It accelerates these in the opposite direction to the particles generating the existing current whilst simultaneously decelerating the particles in that current that continues until the particle current vanishes, at which point the net field has acquired its largest value, particle production continues and with that a negative net current forms and grows. Moreover it shows that the colour field in the flux tube is time-dependent (colour particles are coupled to each other via gauge fields) and the characteristic frequency of oscillation depends on the amplitude of the oscillation, which is one of the hallmarks of non-linear as opposed to linear oscillators.

Here we wish to calculate the quark–anti-quark pair production amplitude from mesons (scalars) in QGP for which we have to solve the Klein–Gordon (KG) equation which is the governing equation scalar particles. For this, we follow the method given by Biswas and Guha [9]. The basic principle in their calculation is the evaluation of the action integral $s(t_1, t_2)$. This is evaluated by solving the colour-coupled Klein–Gordon equation in the external colour potential. This equation is directly obtained from KG equation in the presence of external field in quantum electrodynamics by replacing the coupling constant e by the coupling constant g in quantum chromodynamics and introducing the Pauli spin matrices to respect $SU(2)$. The $SU(2)$ colour coupled KG equation in the external colour potential is evaluated to be (τ_α with $\alpha = 1, 2, 3$ are Pauli spin matrices):

$$\left[\partial_0^2 - \nabla^2 + 2ig\tau_\alpha A_\alpha \partial_3 + g^2 A_\alpha^2 + m^2\right] \begin{bmatrix} \phi_+ \\ \phi_- \end{bmatrix} = 0 \tag{2}$$

with

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and

$$A_\alpha = -\sqrt{2}\frac{E_\alpha}{\omega_0} \tanh \omega_0 t \quad \text{with } E^2 = \sum_{\alpha=0}^{3} E_\alpha^2.$$

Owing to the non-Abelian effect in eq. (2), it may be seen that ϕ_+ and ϕ_- are coupled. Thus, to solve the above equation, it has to be decoupled. It may be shown that the above equation can be decoupled by a unitary transformation in colour space defined by

$$U^+ = \begin{bmatrix} \frac{\sqrt{E^2 - E_3^2}}{E - E_3} & \frac{E_1 - iE_2}{\sqrt{E^2 - E_3^2}} \\ \frac{\sqrt{E^2 - E_3^2}}{E + E_3} & -\frac{E_1 - iE_2}{\sqrt{E^2 - E_3^2}} \end{bmatrix},$$

where $E^2 = E_1^2 + E_2^2 + E_3^2$.

The (column vector) wave function in turn transforms into

$$U^+ \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix} = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}. \tag{3}$$

The decoupled equations are evaluated as

$$\left[\partial_0^2 - \nabla^2 - 2\sqrt{2}\frac{ig}{\omega_0} E \tanh \omega_0 t \partial_3 + \frac{2g^2 E^2}{\omega_0^2} \tanh^2 \omega_0 t + m^2\right] \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix} = 0, \tag{4}$$

using $-\nabla^2 = p_1^2 + p_2^2 + p_3^2$ and $\partial_3 = ip_3$.

Equation (4) can be written in the form:

$$\left[\partial_0^2 + w(t)\right]\Psi_+ = 0 \quad \text{and} \quad \left[\partial_0^2 - w(t)\right]\Psi_- = 0 \tag{5}$$

with

$$w(t) = \left[w^2 + \left(p_3 + \frac{\sqrt{2}gE}{w_0}\tanh\omega_0 t\right)^2\right]^{1/2},$$

where $w^2 = p_1^2 + p_2^2 + m^2 = p_\perp^2 + m^2$.

This is nothing but the Klein–Gordon (KG) equation for one dimension.

Now, we try to solve the decoupled equations using the WKB method. The validity of the approximation can easily be checked [2], i.e.

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{w(t)}\right) \ll 1 \quad \text{or} \quad \left|\frac{\dot{w}(t)}{w^2(t)}\right| \ll 1.$$

To solve our KG equation, we follow the method using the WKB approximation in complex time, given by Biswas and Guha [9].

4. WKB approximation in complex time

In standard WKB approximation with real time t, a wave function is written as

$$\psi = \frac{c_1}{[w(t)]^{1/2}}\mathrm{e}^{\int iw(t)\mathrm{d}t} + \frac{c_2}{[w(t)]^{1/2}}\mathrm{e}^{-\int iw(t)\mathrm{d}t}.$$

In complex time t, we define

$$\int_{t_1}^{t} w(t)\mathrm{d}t = s(t, t_1) = \int_{t_1}^{t}[w^2 + V(t)]^{1/2}\mathrm{d}t,$$

where

$$V(t) = \left(p_3 + \sqrt{2}\frac{gE}{w_0}\tanh\omega_0 t\right)^2.$$

The turning points are determined from $w(t) = 0$, i.e. $[w^2 + V(t)]^{1/2} = 0$. The boundary conditions are chosen such that,

$$\psi = \mathrm{e}^{is(t,t_1)} \sim \mathrm{e}^{\int_{t_1}^{t} w(t)\mathrm{d}t} \quad \text{when } t \to -\infty$$

$$\psi = \mathrm{e}^{is(t,t_1)} + b\mathrm{e}^{-is(t,t_1)},$$

i.e.,

$$\psi = \mathrm{e}^{i\int w(t)\mathrm{d}t} + b\mathrm{e}^{-i\int w(t)\mathrm{d}t} \quad \text{when } \ t \to \infty.$$

Here b is called the reflection coefficient. Consider the pair production as a consequence of reflection in time; then the reflection coefficient is identified as the pair production amplitude [9].

The reflection coefficient is calculated as

$$b = -\frac{i\mathrm{e}^{2is(t_1,t_0)}}{1+\mathrm{e}^{2is(t_1,t_2)}}. \tag{6}$$

From this, the reflection amplitude $|b|^2$ can be calculated, which is our pair creation amplitude according to the complex time WKB approximation.

5. Evaluation of $s(t_1, t_2)$, $s(t_1, t_0)$ and the pair creation amplitude

$$s(t_1,t_2) = \int_{t_1}^{t_2} \left[w^2 + \left(p_3 + \frac{\sqrt{2}gE}{w_0} \tanh \omega_0 t \right)^2 \right]^{1/2} \mathrm{d}t, \tag{7}$$

where

$$t_1 = \frac{1}{w_0} \tanh^{-1} \frac{w_0}{\sqrt{2}gE} (iw - p_3)$$

and

$$t_2 = \frac{-1}{w_0} \tanh^{-1} \frac{w_0}{\sqrt{2}gE} (iw + p_3).$$

To solve eq. (7), let

$$p_3 + \frac{\sqrt{2}gE}{w_0} \tanh \omega_0 t = iw \sin\theta$$

$$s(t_1,t_2) = \frac{iw^2}{\sqrt{2}gE} \int_{-\pi/2}^{+\pi/2} \frac{\cos^2\theta \mathrm{d}\theta}{\left[1 - \frac{w_0^2}{2g^2E^2}(iw\sin\theta - p_3)^2\right]}.$$

This can easily be integrated to give

$$s(t_1,t_2) = -\frac{\pi i w_0}{2\sqrt{gE}} + \frac{\pi i}{2\sqrt{gE}} \left[\sqrt{w^2 + \left(\frac{w_0}{2} + p_3\right)^2} \right] + \left[\sqrt{w^2 + \left(\frac{w_0}{2} - p_3\right)^2} \right]. \tag{8}$$

In the Schwinger limit ($w_0 \to 0$), $S(t_1, t_2)$ becomes

$$s(t_1,t_2) = \pi i \sqrt{\frac{p^2+m^2}{2gE}}. \tag{9}$$

This is so because if we let $w_0 = 0$ in our model solution, we have the static field, that is, the expected Schwinger result. This is justifiable because when ω_0 goes to zero, the coupling constant g goes to zero. When this happens, the self-interaction of gluon fields and colour coupling between quarks vanish. Thus, the non-Abelian effect goes off thereby $SU(2)$ becomes $U(1)$ which is Abelian.

Similarly, $s(t_1, t_0)$ can be evaluated as

$$s(t_1,t_0) = -\frac{\pi i w_0}{4\sqrt{gE}} + \frac{\pi i}{4\sqrt{gE}}\left[\sqrt{w^2+\left(\frac{w_0}{2}+p_3\right)^2}\right] + \left[\sqrt{w^2+\left(\frac{w_0}{2}-p_3\right)^2}\right]. \tag{10}$$

In the Schwinger limit $w_0 \to 0$, $s(t_1, t_0)$ becomes

$$s(t_1,t_0) = \frac{\pi i}{2}\sqrt{\frac{p^2+m^2}{2gE}}. \tag{11}$$

Inserting the values for $s(t_1, t_2)$ and $s(t_1, t_0)$ in eq. (6), we obtain

$$|b| = \frac{\mathrm{e}^{\frac{\pi w_0}{2\sqrt{gE}}} \times \mathrm{e}^{\frac{-\pi}{\sqrt{2gE}}\left[\sqrt{w^2+\left(\frac{w_0}{2}+p_3\right)^2}\right]+\left[\sqrt{w^2+\left(\frac{w_0}{2}-p_3\right)^2}\right]}}{1+\mathrm{e}^{\frac{\pi w_0}{\sqrt{gE}}} \times \mathrm{e}^{\frac{-\pi}{\sqrt{2gE}}\left[\sqrt{w^2+\left(\frac{w_0}{2}+p_3\right)^2}\right]+\left[\sqrt{w^2+\left(\frac{w_0}{2}-p_3\right)^2}\right]}}. \tag{12}$$

The pair creation probability

$$|b|^2 = \frac{\mathrm{e}^{\frac{\pi w_0}{\sqrt{gE}}} \times \mathrm{e}^{\frac{-2\pi}{\sqrt{2gE}}\left[\sqrt{w^2+\left(\frac{w_0}{2}+p_3\right)^2}\right]+\left[\sqrt{w^2+\left(\frac{w_0}{2}-p_3\right)^2}\right]}}{\left|1+\mathrm{e}^{\frac{\pi w_0}{\sqrt{gE}}} \times \mathrm{e}^{\frac{-\pi}{\sqrt{2gE}}\left[\sqrt{w^2+\left(\frac{w_0}{2}+p_3\right)^2}\right]+\left[\sqrt{w^2+\left(\frac{w_0}{2}-p_3\right)^2}\right]}\right|^2}. \tag{13}$$

To estimate the pair creation probability, we let $p_3 = 0$, with the integration range of order of magnitude $2\sqrt{2}(gE/w_0)$, as suggested by the classical equation of motion [2]:

$$|b|^2 = \frac{\mathrm{e}^{\frac{\pi w_0}{2\sqrt{gE}}} \times \mathrm{e}^{\frac{-2\pi}{\sqrt{2gE}}\left[\sqrt{p^2+m^2+\frac{w_0^2}{4}}\right]}}{\left|1+\mathrm{e}^{\frac{\pi w_0}{\sqrt{gE}}} \times \mathrm{e}^{\frac{-\pi}{\sqrt{2gE}}\left[\sqrt{p^2+m^2+\frac{w_0^2}{4}}\right]}\right|^2}. \tag{14}$$

This shows that the pair creation probability depends crucially on the collective oscillation frequency w_0, a result that is expected. Inserting $w_0 = 0$ in eq. (14), we obtain

$$|b|^2 = \frac{\mathrm{e}^{-2\pi\sqrt{\frac{p^2+m^2}{2gE}}}}{\left|1+\mathrm{e}^{-\pi\sqrt{\frac{p^2+m^2}{2gE}}}\right|^2}. \tag{15}$$

This shows that when $w_0 \to 0$ (static limit) independently of gE, we recover the well-known Schwinger result in a better form [1].

From eq. (14) it is evident that when $(\mathrm{d}|b|^2/\mathrm{d}(gE)) = 0$, we obtain $gE \cong m$ and $(\mathrm{d}^2|b|^2/\mathrm{d}^2(gE))$ is negative. This shows that the pair creation probability is maximum for $gE \cong m$. Thus, when the quark–anti-quark mass is comparable with the field strength (gE), the pair creation probability dominates.

From this, we can infer that for the non-Abelian problem of interest, we first have to evaluate the ranges of values for w_0.

6. Discussion on the range of values for gE and ω_0

For the validity of WKB approximation, we have $|\dot{w}(t)/w^2(t)| \ll 1$. A simple calculation shows that $\left|\frac{\dot{w}(t)}{w^2(t)}\right| \cong \frac{gE}{\omega^2} = \frac{gE}{p_\perp^2+m^2}$, i.e. $\frac{gE}{p_\perp^2+m^2} \ll 1$. For gE, we take the estimate discussed by Pavel and Brink [5]. Thus, for p–p collision, $gE \approx 0.2$ GeV^2, for ^{32}s on ^{32}s, $gE < 0.6$ GeV^2 and for U–U collisions, $gE \leq 1.2$ GeV^2. As gE ranges from 0.2 to 1.5 GeV^2 and m for quarks varies from 0.3 to 177 GeV, the condition for the validity of WKB approximation is satisfied. Now we estimate the pair creation probability for our model in the range 0.63–1.732 GeV. It is also evident that the pair creation probability depends crucially on the magnitude of w_0; when $w_0 \to 0$, we recovered the Schwinger limit. From this we conclude that the well-known Schwinger estimate underestimates the pair creation probability by several orders of magnitude, as pointed out by other authors [2,3]. Thus, here we calculate the pair creation probabilities w and w_s for two cases: one (w) for the dynamic case (w_0 depends on gE), and the other (w_s) for the static case $w_0 \to 0$, independently of gE. w and w_s are calculated for a range of mass values: $m_u = m_d = 0.31$ GeV, $m_s = 0.505$ GeV, $m_c = 1.5$ GeV, $m_b = 5$ GeV, $m_t = 177$ GeV, i.e. m varies from 0.3 to 177 GeV. For the calculations we use constituent quark masses rather than current quark masses. On physical grounds we believe that the constituent masses ought to be used, since the colour flux-tube model incorporates the effect of confinement of the colour electric field and of the physical vacuum around it. To materialise pairs from a vacuum, the particles ought to move a distance equal to the Compton wavelength ($\hbar/mc$) under the influence of gE. For the light (u, d quarks) the use of current masses ($m = 10$ MeV) would give ($\hbar/mc$) ~ 20 fm and such length scales appear unreasonable for both p–p and A–A collisions. We therefore argue that pair creation via the flux tube model would be physically meaningful if ($\hbar/mc$) ≤ 1 fm or $m > 200$ MeV.

7. Conclusion

From the pair creation probability values in table 1, we can infer the following;

1. The pair creation probability calculated by the Schwinger mechanism (static limit) shows that the probability of production of $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$ and $b\bar{b}$ increases with increasing field strength. This cannot be justified because the oscillating field, the potential barrier through which the pair tunnels, moves up and down with time. When there is no barrier, the particle may tunnel through with a tunnelling time $\tau_1 = -(m/gE)$, where m is the mass of the particle and gE is the strength of the field. If τ_1 is much less than the period of oscillation of the field, i.e., $\tau_1 \sim \frac{1}{w_0}$, then tunnelling would take place during the half-period when there is no barrier and we essentially have a static situation (Schwinger limit). On the other hand, if $\tau_1 \gg \tau_f$, the pair cannot tunnel before the time interval of the oscillating electric field. In this limit, the pair creation rate depends on the frequency w_0. This is exactly what we obtained from our results.

Table 1. Pair creation probability w and w_s for different values of m and field strength.

m (GeV)	gE (GeV^2)	w_s	w
0.3	0.2	0.1500	0.2494
Up quark or down quark	0.3	0.1820	0.2503
	0.5	0.2018	0.2165
	0.8	0.2183	0.2007
	1.0	0.2242	0.1949
	1.3	0.2298	0.1894
	1.5	0.2324	0.1875
0.5	0.2	0.0711	0.1545
Strange quark	0.3	0.1020	0.1864
	0.5	0.1426	0.2499
	0.8	0.1739	0.2389
	1.0	0.1864	0.2298
	1.3	1.9890	0.2190
	1.5	0.2049	0.2136
1.5	0.2	5.79×10^{-4}	2.06×10^{-5}
Charm quark	0.3	6.42×10^{-4}	3.12×10^{-5}
	0.5	8.84×10^{-4}	4.05×10^{-3}
	0.8	0.0230	0.0246
	1.0	0.0333	0.0499
	1.3	0.0489	0.0896
	1.5	0.0580	0.1174
5.0	0.2	1.63×10^{-11}	2.09×10^{-19}
Bottom quark	0.3	1.79×10^{-9}	4.32×10^{-15}
	0.5	1.52×10^{-7}	1.67×10^{-12}
	0.8	4.07×10^{-6}	1.14×10^{-9}
	1.0	1.51×10^{-5}	1.52×10^{-8}
	1.3	5.89×10^{-5}	8.57×10^{-7}
	1.5	1.16×10^{-4}	3.37×10^{-5}
	5.0	–	7.69×10^{-3}

2. Comparison of w and w_s shows that estimation of the pair creation probability by the Schwinger mechanism is an underestimate by several orders of magnitude, which confirms the observations made by other authors [2,3].
3. Table 1 shows that according to our model the pair creation probability neither increases nor decreases with field strength (gE), but dominates when the quark–anti-quark mass is comparable to the strength of the field. In the case of $u\bar{u}$ quark–anti-quark production ($m = 0.31$ GeV) the pair creation probability is maximum when $gE \approx 0.3$ GeV^2. This shows that p–p collision is relevant for the creation of $u\bar{u}$. In the case of $s\bar{s}$ quark–anti-quark production ($m = 0.5$ GeV), the pair creation probability is maximum when $gE \approx 0.5$ GeV^2. This shows that collision such as ^{3}2s on ^{3}2s is relevant for $s\bar{s}$ creation. In the case of charm quark–anti-quark production, U–U is relevant.

4. To conclude, we described an important non-Abelian effect on the pair creation probability in collisions in high-energy physics. Once again we established that the colour field should be time-dependent due to non-Abelian interactions and should oscillate with a collective frequency w_0, which should depend on the field strength. It is evident that the collective frequency ranges from 0.63 to 1.732 GeV. Our model has several advantages over other models [2–5]. First, we assumed a non-Abelian colour field model, which is an exact solution of the Yang–Mills equation. Second, our calculation is completely analytical and no assumption is made to simplify it. Third, there is no need for any of the cumbersome calculations that other authors adopted [1–4]. Evaluation of the action integral $s(t_1, t_2)$, $s(t_1, t_0)$ is simple, albeit lengthy. Our result does not violate any of the former results in this field; moreover, it recovered the well-known Schwinger result. Finally, we obtained a very consistent result. If our model is good and the pair creation probability given by eq. (14) is true, it should be possible to test it in heavy ion collision experiments.

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