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Bianchi Type-I, V and VIo models in modified generalized scalar–tensor theory

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Abstract. In modified generalized scalar–tensor (GST) theory, the cosmological term Λ is a function of the scalar field ϕ and its derivatives $\dot{\phi}^2$. We obtain exact solutions of the field equations in Bianchi Type-I, V and VIo space–times. The evolution of the scale factor, the scalar field and the cosmological term has been discussed. The Bianchi Type-I model has been discussed in detail. Further, Bianchi Type-V and VIo models can be studied on the lines similar to Bianchi Type-I model.

Keywords. Bianchi type; scalar–tensor theory; cosmological term; scalar field.

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1. Introduction

Theoretical predictions for the cosmological constant Λ in the very early Universe exceed observational limits by some 120 orders of magnitude. Although one possible mechanism for making the cosmological constant vanish is to assume wormholes [1,2], the mechanism of the decaying cosmological term is usually formulated in terms of a scalar field [3–5]. The generalized scalar–tensor (GST) theory [6–8] has been revived recently and has been used to solve the cosmological constant problem [9,10]. It might be modified, however, to explain the graceful exit problem [11].

The nucleosynthesis has been investigated in modified Brans–Dicke (BD) theory [12,13] with the assumption $\Lambda = \Lambda(\phi)$. The assumption resulted in imposing a constraint on the dependence of ϕ on time. Fukui *et al* [14,15] have considered a modified GST to loosen the constraint. The theory is called a modified GST, because Λ is a function not only of ϕ but also of $(\phi_{,i}\phi^{,i})$. Thus Λ depends on the kinetic energy of the Brans–Dicke-type scalar field. A similar idea has been considered by Moffat [16] in connection with the age problem. Recent k-inflation has a large class of higher-order scalar kinetic term [17]. This modification might be suitable for studying the exit problem too. Fukui *et al* [15] have considered the Robertson–Walker Universe and obtained exact solutions for the scale factor, the scalar field and the cosmological term which evolve with a different dependence on time for each era of the Universe. The 'cosmological constant' and flatness problems are examined in the simple case of a flat Universe. A correspondence with solutions of five-dimensional space–time–matter (STM) theory [18,19] has also been pointed out.

In the present work we have considered exact solutions of the field equations in Bianchi Type-I, V and VIo space–times. The evolution of scale factor, the scalar field and the cosmological term has been examined. Bianchi Type-III and Kantowski–Sachs models can be studied on similar lines, but we have not been able to find a solution for scalar field ϕ.

2. Basic field equations

The action principle of the modified GST theory [14] is

$$0 = \delta \int \left\{ \phi[R + 2\Lambda(\phi, \phi_{,i}\phi^{,i})] + \left(\frac{16\pi}{c^4}\right) L_m - \omega(\phi)\left(\frac{\phi_{,i}\phi^{,i}}{\phi}\right) \right\} \sqrt{-g}\, \mathrm{d}\Omega \tag{2.1}$$

where $i = 0, 1, 2, 3$. The variation leads to the following field equations for the metric fields g_{ij}:

$$R_{ij} - \frac{1}{2} g_{ij} R = \left(\frac{8\pi}{c^4}\right)(T_{ij} + T_{ij}(\phi)). \tag{2.2}$$

Here R_{ij} is the Ricci tensor and $R = g^{ij} R_{ij}$. T_{ij} is the energy–momentum tensor of matter, and the energy–momentum tensor of the scalar field is defined as [3]

$$\frac{8\pi}{\phi c^4} T_{ij}(\phi) \equiv \frac{\omega}{\phi^2}\left(\phi_{,i}\,\phi_{,j} - \frac{1}{2} g_{ij} b\right) + \frac{1}{\phi}(\phi_{;ij} - g_{ij}\Box\phi) + g_{ij}\Lambda - 2\frac{\partial\Lambda}{\partial b}\phi_{,i}\,\phi_{,j} \tag{2.3}$$

where $b = \phi_{,i}\phi^{,i}$. The field equation for ϕ is

$$R + 2\Lambda + 2\phi\frac{\partial\Lambda}{\partial\phi} - 4\frac{\partial}{\partial x^l}\left(\phi\frac{\partial\Lambda}{\partial b}\right)\phi^l - 4\phi\frac{\partial\Lambda}{\partial b}\Box\phi = \frac{\omega}{\phi^2} b - \frac{2\omega}{\phi}\Box\phi - \frac{b}{\phi}\frac{\mathrm{d}\omega}{\mathrm{d}\phi}. \tag{2.4}$$

Equation (2.4) ensures that the conservation law ${T_i}^k{}_{;k} = 0$ holds, as required. When $\Lambda = \Lambda(\phi)$ only, the field equations reduce to those obtained by Endo and Fukui [12] (see also Singh and Singh [20]). From eqs (2.2) and (2.4) we have

$$\begin{aligned} R_{ij} = \Lambda g_{ij} &+ 2\frac{\partial\Lambda}{\partial b}\left[\phi_{,i}\,\phi_{,j} - \frac{1}{2} g_{ij}(\phi_{,l}\,\phi^{,l})\right] + \frac{\omega}{\phi^2}\phi_{,i}\,\phi_{,j} \\ &+ \frac{8\pi}{\phi c^4}\left(T_{ij} - \frac{1}{2} g_{ij} T\right) + \frac{1}{\phi}\phi_{;i\,;j} + \frac{1}{2\phi} g_{ij}\Box\phi \end{aligned} \tag{2.5}$$

3. Bianchi Type-I model

The Bianchi Type-I metric is

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 dy^2 - a_3^2 dz^2, \tag{3.1}$$

where a_i $(i = 1, 2, 3)$ are functions of t only. We take $\phi = \phi(t)$. For the metric (3.1) the field eqs (2.4) and (2.5) lead to

$$3\dot{H} + H_1^2 + H_2^2 + H_3^2 = \Lambda + \frac{\partial \Lambda}{\partial b}\dot{\phi}^2 + \omega\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{4\pi}{\phi}(\rho + 3p) + \frac{3}{2}\frac{\ddot{\phi}}{\phi} + \frac{3H}{2}\left(\frac{\dot{\phi}}{\phi}\right) \tag{3.2}$$

$$3HH_1 + \dot{H}_1 = \Lambda - \frac{\partial \Lambda}{\partial b}\dot{\phi}^2 - \frac{4\pi}{\phi}(\rho - p) + H_1\left(\frac{\dot{\phi}}{\phi}\right) + \frac{1}{2}\frac{\ddot{\phi}}{\phi} + \frac{3H}{2}\left(\frac{\dot{\phi}}{\phi}\right) \tag{3.3}$$

$$3HH_2 + \dot{H}_2 = \Lambda - \frac{\partial \Lambda}{\partial b}\dot{\phi}^2 - \frac{4\pi}{\phi}(\rho - p) + H_2\left(\frac{\dot{\phi}}{\phi}\right) + \frac{1}{2}\frac{\ddot{\phi}}{\phi} + \frac{3H}{2}\left(\frac{\dot{\phi}}{\phi}\right) \tag{3.4}$$

$$3HH_3 + \dot{H}_3 = \Lambda - \frac{\partial \Lambda}{\partial b}\dot{\phi}^2 - \frac{4\pi}{\phi}(\rho - p) + H_3\left(\frac{\dot{\phi}}{\phi}\right) + \frac{1}{2}\frac{\ddot{\phi}}{\phi} + \frac{3H}{2}\left(\frac{\dot{\phi}}{\phi}\right) \tag{3.5}$$

$$2\Lambda + 2\phi\frac{\partial \Lambda}{\partial \phi} + 2\frac{\partial}{\partial t}\left(\frac{\omega}{\phi}\right)\dot{\phi} + \omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\dot{\phi}^2}{\phi}\frac{\partial \omega}{\partial \phi} = R. \tag{3.6}$$

Here $H_i = \dot{a}_i/a_i,\ i = 1, 2, 3$ are Hubble parameters and $V = (a_1 a_2 a_3)$. We assume that

$$\frac{\partial \Lambda}{\partial \dot{\phi}^2} = -\frac{\omega}{2\phi^2}. \tag{3.7}$$

Using eq. (3.7) in (3.6) we have

$$R = 6\dot{H} + H_1^2 + H_2^2 + H_3^2 + 9H^2 = 0 \tag{3.8}$$

Now adding eqs (3.3)–(3.5) we get

$$9H^2 + \dot{H}_1 + \dot{H}_2 + \dot{H}_3 = 3\Lambda - 3\frac{\partial \Lambda}{\partial b}\dot{\phi}^2 - \frac{12\pi}{\phi}(\rho - p) + \frac{3}{2}\frac{\ddot{\phi}}{\phi} + \frac{15H}{2}\frac{\ddot{\phi}}{\phi}. \tag{3.9}$$

We now take $\omega = \text{const.}$, $\Lambda = -\frac{\omega}{2\phi^2}\dot{\phi}^2$. For eq. (3.8) we can have the Kasner metric coefficients

$$a_1(t) = t^{p_1},\ a_2(t) = t^{p_2},\ a_3(t) = t^{p_3}, \tag{3.10}$$

where p_i $(i = 1, 2, 3)$ satisfy

$$p_1 + p_2 + p_3 = 1, \quad p_1^2 + p_2^2 + p_3^2 = 1. \tag{3.11}$$

The energy conservation equation leads to

$$\dot{\rho} + (\rho + p)\frac{\dot{V}}{V} = 0. \tag{3.12}$$

For the perfect fluid we take equation of state as

$$p = \gamma\rho, \quad \gamma = \text{const.} \tag{3.13}$$

Using eq. (3.13) in (3.12), we get

$$\rho = \rho_0\ V^{-(1+\gamma)}, \quad \rho_0 = \text{const.} \tag{3.14}$$

From eqs (3.10) and (3.11) we have $V = t$. Then eq. (3.14) reduces to

$$\rho = \rho_0\ t^{-(1+\gamma)}. \tag{3.15}$$

Using eqs (3.10), (3.11) and (3.15) in eq. (3.2) we get

$$3t^2\frac{\mathrm{d}^2\phi}{\mathrm{d}t^2} + t\frac{\mathrm{d}\phi}{\mathrm{d}t} = -8\pi\rho_0(1 + 3\gamma)t^{1-\gamma}. \tag{3.16}$$

Now we consider the solution of eq. (3.16) in some physically interesting cases.

Case I. $\gamma \neq 1,\ \gamma \neq 1/3$. Then $\phi = C_1 + C_2 t^{2/3} - \frac{8\pi\rho_0(1+3\gamma)}{(3\gamma^2-4\gamma+1)}t^{1-\gamma}$.
Case II. $\gamma = 1$ (Zeldovich fluid). Then $\phi = C_3 - 12\pi\rho_0 + 16\pi\rho_0 \log t$.
Case III. $\gamma = 1/3$ (disordered radiation). Then $\phi = C_4 + 24\pi\rho_0 t^{2/3}$.
Case IV. $\gamma = 0$ (dust). $\phi = C_5 + 16\pi\rho_0 t$.

With the increase of t, the volume V and scalar field ϕ increase but pressure p and density ρ decrease.

4. Bianchi Type-V model

The Bianchi Type-V metric is

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a_1^2\mathrm{d}x^2 - a_2^2\mathrm{e}^{-2mx}\mathrm{d}y^2 - a_3^2\mathrm{e}^{-2mx}\mathrm{d}z^2, \tag{4.1}$$

where $m = \text{const.}$ and a_1, a_2, a_3 are functions of t only. We take $\phi = \phi(t)$. For the metric (4.1) the field eqs (2.4) and (2.5) lead to

$$3\dot{H} + H_1^2 + H_2^2 + H_3^2 = \Lambda + \frac{\partial\Lambda}{\partial b}\dot{\phi}^2 + \omega\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{4\pi}{\phi}(\rho + 3p) + \frac{3}{2}\frac{\ddot{\phi}}{\phi} + \frac{3H}{2}\left(\frac{\dot{\phi}}{\phi}\right) \tag{4.2}$$

$$3HH_1+\dot{H}_1-\frac{2m^2}{a_1^2}=\Lambda-\frac{\partial\Lambda}{\partial b}\dot{\phi}^2-\frac{4\pi}{\phi}(\rho-p)+H_1\left(\frac{\dot{\phi}}{\phi}\right)+\frac{1}{2}\frac{\ddot{\phi}}{\phi}+\frac{3H}{2}\left(\frac{\dot{\phi}}{\phi}\right) \quad (4.3)$$

$$3HH_2+\dot{H}_2-\frac{2m^2}{a_1^2}=\Lambda-\frac{\partial\Lambda}{\partial b}\dot{\phi}^2-\frac{4\pi}{\phi}(\rho-p)+H_2\left(\frac{\dot{\phi}}{\phi}\right)+\frac{1}{2}\frac{\ddot{\phi}}{\phi}+\frac{3H}{2}\left(\frac{\dot{\phi}}{\phi}\right) \quad (4.4)$$

$$3HH_3+\dot{H}_3-\frac{2m^2}{a_1^2}=\Lambda-\frac{\partial\Lambda}{\partial b}\dot{\phi}^2-\frac{4\pi}{\phi}(\rho-p)+H_3\left(\frac{\dot{\phi}}{\phi}\right)+\frac{1}{2}\frac{\ddot{\phi}}{\phi}+\frac{3H}{2}\left(\frac{\dot{\phi}}{\phi}\right) \quad (4.5)$$

$$2H_1=H_2+H_3. \quad (4.6)$$

Here $H_i=\dot{a}_i/a_i,\ i=1,2,3$ and $H=(H_1+H_2+H_3)/3$ and $b=\phi_{,i}\phi^{,i}=\dot{\phi}^2,\ \dot{\phi}=\frac{\mathrm{d}\phi}{\mathrm{d}t}$. From eq. (2.5), we obtain

$$R-2\Lambda-2\phi\frac{\partial\Lambda}{\partial\phi}+4\frac{\partial}{\partial t}\left(\phi\frac{\partial\Lambda}{\partial\dot{\phi}^2}\right)\dot{\phi}+4\phi\frac{\partial\Lambda}{\partial\dot{\phi}^2}(\ddot{\phi}+3H\dot{\phi})$$
$$=\omega\left(\frac{\dot{\phi}}{\phi}\right)^2-2\omega\left(\frac{\ddot{\phi}}{\phi}\right)-6H\omega\frac{\dot{\phi}}{\phi}-\frac{\dot{\phi}^2}{\phi}\frac{\partial\omega}{\partial\phi}. \quad (4.7)$$

We assume that $\frac{\partial\Lambda}{\partial\dot{\phi}^2}=-\frac{\omega}{2\phi^2}$ where $\omega=$ const., $\Lambda=-\frac{\omega}{2\phi^2}\dot{\phi}^2$. Then we have

$$R=6\dot{H}+H_1^2+H_2^2+H_3^2+9H^2-\frac{6m^2}{a_1^2}=0 \quad (4.8)$$

and

$$2\Lambda+2\phi\frac{\partial\Lambda}{\partial\phi}+2\frac{\partial}{\partial t}\left(\frac{\omega}{\phi}\right)\dot{\phi}+\omega\left(\frac{\dot{\phi}}{\phi}\right)^2-\frac{\dot{\phi}^2}{\phi}\frac{\partial\omega}{\partial\phi}=0. \quad (4.9)$$

From eq. (4.6) we get $a_2a_3=a_1^2$. By the transformation $\mathrm{d}t=a_1\,\mathrm{d}\eta$, eq. (4.8) reduces to

$$\frac{a_1''}{a_1}+\frac{a_2''}{a_2}+\frac{a_3''}{a_3}+\frac{a_1'^2}{a_1^2}+\frac{a_2'a_3'}{a_2a_3}-3m^2=0. \quad (4.10)$$

Here $'\equiv\mathrm{d}/\mathrm{d}\eta$. The solution of eq. (4.10) is given by

$$a_1^2=\sinh(2m\eta),\quad a_2^2=\sinh(2m\eta)[\tanh(m\eta)]^{\sqrt{3}},$$
$$a_3^2=\sinh(2m\eta)[\tanh(m\eta)]^{-\sqrt{3}}. \quad (4.11)$$

From the conservation equation $T^i_j;_i=0$ we have

$$\rho=\rho_0[\sinh^{3/2}(2m\eta)]^{-(1+\gamma)},\quad \rho_0=\text{const.} \quad (4.12)$$

Using eqs (4.11) and (4.12) in eq. (4.2) we get

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} = -\frac{8\pi\rho_0(1+\gamma)}{3}[\sinh(2m\eta)]^{-(1+3\gamma)/2}. \tag{4.13}$$

We consider the solution of eq. (4.13) in some physically interesting cases.

Case I. $\gamma = 1/3$ (disordered radiation). Then $\phi = -\frac{4\pi\rho_0}{3m^2}[2m^2\eta^2\sin^{-1}(2m\eta) - 3m\eta\sqrt{1+4m^2\eta^2} - \log|2m\eta + \sqrt{1+4m^2\eta^2}|] + C_1 m\eta + C_2$.

Case II. $\gamma = -1$ (vacuum fluid). Then $\phi = \frac{4\pi\rho_0}{3m^2}\sinh(2m\eta) + C_3 m\eta + C_4$.

With the increase of t, the volume V and scalar field ϕ increase but pressure p and density ρ decrease.

5. Bianchi Type-VIo model

The Bianchi Type-VIo metric is

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a_1^2\mathrm{d}x^2 - a_2^2\mathrm{e}^{-2m^2x}\mathrm{d}y^2 - a_3^2\mathrm{e}^{2m^2x}\mathrm{d}z^2, \tag{5.1}$$

where $m =$ const. and a_1, a_2, a_3 are functions of t only. For the metric (5.1) the field eqs (2.4) and (2.5) lead to

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} = \Lambda + \frac{\partial\Lambda}{\partial b}\dot{\phi}^2 + \omega\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{4\pi}{\phi}(\rho+3p) + \frac{3}{2}\frac{\ddot{\phi}}{\phi} + \frac{1}{2}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\frac{\dot{\phi}}{\phi} \tag{5.2}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1}{a_1}\left(\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) - \frac{2m^4}{a_1^2} = \Lambda - \frac{\partial\Lambda}{\partial b}\dot{\phi}^2 - \frac{4\pi}{\phi}(\rho-p) + \frac{\dot{a}_1}{a_1}\frac{\dot{\phi}}{\phi} + \frac{1}{2}\frac{\ddot{\phi}}{\phi} + \frac{1}{2}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\frac{\dot{\phi}}{\phi} \tag{5.3}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_2}{a_2}\left(\frac{\dot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1}\right) = \Lambda - \frac{\partial\Lambda}{\partial b}\dot{\phi}^2 - \frac{4\pi}{\phi}(\rho-p) + \frac{\dot{a}_2}{a_2}\frac{\dot{\phi}}{\phi} + \frac{1}{2}\frac{\ddot{\phi}}{\phi} + \frac{1}{2}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\frac{\dot{\phi}}{\phi} \tag{5.4}$$

$$\frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_3}{a_3}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2}\right) = \Lambda - \frac{\partial\Lambda}{\partial b}\dot{\phi}^2 - \frac{4\pi}{\phi}(\rho-p) + \frac{\dot{a}_3}{a_3}\frac{\dot{\phi}}{\phi} + \frac{1}{2}\frac{\ddot{\phi}}{\phi} + \frac{1}{2}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\frac{\dot{\phi}}{\phi} \tag{5.5}$$

$$\frac{\dot{a}_2}{a_2} = \frac{\dot{a}_3}{a_3}. \tag{5.6}$$

From eq. (2.4) we obtain

$$R - 2\Lambda - 2\phi\frac{\partial\Lambda}{\partial\phi} + 4\frac{\partial}{\partial t}\left(\phi\frac{\partial\Lambda}{\partial\dot{\phi}^2}\right)\dot{\phi} + 4\phi\frac{\partial\Lambda}{\partial\dot{\phi}^2}\left\{\ddot{\phi} + \dot{\phi}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right)\right\}$$

$$= \omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - 2\omega\left(\frac{\ddot{\phi}}{\phi}\right) - 2\omega\frac{\dot{\phi}}{\phi}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) - \frac{\dot{\phi}^2}{\phi}\frac{\partial\omega}{\partial\phi}, \tag{5.7}$$

where

$$R = 2\left[\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} - \frac{m^4}{a_1^2}\right]. \tag{5.8}$$

Now we have $(\partial\Lambda/\partial\dot{\phi}^2) = -(\omega/2\phi^2)$. Then eq. (5.7) reduces to

$$2\Lambda + 2\phi\frac{\partial\Lambda}{\partial\phi} + \frac{\partial}{\partial t}\left(\frac{\omega}{\phi}\right)\dot{\phi} + \omega\left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{\dot{\phi}^2}{\phi}\frac{\partial\omega}{\partial\phi} = R. \tag{5.9}$$

Now we take $\omega = \text{const.}$, $\Lambda = -\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}$. Then eq. (5.9) reduces to

$$R = \frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} - \frac{m^4}{a_1^2} = 0. \tag{5.10}$$

From eq. (5.6) we have

$$a_2 = a_3. \tag{5.11}$$

We take $p = \gamma\rho$, $0 \le \gamma \le 1$. Then conservation equation $T^i_j;_i = 0$ leads to $\rho = \rho_0(a_1a_2a_3)^{-(1+\gamma)}$. Now using eq. (5.11) and making a change of variable $t \to \eta$ by $\mathrm{d}t = a_1\,\mathrm{d}\eta$, eqs (5.2)–(5.5) reduce to

$$\frac{a_1''}{a_1} - \frac{a_1'^2}{a_1^2} + 2\frac{a_2''^2}{a_2^2} - \frac{a_1'a_2'}{a_1a_2} = \frac{4\pi\rho}{\phi}(1+3\gamma) + \frac{3}{2}\frac{\phi''}{\phi} + \left(\frac{a_2'}{a_2} - \frac{a_1'}{a_1}\right)\frac{\phi'}{\phi} \tag{5.12}$$

$$\frac{a_1''}{a_1} - \frac{a_1'^2}{a_1^2} + 2\frac{a_1'a_2'}{a_1a_2} - 2m^4 = \frac{4\pi\rho}{\phi}(1-\gamma) + \frac{1}{2}\frac{\phi''}{\phi} + \left(\frac{a_2'}{a_2} + \frac{a_1'}{a_1}\right)\frac{\phi'}{\phi} \tag{5.13}$$

$$\frac{a_2''}{a_2} - \frac{a_2'^2}{a_2^2} = \frac{4\pi\rho}{\phi}(1-\gamma) + \frac{1}{2}\frac{\phi''}{\phi} + 2\frac{a_2'}{a_2}\frac{\phi'}{\phi}. \tag{5.14}$$

Here $' \equiv \mathrm{d}/\mathrm{d}\eta$. Then eq. (5.10) becomes

$$\frac{a_1''}{a_1} - \frac{a_1'^2}{a_1^2} + 2\frac{a_2''}{a_2} + \frac{a_2'^2}{a_2^2} - m^4 = 0. \tag{5.15}$$

A solution of eq. (5.15) is $a_1 = \eta^{-1/4}\,\mathrm{e}^{\eta^2/4}$, $a_2 = a_3 = \sqrt{2}\eta^{1/2}$, $m = (1/2)^{1/4}$. Then $\rho = \rho_0(2\eta^{3/4}\mathrm{e}^{\eta^2/4})^{-(1+\gamma)}$. From eqs (5.12) and (5.14) we have

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\eta^2} + \frac{1}{\eta}\frac{\mathrm{d}\phi}{\mathrm{d}\eta} + 8\pi\rho_0\left(\gamma - \frac{1}{3}\right)[2\eta^{3/4}\mathrm{e}^{\eta^2/4}]^{-(1+\gamma)} = 0. \tag{5.16}$$

Now we consider some physically interesting cases.

Case I. $\gamma = 1/3$. Then $\phi = C_1 + C_2\log\eta$.

Case II. $\gamma = -1$. Then $\phi = C_3 + C_4\log\eta + \frac{8}{3}\pi\rho_0\eta^2$.

With the increase of t, the volume V and scalar field ϕ increase but pressure p and density ρ decrease.

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