

## Preface

In 1953 something extraordinary happened at the Indian Association for the Cultivation of Science in Calcutta. Amal Kumar Raychaudhuri, twenty-seven years of age and employed ungainfully as a scientific assistant at the Experimental X-ray Section, made a startling theoretical discovery in General Relativity (GR). Without assuming any symmetry constraint on the underlying spacetime, he derived an equation which showed the unavoidable occurrence of spacetime singularities in GR under quite general conditions. Nearly a decade later, by global topological arguments utilizing the causal structure of spacetime and Einstein's equations, this result was given a complete mathematical generalization and proved rigorously in terms of a set of precisely enunciated theorems, now very well-known as singularity theorems, by Hawking, Penrose and Geroch. The equation of Raychaudhuri was the critical starting point for these theorems which held under more general conditions of which Raychaudhuri's conditions were a subset. Its import and significance were immediately recognized as was evident from the fact that Prof. Charles Misner could obtain a grant from NSF for an year's visit of AKR to the University of Maryland in 1964.

The Raychaudhuri equation has three aspects which are logically sequential. First and foremost, it is a geometric statement on the congruence of non-spacelike paths, including geodesics, in an arbitrary spacetime manifold. Second, on introducing the Principle of Equivalence, it becomes a statement on the congruence of the trajectories of material particles and photons in an arbitrary gravitational field. Finally, the use of Einstein's equations and of the energy conditions leads to the result that in a generally nonflat spacetime manifold there exist trajectories which are necessarily incomplete in the sense that they and their neighbouring trajectories inevitably focus into singularities at finite comoving times. The equation describes how trajectories behave during the course of their dynamical evolution, i.e. how they expand, reconverge, get distorted under shearing effects of gravitational fields and rotate under the influence of the energy density and matter fields present. The scope of the Raychaudhuri equation is very wide since it is a geometric statement on the evolution of paths in a general (not necessarily spacetime) manifold. For gravitational dynamics, it encompasses all spacetime singularities from the cosmological big bang to black holes and naked singularities that could arise in astrophysics from collapsing stars.

More than fifty years have passed since this powerful equation was written down. In this intervening half-century, it has influenced different types of research, not only in classical GR as well as in still incomplete theories of quantum gravity, but also in string theory and even in hydrodynamics. There is every likelihood that research involving the Raychaudhuri equation will take

*Preface*

new directions in future. Just to illustrate this point, let us mention that in the currently fashionable Loop Quantum Cosmology, this equation is needed in a new avatar in the possible avoidance of the cosmological big bang singularity. Standing at the crossroads, we feel this to be an appropriate juncture to view this equation in perspective. To this end, we have invited essays from several experts, working in different areas, whose current research not only derives inspiration from this equation, but in fact makes use of it in some way or the other. We feel that, as a celebration of the golden jubilee of the birth of this amazing equation, this is the best tribute that we can offer to the memory of its deceased creator.

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