PRAMANA — journal of physics

Vol. 69, No. 2
August 2007
pp. 167–180

Regge behaviour of distribution functions and t and x-evolutions of gluon distribution function at low-x

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MS received 20 October 2005; revised 6 December 2006; accepted 11 April 2007

Abstract. In this paper, t and x-evolutions of gluon distribution function from Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) evolution equation in leading order (LO) at low-x are presented assuming the Regge behaviour of quarks and gluons at this limit. We compare our results of gluon distribution function with MRST 2001, MRST 2004 and GRV 1998 parametrizations and show the compatibility of Regge behaviour of quark and gluon distribution functions with perturbative quantum chromodynamics (PQCD) at low-x. We also discuss the limitations of Taylor series expansion method used earlier to solve DGLAP evolution equations in the Regge behaviour of distribution functions.

Keywords. Regge behaviour; DGLAP evolution equations; low-x; structure functions; gluon distribution function.

PACS Nos 11.55.Jy; 12.38.-t; 14.70.Dj; 25.30.-c

1. Introduction

The measurement of proton and deuteron structure functions by deep inelastic scattering (DIS) processes in the low-x region, where x is the Bjorken variable, have been of great importance for understanding the quark-gluon substructure of hadrons [1,2]. In addition to this, it is also important to have the knowledge of the gluon distribution inside hadrons at low-x, because gluons are expected to be dominant in that region. Moreover, gluon distributions are important inputs in many high energy processes and also important for examining PQCD [3], the underlying dynamics of quarks and gluons. In PQCD, the high-Q^2 behaviour of DIS can be explained by the DGLAP evolution equations [4,5]. These equations introduced the parton distribution functions which can be interpreted as the probability of finding, say in a proton, respectively a quark, an antiquark or a gluon with virtuality less than Q^2 and with momentum fraction x. When $Q^2 \to \infty$, i.e., very large, the Q^2-evolution of these densities (at fixed-x) are given by the DGLAP equations and these are often considered as a very good test of PQCD. The solutions of the

DGLAP equations can be calculated either by numerical integration in steps or by taking the moments of the distributions [6]. The approximate solutions of DGLAP evolution equations have been reported in recent years [7–11] with considerable phenomenological success. Structure functions thus calculated are expected to rise approximately with a power of x towards low-x which is supported by Regge theory [12,13].

The low-x region of DIS offers a unique possibility to explore the Regge limit [12] of PQCD. The low-x behaviour of parton distributions can be considered by a triple pole pomeron model [13,14] at the initial scale Q_0^2 and then evolved using DGLAP evolution equations. It is possible to have the same singularity structure for quarks and gluons, in agreement with Regge theory. The Regge behaviour of the structure function F_2 in the high-Q^2 region reflects itself in the low-x behaviour of the quark and antiquark distributions. Thus the Regge behaviour of the sea quark and antiquark distributions for low-x is given by $q_{\rm sea}(x) \sim x^{\lambda p}$ with pomeron exchange [13] of intercept $\lambda p = -1$. But the valence quark distribution for low-x given by $q_{\rm val}(x) \sim x^{-\lambda r}$ corresponds to a reggeon exchange of intercept $\lambda r = 1/2$.

In our present work, we have derived the solution of DGLAP evolution equation for gluon distribution function at low-x in leading order (LO) considering Regge behaviour of distribution functions. The t and x-evolutions of LO gluon distribution functions thus obtained have been compared with global MRST 2001, MRST 2004 and GRV 1998 parametrizations. Here we overcome the limitations that arise from Taylor series expansion method [15,16] and this method is also mathematically simple. In this paper, §1, 2, 3 and 4 are the introduction, theory, results and discussions and conclusions respectively.

2. Theory

The LO DGLAP evolution equation for gluon distribution function has the standard form [17,18]

$$\begin{aligned} Q^2 \frac{\partial}{\partial Q^2} G(x,Q^2) = \frac{3\alpha_{\rm s}(Q^2)}{\pi} \Bigg\{ & \left(\frac{11}{12} - \frac{N_{\rm f}}{18} + \ln(1-x) \right) G(x,Q^2) \\ & + \int_x^1 {\rm d}\omega \Bigg[\frac{\omega G(x/\omega, Q^2) - G(x,Q^2)}{1-\omega} \\ & + \left(\omega(1-\omega) + \frac{1-\omega}{\omega} \right) G(x/\omega, Q^2) \\ & + \frac{2}{9} \left(\frac{1+(1-\omega)^2}{\omega} \right) F_2^{\rm s}(x/\omega, Q^2) \Bigg] \Bigg\}, \end{aligned} \tag{1}$$

where $\alpha_{\rm s}(Q^2) = 12\pi/[(33-2N_{\rm f})\ln(Q^2/\Lambda^2)]$, Λ being the QCD cut-off parameter and $N_{\rm f}$ is the number of flavours.

After changing the variable Q^2 by t, where $t = \ln(Q^2/\Lambda^2)$, we get

$$\frac{\partial G(x,t)}{\partial t} - \frac{A_{\rm f}}{t} \left\{ \left(\frac{11}{12} - \frac{N_{\rm f}}{18} + \ln(1-x) \right) G(x,t) + I_{\rm g} \right\} = 0, \tag{2}$$

where

$$I_{\mathrm{g}} = \int_x^1 \mathrm{d}\omega \left[\frac{\omega G(x/\omega,t) - G(x,t)}{1-\omega} + \left(\omega(1-\omega) + \frac{1-\omega}{\omega}\right) G(x/\omega,t) \right.$$
$$\left. + \frac{2}{9}\left(\frac{1+(1-\omega)^2}{\omega}\right) F_2^{\mathrm{s}}(x/\omega,t)\right],$$

and $A_{\mathrm{f}} = 36/(33-2N_{\mathrm{f}})$.

As the gluons are expected to be dominant at low-x, we can neglect the quark contribution to the evolution equation of gluon distribution function and we get the amount of contribution of quark to the gluon distribution function at different x and Q^2 phenomenologically.

Among the various methods to solve DGLAP equations, one simple method is to use Taylor expansion [19] to transform the integro-differential equations into partial differential equations and thus to solve them by standard methods [20,21]. But when we consider Regge behaviour of structure functions to solve these evolution equations, the use of Taylor expansion becomes limited. In this method, we introduce the variable $u = 1-\omega$ and we get $(x/\omega) = x/(1-u) = x\sum_{k=0}^{\infty} u^k$. Since $0 < u < (1-x)$, $|u| < 1$. It implies that $x/(1-u) = x\sum_{k=0}^{\infty} u^k$ is convergent. Applying the Taylor expansion, for example, for the gluon distribution function, we get

$$\begin{aligned} G\left(\frac{x}{\omega},t\right) &= G\left(\frac{x}{1-u},t\right) = G\left(x + x\sum_{k=1}^{\infty} u^k, t\right) \\ &= G(x,t) + x\sum_{k=1}^{\infty} u^k \frac{\partial G(x,t)}{\partial x} \\ &\quad + \frac{1}{2}x^2\left(\sum_{k=1}^{\infty} u^k\right)^2 \frac{\partial^2 G(x,t)}{\partial x^2} + \cdots . \end{aligned} \tag{3}$$

When we apply the Regge behaviour, we take the form of gluon distribution function as $G(x,t) = A(t)x^{-\lambda}$, where $A(t)$ is a function of t and λ is the intercept. Then

$$\frac{\partial G(x,t)}{\partial x} = A(t)(-\lambda)x^{-\lambda-1} = A(t)x^{-\lambda}(-\lambda)x^{-1} = (-1)\lambda x^{-1}G(x,t),$$

$$\begin{aligned} \frac{\partial^2 G(x,t)}{\partial x^2} &= A(t)(-\lambda)(-\lambda-1)x^{-\lambda-2} = A(t)x^{-\lambda}(-\lambda)x^{-1} \\ &= (-1)^2\lambda(\lambda+1)x^{-2}G(x,t), \end{aligned}$$

$$\frac{\partial^3 G(x,t)}{\partial x^3} = (-1)^3\lambda(\lambda+1)(\lambda+2)x^{-3}G(x,t)$$

and so on. So eq. (3) becomes

$$G\left(\frac{x}{\omega},t\right) = G(x,t) + \left(\sum_{k=1}^{\infty} u^k\right)(-1)\lambda G(x,t)$$
$$+\frac{1}{2}\left(\sum_{k=1}^{\infty} u^k\right)^2 (-1)^2\lambda(\lambda+1)G(x,t) + \cdots.$$

In the expansion series, we will get terms with alternate positive and negative signs and contribution from λ to each term increases. So in this case, it is not possible to truncate this infinite series into finite number of terms by applying boundary condition such as low-x [22] or so and also this is not a convergent series [19]. So, in solving DGLAP evolution equation applying Regge behaviour of distribution functions, we cannot apply Taylor series expansion method.

Now let us consider the Regge behaviour of gluon distribution function [13,23] as

$$G(x,t) = T(t)x^{-\lambda_{\rm g}}, \tag{4}$$

where $T(t)$ is a function of t and $\lambda_{\rm g}$ is the Regge intercept for gluon distribution function. But according to Regge theory, the high energy (low-x) behaviour of both gluons and sea quarks is controlled by the same singularity factor in the complex angular momentum plane [13], and so we would expect $\lambda_{\rm s} = \lambda_{\rm g} = \lambda$, where λ is taken as a constant factor throughout the calculation. So eq. (4) becomes

$$G(x,t) = T(t)x^{-\lambda} \tag{5}$$

and

$$G(x/\omega,t) = T(t)\omega^{\lambda}x^{-\lambda} = \omega^{\lambda}G(x,t). \tag{6}$$

Since the DGLAP evolution equations of gluon and singlet structure functions in leading order are in the same forms of derivative with respect to t, we consider the ansatz [7,15,24,25]

$$G(x,t) = K(x)F_2^{\rm s}(x,t) \tag{7}$$

for simplicity, where $K(x)$ is a parameter to be determined from phenomenological analysis and we assume $K(x) = k$, ax^b or ce^{dx}, where k, a, b, c and d are constants. Though we have assumed some simple standard functional forms of $K(x)$, we cannot rule out the other possibilities. So, we have to consider k, a, b, c and d as some parameters. Actual functional form of $K(x)$ can be determined by simultaneous solution of coupled equations of gluon and singlet structure functions.

Now,

$$F_2^{\rm s}(x/\omega,t) = \frac{1}{K(x/\omega)}G(x/\omega,t) = \frac{\omega^{\lambda}}{K(x/\omega)}G(x,t). \tag{8}$$

Putting eqs (6) and (8) in eq. (2), we get

$$\frac{\partial G(x,t)}{\partial t} - \frac{G(x,t)}{t}P(x) = 0, \tag{9}$$

where

$$P(x) = A_{\mathrm{f}}\{A + \ln(1-x) + f(x)\},$$

$$A = \frac{11}{12} - \frac{N_{\mathrm{f}}}{18}$$

and

$$f(x) = \int_x^1 \mathrm{d}\omega \left[\frac{(\omega^{\lambda+1}-1)}{1-\omega} + \left(\omega(1-\omega) + \frac{1-\omega}{\omega}\right)\omega^{\lambda} + \frac{2}{9}\left(\frac{1+(1-\omega)^2}{\omega}\right)\frac{\omega^{\lambda}}{K(x/\omega)}\right].$$

Integrating eq. (9) [26] we get

$$G(x,t) = C.t^{P(x)}, \tag{10}$$

where C is a constant of integration.

Applying initial conditions at $x = x_0, G(x,t) = G(x_0,t)$, and at $t = t_0$, $G(x,t) = G(x,t_0)$, we found the t and x-evolution equations for the gluon distribution function respectively as

$$G(x,t) = G(x,t_0)(t/t_0)^{P(x)} \tag{11}$$

and

$$G(x,t) = G(x_0,t)t^{\{P(x)-P(x_0)\}}. \tag{12}$$

Now ignoring the quark contribution to the gluon distribution function we get from the standard DGLAP evolution eq. (2)

$$\frac{\partial G(x,t)}{\partial t} - \frac{A_{\mathrm{f}}}{t}\left\{\left(\frac{11}{12} - \frac{N_{\mathrm{f}}}{18} + \ln(1-x)\right)G(x,t) + I'_{\mathrm{g}}\right\} = 0, \tag{13}$$

where

$$I'_{\mathrm{g}} = \int_x^1 \mathrm{d}\omega \left[\frac{\omega G(x/\omega,t) - G(x,t)}{1-\omega} + \left(\omega(1-\omega) + \frac{1-\omega}{\omega}\right)G(x/\omega,t)\right].$$

Now pursuing the same procedure as above, we get the t and x-evolution equations for the gluon distribution function ignoring the quark contribution in LO respectively as

$$G(x,t) = G(x,t_0)(t/t_0)^{B(x)} \tag{14}$$

and

$$G(x,t) = G(x_0,t)t^{\{B(x)-B(x_0)\}}. \tag{15}$$

Here

$$B(x) = A_{\mathrm{f}}\left\{(A + \ln(1-x)) + \int_x^1 \mathrm{d}\omega \left[\frac{(\omega^{\lambda+1}-1)}{1-\omega} + \left(\omega(1-\omega) + \frac{1-\omega}{\omega}\right)\omega^{\lambda}\right]\right\}.$$

3. Results and discussions

In this paper, we have obtained a new description of t and x-evolutions of gluon distribution function considering Regge behaviour of distribution functions given by eqs (11) and (12) respectively. We are also interested to see the contribution of quark to gluon distribution functions at low-x and high-Q^2, theoretically which should decrease for $x \to 0$, $Q^2 \to \infty$ [2,27]. We have obtained this description of gluon distribution function ignoring the quark contribution in the evolution equation given by eqs (14) and (15). We compare our results of t and x-evolutions of gluon distribution function in LO with MRST 2001, MRST 2004 and GRV 1998 global parametrizations. We have taken the MRST 2001 fit [28] to the CDFIB data [29] for $Q^2 = 20$ GeV^2, in which they obtained the optimum global NLO fit with the starting parametrization of the partons at $Q_0^2 = 1$ GeV^2 given by $xg = 123.5x^{1.16}(1-x)^{4.69}(1-3.57x^{0.5}+3.41x) - 0.038x^{-0.5}(1-x)^{10}$. The optimum fit corresponds to $\alpha_s(M_z^2) = 0.119$, i.e. $\Lambda_{\overline{\rm MS}}(N_f = 4) = 323$ MeV. We have also taken the MRST 2004 fit [30] to the ZEUS [31] and H1 [32] data with $x < 0.01$ and $2 < Q^2 < 500$ GeV^2 for $Q^2 = 100$ GeV^2, in which they have taken parametric form for the starting distribution at $Q_0^2 = 1$ GeV^2 given by $xg = A_g x^{-\lambda g}(1-x)^{3.7}(1+\varepsilon_g\sqrt{x}+\gamma_g x) - Ax^{-\delta}(1-x)^{10}$, where the powers of the $(1-x)$ factors are taken from MRST 2001 fit. The λ_g, ε_g, A and δ are taken as free parameters. The value of $\alpha_s(M_z^2)$ is taken to be the same as in the MRST 2001 fit. We have taken the GRV 1998 parametrization [33] for $10^{-2} \le x \le 10^{-5}$ GeV^2 and $20 \le Q^2 \le 40$ GeV^2, where they used H1 [34] and ZEUS [35] high precision data on $G(x, Q^2)$. They have chosen $\alpha_s(M_z^2) = 0.114$, i.e. $\Lambda_{\overline{\rm MS}}$ $(N_f = 4) = 246$ MeV. The input densities have been fixed using the data sets of HERA [34], SLAC [36], BCDMS [37], NMC [38] and E665 [39]. The resulting input distribution at $Q^2 = 0.40$ GeV^2 is given by $xg = 20.80x^{1.6}(1-x)^{4.1}$.

We compare our results from eqs (11) and (12) for $K(x) = k$, ax^b and $c\mathrm{e}^{-dx}$, where k, a, b, c and d are constants. In our work, we have found that the values of the gluon distribution function remain almost same for $b < 0.00001$ and $d < 0.0001$. We have chosen $b = 0.00001$ and $d = 0.0001$ for our calculation and the best-fit graphs are observed by changing the values of k, a and c. As the value of λ should be close to 0.5 in quite a broad range of low-x [13,40], we have taken $\lambda = 0.5$ in our calculation.

In figures 1a–f, we compare our result of t-evolution of gluon distribution function with GRV 1998 gluon distribution parametrization at $x = 10^{-5}$ and 10^{-4} respectively. At both the x values, we compare our results for $K(x) = k, ax^b$ and $c\mathrm{e}^{-dx}$. At $x = 10^{-5}$, the best-fit results are for $k = 1.55$, $a = 1.55$ and $c = 1.55$ and at $x = 10^{-4}$, the best-fit results are for $k = 3$, $a = 3$ and $c = 3$. The figures show good agreement of our result with GRV 1998 parametrization at low-x.

In figures 2a–d, we compare our result of x-evolution of gluon distribution function for $K(x) = k$ with MRST 2001, MRST 2004 and GRV 1998 global parametrizations. Figure 2a shows the comparison of our result with MRST 2001 parametrization at $Q^2 = 20$ GeV^2. The best-fit result corresponds to $k = 0.8$. Figure 2b shows the comparison with MRST 2004 parametrization at $Q^2 = 100$ GeV^2. Here the best-fit result corresponds to $k = 0.33$. Figure 2c shows the comparison with GRV 1998 parametrization at $Q^2 = 20$ GeV^2. Here the best-fit result corresponds

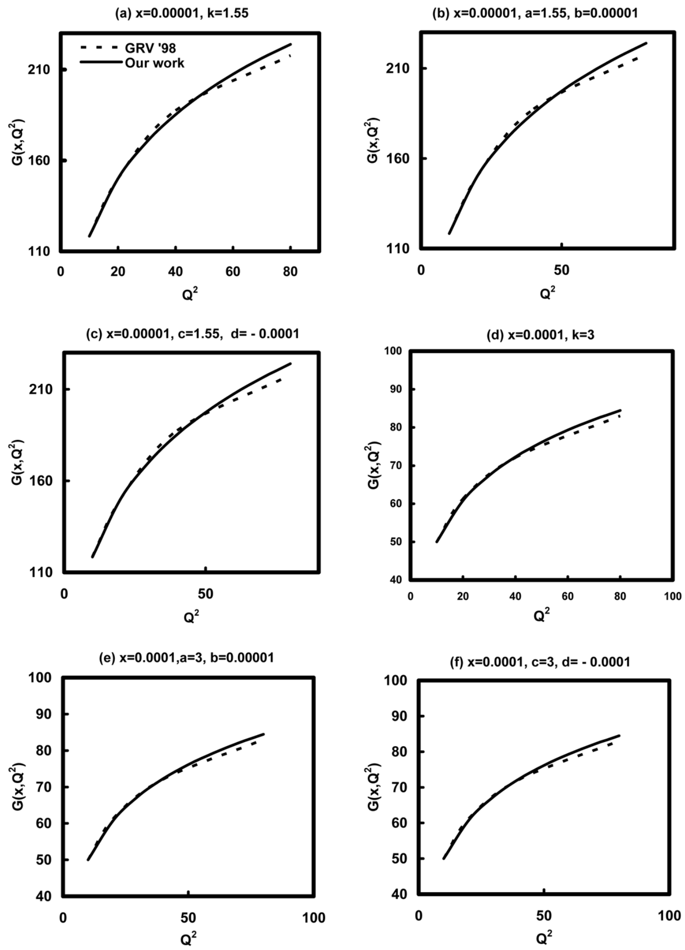

Figure 1. t-evolution of gluon distribution function in LO for $\lambda = 0.5$ and $K(x) = k, ax^b$ and ce^{dx} for the representative values of x. Data points at lowest-Q^2 values are taken as input to test the evolution eq. (11). Here figures 1a–f are the best-fit graphs of our results with GRV 1998 parametrization corresponding to $K(x) = k, ax^b$ and ce^{dx} respectively.

to $k = 0.11$. Figure 2d shows the comparison with GRV 1998 parametrization at $Q^2 = 40$ GeV^2. Here the best-fit result corresponds to $k = 0.12$. In some recent papers [40], Choudhury and Saharia presented a form of gluon distribution function at low-x obtained from a unique solution with one single initial condition through the application of the method of characteristics [41]. They have overcome the limitations of non-uniqueness of some of the earlier approaches [15]. So, it is theoretically and phenomenologically favoured over the earlier approximations. We have presented these results with GRV 1998 parametrizations and our results, and found that with decreasing x we get a better fit of our result to GRV 1998

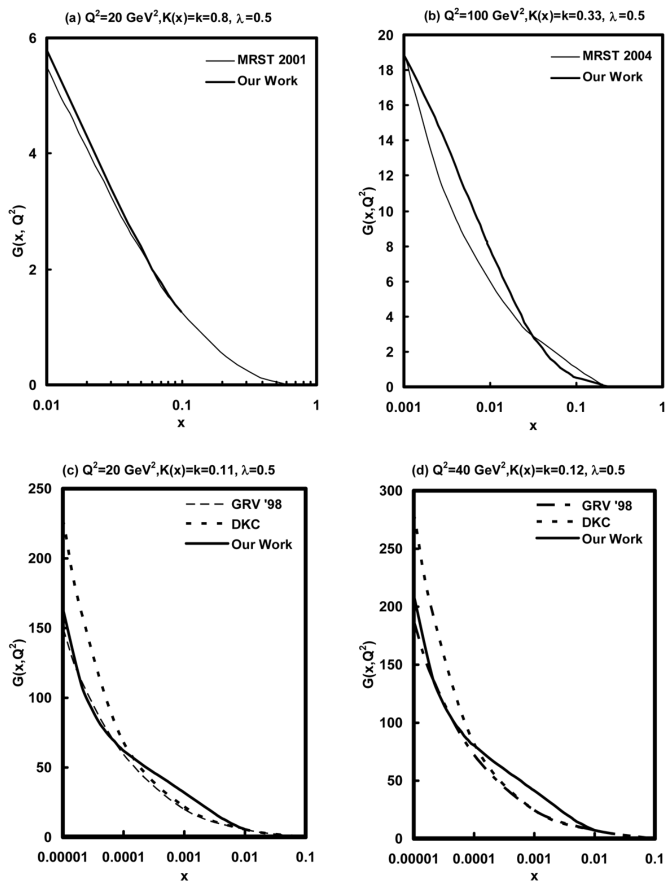

Figure 2. x-evolution of gluon distribution function in LO for $\lambda = 0.5$ and $K(x) = k$. Here figure 2a is the best-fit graph of our result with MRST 2001 parametrization for $Q^2 = 20$ GeV^2. Data points at $x = 0.099$ is taken as input to test the evolution eqs (12). Figures 2b is the best-fit graph of our result with MRST 2004 parametrization for $Q^2 = 100$ GeV^2. Data points at $x = 0.2$ is taken as input to test the evolution eqs (12). Figure 2c and 2d are the best-fit graphs of our result with GRV 1998 parametrization for $Q^2 = 20$ GeV^2 and 40 GeV^2 respectively. Data points at $x = 0.01$ are taken as input to test the evolution eqs (12).

parametrization in comparison with those results. We have compared our result of x-evolution of gluon distribution function with MRST 2001, MRST 2004 and GRV 1998 global parametrizations for $K(x) = ax^b$ and $c\mathrm{e}^{-dx}$ also and found the same graphs as for $K(x) = k$. The best-fit results correspond to $a = c = 0.8$ with MRST 2001 parametrization at $Q^2 = 20$ GeV^2, $a = c = 0.33$ with MRST 2004 parametrization at $Q^2 = 100$ GeV^2, $a = c = 0.11$ with GRV 1998 parametrization at $Q^2 = 20$ GeV^2 and $a = c = 0.12$ with GRV 1998 parametrization at $Q^2 = 40$ GeV^2.

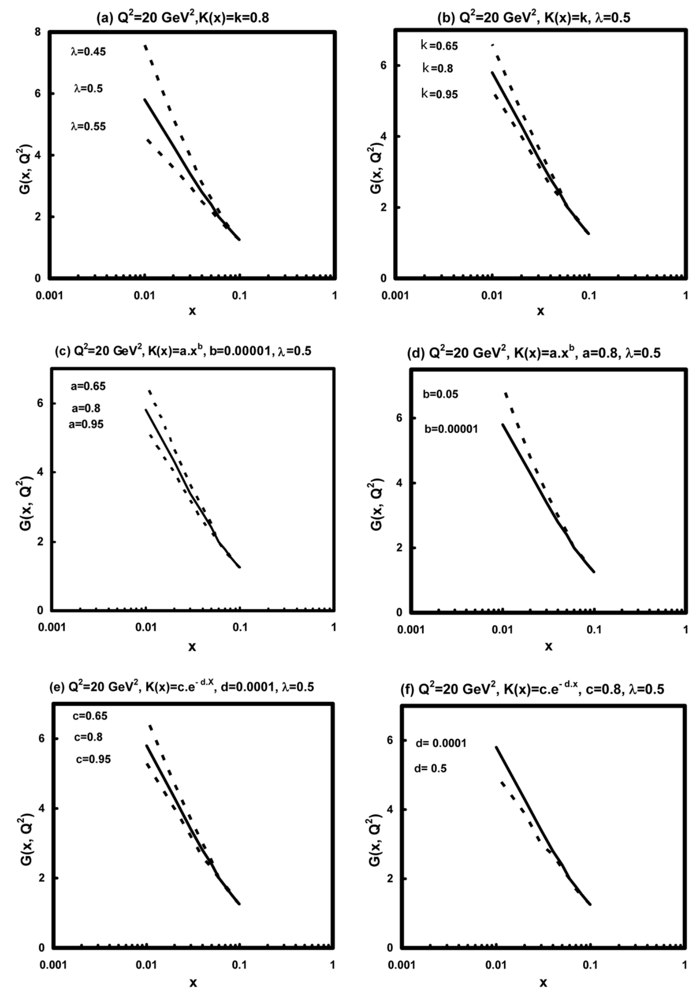

Figure 3. Figures 3a–f show the sensitivity of the parameters λ, k, a, b, c and d respectively at $Q^2 = 20$ GeV^2 with the best-fit graph of our results with MRST 2001 parametrization.

Figures 3a–f show the sensitivity of the parameters λ, k, a, b, c and d respectively. Taking the best-fit figures to the x-evolution of gluon distribution function with MRST 2001 parameterization at $Q^2 = 20$ GeV^2, we have given the ranges of the parameters as $0.45 \leq \lambda \leq 0.55$, $0.65 \leq k \leq 0.95$, $0.65 \leq a \leq 0.95$, $0.00001 \leq b \leq 0.05$, $0.65 \leq c \leq 0.95$ and $0.0001 \leq d \leq 0.5$.

4. Conclusions

We have considered the Regge behaviour of singlet structure function and gluon distribution function to solve DGLAP evolution equations. Here we find the t and

x-evolutions of gluon distribution function in LO. We see that our results are in good agreement with MRST 2001, MRST 2004 and GRV 1998 global parametrizations especially at low-x and high-Q^2 region. We can conclude that Regge behaviour of quark and gluon distribution functions is compatible with PQCD at that region assuming the Regge intercept to be the same for both quark and gluon. We were also interested to see the amount of contribution of quark to the gluon distribution function at different x and Q^2 but it has been observed that in our x–Q^2 region of discussion quark contributes appreciably to gluon distribution function. So we cannot ignore the contribution of quark in that region. Here we have overcome the limitations that arise from Taylor series expansion method. Considering Regge behaviour of distribution functions DGLAP equations become quite simple to solve and so this method is a viable simple alternative to other methods. But here also the problem of ad hoc assumption of the function $K(x)$, the relation between singlet structure function and gluon distribution function could not be overcome. Work is going on to obtain the simultaneous solution of coupled DGLAP evolution equations for singlet structure function and gluon distribution function. Moreover, here we solve only leading order evolution equations. We expect that next-to-leading order equations are more correct and their solutions will give better fit to global data and parametrizations.

Appendix

Computer programmes for calculation of t and x-evolutions of gluon structure function

```
//*t-evolution of gluon structure function for k=constant*//
#include<math.h>
#include<stdio.h>
#include<conio.h>
#define Nf       4
#define t        (log(Q^2)-log(Z^2))
#define t0       (log(Q0^2)-log(Z^2))
#define Af       0.16
#define C        0.694444
#define Z        0.323
#define K        0.11
#define λ        0.5
#define x        0.01
#define Q0^2     10
#define G(x0,t)  5.67
#define div      10000
#define ul       0.9999
#define f(w)     (K*((pow(w,(λ+1))-1)/(1-w))+(1-w)*(w+pow(w,-
1))*K*pow(w,λ)+0.222222*((1+pow((1-w),2))/w)*pow(w,λ))
#define p        (U/K)
#define B        (Af*(C+log(1-x)+p))
#define G(x,t)   (G(x0,t)*(pow((t/t0),B)))
```

```
main()
{
int i;
double h,s,sa,sb,U,Q²;
clrscr();
h=(ul-x)/div;
s=f(ul)+f(x);
for(i=1;i<=div-1;i=i+1)
{
sa=(x+(i*h));
s=s+(2*(f(sa)));
}
for(i=1;i<=div-1;i=i+2)
{
sb=(x+(i*h));
s=s+(2*(f(sb)));
}
U=(s*h)/3.0;
printf("\n integral=%lf", U);
printf("\n value of p=%lf", p);
printf("\n value of Q²:  ");
scanf("%lf", &Q²);
printf("\n value of G(x,t)=%lf", G(x,t));
getch();
return(0);
}

//*x-evolution of gluon structure function for k=constant*//
#include<math.h>
#include<stdio.h>
#include<conio.h>
#define Nf       4
#define Af       0.16
#define C        0.694444
#define Z        0.323
#define K        0.16
#define λ        0.6
#define x0       0.1
#define Q²       100
#define G(x0,t)  1.1765
#define div      1000
#define ul       0.9999
#define f(w)     (K*((pow(w,(λ+1))-1)/(1-w))+(1-w)*(w+pow(w,-
1))*K*pow(w,lam)+0.222222*((1+pow((1-w),2))/w)*pow(w,λ))
```

```
#define p      (U/K)
#define B      (Af*(C+log(1-x)+p))
#define p0     (U0/K)
#define B0     (Af*(C+log(1-x0)+p0))
#define G(x,t) (G(x0,t)*(pow((log(Q^2)-log(Z^2)),(B-
B0)))*(pow((x0/x),λ)))
main()
{
int i;
double h,s,sa,sb,U,h0,s0,sa0,sb0,U0,x;
clrscr();
h0=(ul-x0)/div;
s0=f(ul)+f(x0);
for(i=1;i<=div-1;i=i+1)
{
sa0=(x0+(i*h0));
s0=s0+(2*(f(sa0)));
}
for(i=1;i<=div-1;i=i+2)
{
sb0=(lm0+(i*h0));
s0=s0+(2*(f(sb0)));
}
U0=(s0*h0)/3.0;
printf("\n integral=%lf", U0);
printf("\n value of p0=%lf", p0);
printf("\n value of x:  ");
scanf("%lf", &x);
h=(ul-x)/div;
s=f(ul)+f(x);
for(i=1;i<=div-1;i=i+1)
{
sa=(x+(i*h));
s=s+(2*(f(sa)));
}
for(i=1;i<=div-1;i=i+2)
{
sb=(x+(i*h));
s=s+(2*(f(sb)));
}
U=(s*h)/3.0;
printf("\n integral=%lf", U);
printf("\n value of p=%lf", p);
printf("\n value of G(x,t)=%lf", G(x,t));
getch();
return(0);
}
```

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