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Cluster model of s- and p-shell ΛΛ hypernuclei

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Abstract. The ΛΛ binding energy ($B_{\Lambda\Lambda}$) of the s- and p-shell hypernuclei are calculated variationally in the cluster model and multidimensional integrations are performed using Monte Carlo. A variety of phenomenological Λ-core potentials consistent with the Λ-core energies and a wide range of simulated s-state ΛΛ potentials are taken as input. The $B_{\Lambda\Lambda}$ of ${}_{\Lambda\Lambda}^{6}$He is explained and ${}_{\Lambda\Lambda}^{5}$He and ${}_{\Lambda\Lambda}^{5}$H are predicted to be particle stable in the ΛΛ-core model. The results for s-shell hypernuclei are in excellent agreement with those of non-VMC calculations. The ${}_{\Lambda\Lambda}^{10}$Be in ΛΛαα model is overbound for combinations of ΛΛ and Λα potentials. A phenomenological dispersive three-body force, $V_{\Lambda\alpha\alpha}$, consistent with the B_Λ of ${}_{\Lambda}^{9}$Be in the Λαα model underbinds ${}_{\Lambda\Lambda}^{10}$Be. The incremental $\Delta B_{\Lambda\Lambda}$ values for the s- and p-shell cannot be reconciled, consistent with the finding of earlier analyses.

Keywords. Variational Monte Carlo; ΛΛ hypernuclei; cluster model; Urbana-type simulated ΛΛ potentials, αα potential; Λα potential.

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1. Introduction

The KEK hybrid emulsion experiment [1] E373 has unambiguously identified the ${}_{\Lambda\Lambda}^{6}$He, named as NAGARA events and its measured $B_{\Lambda\Lambda}$ value $7.25\pm0.19^{+0.18}_{-0.11}$ MeV suggests a weak ΛΛ interaction. The NAGARA and the other events [2] conjectured as ${}_{\Lambda\Lambda}^{4}$H at Brookhaven AGS experiment E906 have revived the interest in extracting the nature of ΛΛ potential and therefore, giving rise to the hope of constraining the parameters of meson interaction models. Consequently, in the recent past a number of few-body [3–7] and cluster model [5,8–13] calculations of hypernuclei in the $S = -2$ sector have been performed. These two types of approaches supplement each other.

There are three recent theoretical studies [3,6,7] on the issue of the stability of the lightest $S = -2$ hypernucleus ${}_{\Lambda\Lambda}^{4}$H. All these use the same NN, ΛN and ΛΛ potentials in the four-body ΛΛnp model. The Faddeev–Yakubovsky method [3] does not bind ${}_{\Lambda\Lambda}^{4}$H but the stochastic variational method (SVM) [6] and variational Monte Carlo (VMC) calculation [7] produce a weakly bound system. The three-body ΛΛd (d=deuteron) cluster model calculation using Faddeev method [5] and VMC approach [7] also favours a particle stable system.

The use of the cluster model, where nucleonic degrees of freedom are frozen, offers a reasonably good approximation in reducing the many-body problem to a few-body one and thus surmounting to a limited extent the complexities arising out of the dynamical correlations. Inherent to the model used here and in earlier work [5,8–13] is the input of Λ-cluster and cluster–cluster correlations and interactions which are mainly an average over the properties of many baryons. Despite these simplifications the use of cluster model to $S = -2$ systems has given important insight not only in exploring the ΛΛ interaction but also in predicting new species which are yet to be identified or confirmed experimentally. In the past, Filikhin and Gal [5] and their coworkers [8,10,11] in a series of papers have analyzed the s-shell ΛΛ hypernuclei in the ΛΛ-core model using Faddeev method and the p-shell ΛΛ hypernuclei in the Faddeev–Yakubovsky framework in the $\Lambda\Lambda\alpha\alpha$ model. The $B_{\Lambda\Lambda}$ of ${}_{\Lambda\Lambda}^{6}$He is explained and the s-shell hypernuclei ${}_{\Lambda\Lambda}^{5}$He and ${}_{\Lambda\Lambda}^{5}$H are predicted to be particle stable [5,8] for a wide range of simulated ΛΛ interaction models of Nijmegen group. The recent variational calculation of Myint *et al* [9] agrees with the Faddeev results [8] for $A = 5$ and 6 ΛΛ hypernuclei.

Filikhin and Gal [5] have found that Λ-binding (B_Λ) of ${}_{\Lambda}^{9}$Be in the $\Lambda\alpha\alpha$ model in the Faddeev approach is explained using s-state Isle $\Lambda\alpha$ potential either without normalizing it or without the introduction of the dispersive three-body ΛNN force. Subsequent investigations [10,11] demonstrated this to be the effect of ignoring the contributions of higher partial waves ($l > 0$). The use of weak p-state $\Lambda\alpha$ interaction, derived using NSC97e potential model, reduces significantly the B_Λ-binding compared to the s-state model in these analyses. However, the $\Lambda\alpha$ potential which fits B_Λ of ${}_{\Lambda}^{9}$Be in [5] gives binding for ${}_{\Lambda\Lambda}^{10}$Be in $\Lambda\Lambda\alpha\alpha$ model close to the experimental value for a very strong ΛΛ model ND, a consequence of restricting the hadron–hadron potential to s-wave only. A cluster model calculation of ${}_{\Lambda\Lambda}^{10}$Be by Hiyama *et al* [14] where $l = 1$ state $\Lambda\alpha$ potential adjusted to fit B_Λ of ${}_{\Lambda}^{9}$Be gives $B_{\Lambda\Lambda}$ much lower than the currently accepted experimetal value. From these analyses it appears that dispersive ΛNN forces do not play any role in these systems.

In the earlier VMC cluster model analyses [12,13] the microscopically calculated repulsive contribution of the dispersive ΛNN potential to the ${}_{\Lambda}^{9}$Be in the cluster model from $\Lambda\alpha\alpha$ channel turned out to be important in explaining its B_Λ. The dispersive energy was also found to be contributing significantly to the $B_{\Lambda\Lambda}$ of ${}_{\Lambda\Lambda}^{10}$Be. In the light of these remarks we may add that effective l-dependent potentials in these analyses [5,9–11,14] may be assumed to be simulating the effect of the dispersive force. But dispersive energy from $\Lambda\alpha\alpha$ channel in ${}_{\Lambda}^{9}$Be and ${}_{\Lambda\Lambda}^{10}$Be could not be incoporated in the analyses [5,10]. Here we may emphasize that $\Lambda\alpha$ potentials which are constrained to fit the limited data of ${}_{\Lambda}^{5}$He are uncertain in the shape and the choice of parameters. Further, the shape and the parameters of weaker p-state $\Lambda\alpha$ potential [9] derived using NSC97e is yet to stand scrutiny of the relevant experimental data. Therefore, we prefer the s-state $\Lambda\alpha$ potentials along with the phenomenological dispersive $\Lambda\alpha\alpha$ force (to be discussed in the next section) to explain the $S = -1$ and -2 data of p-shell hypernuclei. The dispersive force so parameterized can be conveniently used in analyzing the ground and excited state properties of α-cluster hypernuclei. Similar type of force among the α-clusters has been successfully employed in explaining the spectra of α-cluster nuclei [15]. At present full N-body VMC calculations for p- and s–d shells are not feasible and therefore, cluster model calculations seem to be an alternative to microscopic ones for the detailed study. In addition to the above remark, the motivation for performing the VMC calculation of the $B_{\Lambda\Lambda}$ of s- and p-shell hypernuclei in the cluster model arose due to the following reasons:

- the unambiguous observation of ${}^{\,6}_{\Lambda\Lambda}\mathrm{He}$, called NAGARA event, with accurate $B_{\Lambda\Lambda}$ is expected to constrain the $\Lambda\Lambda$ potential [11]
- various types of the $\Lambda\alpha$ potentials, e.g. Isle type [5], Myint *et al* [9], Gibson [16], and Tang and Herndon [17] and a wide range of simulated $\Lambda\Lambda$ potentials are available. It would be interesting to investigate their effects on binding energies of s- and p-shell $S = -2$ hypernuclei
- to compare the results and check the accuracy of our VMC calculation with those of the other exact ones [5,10,11] and the variational approach [9], which were not available at the time of earlier cluster model analyses [12,13].

The work presented here differs from ref. [12], in that a wide range of simulated $\Lambda\Lambda$, the phenomenological Λ-cluster and $\Lambda\alpha\alpha$ potentials are employed and all the $S = -2$ systems are included to make the analysis more elaborate.

The paper is organized as follows: In the next section we discuss the cluster model for the s- and p-shell hypernuclei. The potential models and the trial wave functions are discussed in §3. The calculations, results and discussion are presented in the §4. Conclusions of our study are given in the last section.

2. Hamiltonian of ΛΛ hypernuclei in the cluster model

In the VMC calculation the systems included are: ${}^{\,6}_{\Lambda\Lambda}\mathrm{He}$, ${}^{\,5}_{\Lambda\Lambda}\mathrm{He}$, ${}^{\,5}_{\Lambda\Lambda}\mathrm{H}$, ${}^{10}_{\Lambda\Lambda}\mathrm{Be}$ and ${}^{13}_{\Lambda\Lambda}\mathrm{B}$. The s-shell $\Lambda\Lambda$ hypernuclei are treated as $\Lambda\Lambda + x$ three-body system, where $x = {}^4\mathrm{He}$, ${}^3\mathrm{He}$ and ${}^3\mathrm{H}$. The ${}^{10}_{\Lambda\Lambda}\mathrm{Be}$ along with ${}^{13}_{\Lambda\Lambda}\mathrm{B}$ are also analyzed in a simplified picture $\Lambda\Lambda + C$ (where $C = {}^8\mathrm{Be}$ and ${}^{11}\mathrm{B}$). For the $\Lambda\Lambda + \mathbf{C}$ ($\mathbf{C} = x$ and C) model of s- and p-shell hypernuclei, the Hamiltonian takes the form

$$H^{\mathbf{C}}_{\Lambda\Lambda} = K_{\mathbf{C}}(1) + \sum_{i=2}^{3}(K_\Lambda(i) + V_{\Lambda\mathbf{C}}(r_{1i})) + V_{\Lambda\Lambda}(r_{23}), \tag{1}$$

where 1 denotes the core and 2, 3 stand for the two lambdas, K_Y is the kinetic energy operator for the particle Y (=$\mathbf{C}$ or Λ), V_{hh} ($hh = \Lambda\mathbf{C}, \Lambda\Lambda$) is the hadron–hadron potential and $V_{\Lambda\mathbf{C}}$ is the two-body potential that fits the binding energy of $\Lambda\mathbf{C}$ system.

For the p-shell hypernucleus ${}^{10}_{\Lambda\Lambda}\mathrm{Be}$, the Hamiltonian in a four-body $\Lambda\Lambda\alpha\alpha$ model is given by

$$H_{\Lambda\Lambda}^{\alpha\alpha} = \sum_{i=1}^{2}(K_\alpha(i) + K_\Lambda(i+2)) + V_{\alpha\alpha}(r_{12}) + \sum_{i<j_{i=1,2,j=3,4}} V_{\Lambda\alpha}(r_{ij})$$
$$+ \sum_{i=3}^{4} V_{\Lambda\alpha\alpha}(r_{i1}, r_{i2}) + V_{\Lambda\Lambda}(r_{34}), \quad (2)$$

where 1, 2 indicate the two α clusters and 3, 4 label the two Λ particles. $V_{\alpha\alpha}$ denotes the $\alpha\alpha$ potential that fits low-energy $\alpha\alpha$ phase shifts data. The $V_{\Lambda\alpha\alpha}$ is the phenomenological dispersive three-body potential which may be thought of arising due to 16 pairs of nucleons in the triad $\Lambda N_1 N_2$ and a nucleon coming from each α. The contribution of $\langle V_{\Lambda\alpha\alpha}\rangle$ to the energy is quite significant as shown in ref. [12], neglecting it among the $\Lambda\alpha\alpha$ clusters over binds the ${}^{9}_{\Lambda}$Be and ${}^{10}_{\Lambda\Lambda}$Be. It may be noted that $V_{\Lambda\mathbf{C}}$ which reproduces the experimental energy of the $\Lambda\mathbf{C}$ hypernucleus simulates the dispersive $\Lambda\alpha\alpha$ potential energy and other smaller effects for all the systems treated in the $\Lambda\Lambda\mathbf{C}$ model. However, in the $\Lambda\alpha\alpha$ and $\Lambda\Lambda\alpha\alpha$ model of two Be hypernuclei there seems to be no mechanisms to incorporate the dispersive energy among the $\Lambda\alpha\alpha$ clusters [5,10,11]. Following ref. [15] we propose to simulate phenomenologically the dispersive energy in the triad $\Lambda\alpha\alpha$ through a simple form

$$V_{\Lambda\alpha\alpha} = W_0 f^2(r_{\Lambda\alpha_1}) f^2(r_{\Lambda\alpha_2}), \quad (3)$$

where the strength $W_0 > 0$ gives repulsion. The radial factor $f(r)$ is of Yukawa form:

$$f^2(r) = \exp(-ar)/ar,$$

where a is the range parameter. We may mention that the contribution of $V_{\Lambda\alpha\alpha}$ calculated here turns out to be close to the microscopic one [12] and justifies the above choice. The purely attractive $\Lambda\alpha$ potentials in the analysis of Be hypernuclei are excluded from our consideration, as these require an attractive dispersive three-body force which is ruled out on physical ground and also an earlier microscopic analysis [12] does not favour it.

3. Potential models and wave functions

The phenomenological s-state V_{hh} ($hh = \Lambda x, \alpha\alpha$) interactions which have been taken from refs [5,8] are the sum of two-range Gaussian with the coefficients V_{rep}^{hh} and V_{att}^{hh} multiplying the repulsive and attractive parts respectively as

$$V_{hh}(r) = V_{\text{rep}}^{hh} \exp\left(\frac{-r^2}{\beta_{\text{rep}}^2}\right) - V_{\text{att}}^{hh} \exp\left(\frac{-r^2}{\beta_{\text{att}}^2}\right) + V_{\text{Coul}}(r), \quad (4)$$

where β_i is the range for $i = \text{rep(att)}$, and $V_{\text{Coul}}(r)$ is finite size Coulomb $\alpha\alpha$ potential which is zero for other pairs.

The potential parameters for $hh = \Lambda x$ ($x = \alpha$, ^{3}H and ^{3}He) taken from refs [5,9,16,17] are shown in table 1. In order to have direct comparison with the work of ref. [9] we use the spin average Λx potential

Table 1. The parameters of s-state Λx (=α, ^{3}He, ^{3}H) potentials used in the present work. In the next table we have used the abbreviations to denote Λx potentials: Isle, MSA, Gib and TH for refs [5,9,16] and [17], respectively.

Λx potential	$V_{\rm rep}^{\Lambda x}$ (MeV)	$\beta_{\rm rep}$ (fm)	$V_{\rm att}^{\Lambda x}$ (MeV)	$\beta_{\rm att}$ (fm)
$\Lambda\alpha$				
Isle [5]	450.4	1.25	404.9	1.41
Ref. [9]	91.0	1.30	95.0	1.70
Ref. [16]	–	–	43.48	1.5764
Ref. [17]	–	–	60.17	1.2729
Λ ^{3}He				
Isle [5]	450.4	1.2532	404.9	1.41
Ref. [9]	58.1	1.4	78.4	1.72
Λ ^{3}H				
Isle [5]	450.4	1.2573	404.9	1.41
Ref. [9]	58.1	1.4	76.8	1.72

$$V_{\Lambda x}(r) = \frac{3V_{\Lambda x}(0^+) + V_{\Lambda x}(1^+)}{4} \tag{5}$$

for predicting ${}_{\Lambda\Lambda}^{5}$He and ${}_{\Lambda\Lambda}^{5}$H. In ref. [9] the Λx folding potential is obtained from the ΛN Nijmegen ND by the Brückner–Hartee–Fock method. The resulting potentials which are slightly modified so as to reproduce the experimental B_Λ value of the systems ${}_{\Lambda}^{4}$H, ${}_{\Lambda}^{4}$H*, ${}_{\Lambda}^{4}$He, ${}_{\Lambda}^{4}$He*, and ${}_{\Lambda}^{5}$He have the same radial dependence as Isle potential [5].

The $\alpha\alpha$ potentials of Ali and Bodmer [18] is chosen as this will facilitate a comparison with the earlier work [10]. We have also used Chien and Brown [19] $\alpha\alpha$ potential in case of ${}_{\Lambda}^{9}$Be as results differ marginally from those when Ali and Bodmer [18] potential is used. Therefore, we employ latter potential in subsequent calculation. Very recently, Lanskoy and Yamamoto [20] investigated the effect of channel $\Lambda\Lambda \longrightarrow \Xi N$ on the binding of $A = 5$ double Λ hypernuclei and found it to be significant. However, the quantitative extraction of coupling strength is not possible in view of the poor knowledge of hyperon nucleus potentials and the uncertainties in the determination of effective interaction which is related to Γ-matrix. Therefore, we prefer to work in a single channel approach as the effect of $\Lambda\Lambda \longrightarrow \Xi N$ coupling can be simulated through the modification of the strength of the relevant part of $\Lambda\Lambda$ potential. We have chosen three-range Gaussian simulated $\Lambda\Lambda$ potentials [5] which are phase shift equivalent to the Nijmegen realistic interaction NSC97(b,e)and ND. These have the form

$$V_{\Lambda\Lambda}(r) = \sum_{i=1}^{3} v_i \exp\left(\frac{-r^2}{\alpha_i^2}\right), \tag{6}$$

where symbols have meaning as given in ref. [5]. The other potential (denoted by NAGSIM) has been constrained to reproduce $B_{\Lambda\Lambda}$ of ${}_{\Lambda\Lambda}^{6}$He and is taken from ref. [3]. All these potentials differ in attractive strengths of the mid-range part of $V_{\Lambda\Lambda}$. We have also chosen $\Lambda\Lambda$ single channel effective potentials [9] which are constructed from Nijmegen soft-core NSC97e potential and are denoted as $V^{e1}_{\Lambda\Lambda}$ reproduces $B_{\Lambda\Lambda}$ of ${}^{6}_{\Lambda\Lambda}$He, $V^{e2}_{\Lambda\Lambda}$ is free-space interaction and is expected to include the cannel $\Lambda\Lambda \longrightarrow \Xi N$, and $V^{e}_{\Lambda\Lambda}$ reproduces the scattering parameters of NSC97e $S = -2$ interaction. Urbana-type $\Lambda\Lambda$ potential has been successfully used in the past [4,7,12,13,21–23] to analyze the $S = -2$ s-shell hypernuclei. Therefore, in the present analysis, the Urbana-type potential with strength $V_0^{\Lambda\Lambda}$ has also been included because of its widespread use, and is of the form

$$V_{\Lambda\Lambda} = V_{\rm c}(r) - V_0^{\Lambda\Lambda} T_\pi^2(r), \tag{7}$$

where

$$V_{\rm c}(r) = \frac{V_{\rm c}}{1 + \exp\frac{(r - R_{\rm c})}{a_{\rm c}}}$$

with $V_{\rm c} = 2137$ MeV, $R_{\rm c} = 0.5$ fm and $a_{\rm c} = 0.2$ fm and

$$T_\pi(r) = (1 + 3/x + 3/x^2)(\exp(-x)/x)(1 - \exp(-2r^2))^2,$$

the symbols have the same meaning as in refs [4,12,13,20].

The ΛC ($C = {}^{8}$Be and ^{11}B) potentials for ${}^{9}_{\Lambda}$Be and ${}^{12}_{\Lambda}$B relevant to the study of ${}^{10}_{\Lambda\Lambda}$Be and ${}^{13}_{\Lambda\Lambda}$B are of simple Woods–Saxon (W–S) form:

$$V_{\Lambda C}(r) = \frac{-V_0}{1 + \exp\frac{(r - R)}{c}}$$

with $R = 3.24$ fm and $c = 0.60$ fm. We could have taken slightly different values for R and c in the two cases but the results are not very sensitive to small variation in these parameters. The effective depths $V_0 = 18.12$ MeV and 24.82 MeV reproduce the Λ-binding energies 6.70 MeV and 11.70 MeV for the subsystems ${}^{9}_{\Lambda}$Be and ${}^{12}_{\Lambda}$B, respectively. For ${}^{9}_{\Lambda}$Be the parameter V_0 has the same value as found by Filikhin and Gal [5]. Implicit in the ΛC model is the neglect of finite range effect of ΛN and the weak spin-dependent efffect in the case of ${}^{12}_{\Lambda}$B. These might have been simulated in the potential parameters of ΛC W–S shape. This simple model predicts result for ${}^{10}_{\Lambda\Lambda}$Be close to the result of $\Lambda\Lambda\alpha\alpha$ model discussed in the text. Therefore, a brief discussion of ${}^{10}_{\Lambda\Lambda}$Be and ${}^{13}_{\Lambda\Lambda}$B does not seem to be out of context.

The trial wave functions for the systems included in the study are the product of two-body correlation functions f_{HH} ($HH = \Lambda\Lambda, \Lambda\mathbf{C}$, $\alpha\alpha$) and the appropriate spin function χ_0^0 and ζ_0^0.

(i) $\Lambda\Lambda C$ system

$$\Psi^{\mathbf{C}}_{\Lambda\Lambda} = \left[\prod_{i=1}^{2} f_{\Lambda\mathbf{C}}(r_{i3})\right] f_{\Lambda\Lambda}(r_{12})\chi_0^0. \tag{8}$$

(ii) $\Lambda\Lambda\alpha\alpha$

$$\Psi^{\alpha\alpha}_{\Lambda\Lambda} = \left[\prod_{i=1}^{2}\prod_{j=3}^{4} f_{\Lambda\alpha}(r_{ij})\right] f_{\Lambda\Lambda}(r_{12}) f_{\alpha\alpha}(r_{34})\zeta_0^0. \tag{9}$$

The Hamiltonian and wave function for ${}^{9}_{\Lambda}$Be in $\Lambda\alpha\alpha$ cluster model are given by suppressing a Λ index in eqs (2) and (9) and substituting $V_{\Lambda\Lambda}$=0 and $f_{\Lambda\Lambda}$=1. The correlation functions $f_{HH}(r)$ have the asymptotic behavior $f_{HH}(r) \sim r^{-\nu_{HH}}$ $\exp(-\kappa_{HH} r)$ and are obtained from the solution of the Schrödinger-type equation. For further details we refer the readers to refs [4,7,12,13,20].

4. Variational calculations, results and discussion

The energy $-B_{\Lambda\Lambda}$ (or $-B_{\Lambda}$) for $S = -2$(or -1) system in the cluster model is evaluated using the following relation:

$$-B_{\lambda} = \frac{\langle \Psi^{\mathbf{N}}_{\lambda} | H^{\mathbf{N}}_{\lambda} | \Psi^{\mathbf{N}}_{\lambda} \rangle}{\langle \Psi^{\mathbf{N}}_{\lambda} | \Psi^{\mathbf{N}}_{\lambda} \rangle}, \qquad (10)$$

where symbols λ and $\mathbf{N}$ have the meaning as defined below:

(i) $\Lambda\Lambda C$ model of s- and p-shell hypernuclei: $\lambda = \Lambda\Lambda$ and $\mathbf{N} = \mathbf{C}$.

(ii) $\Lambda\Lambda\alpha\alpha$ model of ${}^{10}_{\Lambda\Lambda}$Be: $\lambda = \Lambda\Lambda$ and $\mathbf{N} = \alpha\alpha$.

(iii) $\Lambda\alpha\alpha$ model of ${}^{9}_{\Lambda}$Be: $\lambda = \Lambda$ and $\mathbf{N} = \alpha\alpha$.

The VMC estimates of the energy were made for 100 000 points. The statistical errors in the energies were less than 1% and therefore, have not been quoted in the tables.

4.1 *s- and p-shell* $\Lambda\Lambda$ *hypernuclei in* $\Lambda\Lambda C$ *model*

s-Shell: In this section we describe the results of calculation for $B_{\Lambda\Lambda}$ of $\Lambda\Lambda$ hypernuclei for baryon number $A = 6$ and 5 using VMC method. Our results (in the bold face) for various combinations of hadron–hadron potentials along with those of non-VMC analyses [5,8,10] for the purpose of comparison are listed in table 2. We discuss the results for ${}^{6}_{\Lambda\Lambda}$He and ${}^{5}_{\Lambda\Lambda}$He, ${}^{5}_{\Lambda\Lambda}$H in the following:

A. ${}^{6}_{\Lambda\Lambda}$*He*

The $B_{\Lambda\Lambda}$ for the simulated three-range Gaussian $\Lambda\Lambda$ potentials corresponding to Nijmegen NSC97 and various s-state $\Lambda\alpha$ are shown in table 2. The calculated $B_{\Lambda\Lambda}$ for NAGSIM is equal to the experimental value, not unexpected as the $\Lambda\Lambda$ potential used, is the one which fitted NAGARA event in the earlier work [4]. The $B_{\Lambda\Lambda}$, for a given $\Lambda\Lambda$ potential, depends on the choice of $\Lambda\alpha$ potential. Myint, Shinmura and Akaishi [9] $\Lambda\alpha$ potential gives more binding compared to Isle [5] and Herndon and Tang [17] binds less compared to Gibson [16]. On the other hand, the binding of ${}^{6}_{\Lambda\Lambda}$He for a given $\Lambda\alpha$ potential increases with the increasing attraction of the $\Lambda\Lambda$ potentials as we go down the table from NSC97b to ND. This increase becomes progressively more prominent as the size of inner repulsive soft core of $\Lambda\alpha$ potential decreases in going from Isle [5] or Myint, Shinmura and Akaishi [9] potential to purely attractive ones [16,17] as we move in a row. This is a consequence of using

Table 2. The ground state energies $-B_{\Lambda\Lambda}$ (in MeV) for s-shell hypernuclei (HN) are listed from the third column onward against each $\Lambda\Lambda$ potentials (in the first column) for a wide range of $\Lambda\alpha$ potentials. Urbana potential has a depth $V_0^{\Lambda\Lambda} = 5.65$ MeV. Our results are in the bold face and the statistical errors in the calculated $B_{\Lambda\Lambda}$ are less than 1% and therefore, are not being quoted. Other quantities are the same as in the preceding table.

$\Lambda\Lambda$ Potential	HN	$B_{\Lambda\Lambda}$							
		Isle	Isle	**MSA**	MSA	**Gib**	Gib	**TH**	TH
NSC97b	${}_{\Lambda\Lambda}^{6}$He	**6.66**	6.59[a] 6.698[b]	**6.74**	6.773[b]	**6.37**	6.541[b]	**6.15**	6.200[b]
NSC97e		**6.87**	6.81[a] 6.903[b]	**6.97**	6.998[b]	**6.76**	6.877[b]	**6.51**	6.593[b]
NAGSIM		**7.27**		**7.41**		**7.46**		**7.30**	
ND		**9.07**	9.09[a]	**9.39**		**9.86**		**10.80**	
Urbana		**7.27**							
$V_{\Lambda\Lambda}^{e}$		**6.972**	7.107[c]						
$V_{\Lambda\Lambda}^{e1}$		**7.327**	7.477[c]						
$V_{\Lambda\Lambda} = 0$		**6.20**	6.27[a]						
NSC97b	${}_{\Lambda\Lambda}^{5}$He	**3.39**	3.44[a]	**3.37**					
NSC97e		**3.53**	3.59[a]	**3.53**					
NAGSIM		**3.82**		**3.87**					
ND		**5.26**	5.31[a]	**5.54**					
Urbana		**3.84**							
$V_{\Lambda\Lambda}^{e}$		**3.610**	3.671[c]						
$V_{\Lambda\Lambda}^{e1}$		**3.859**	3.956[c]						
$V_{\Lambda\Lambda}^{e2}$		**4.12**							
NSC97b	${}_{\Lambda\Lambda}^{5}$H	**2.85**	2.96[a]	**2.82**					
NSC97e		**2.99**	3.09[a]	**2.98**					
NAGSIM		**3.31**		**3.31**					
ND		**4.66**	4.70[a]	**4.87**					
Urbana		**3.32**							
$V_{\Lambda\Lambda}^{e}$		**3.074**	3.115[c]						
$V_{\Lambda\Lambda}^{e1}$		**3.316**	3.376[c]						
$V_{\Lambda\Lambda}^{e2}$		**3.56**							

[a]Ref. [5]; [b]s-wave model for $\alpha\Lambda$ potential (ref. [10]); [c]for $l_{\max} = 6$ (ref. [8]).

increasingly attractive $\Lambda\Lambda$ potentials which push $\Lambda\alpha$ pair more closer for purely attractive potential compared to Isle [5] or Myint, Shinmura and Akaishi [9] and thereby leading to the enhanced binding energy. The difference in the $B_{\Lambda\Lambda}$ for Isle [5] and Myint, Shinmura and Akaishi [9] $\Lambda\alpha$ potentials corresponding to NAGSIM lies within the experimental uncertainty and thus data are unable to discriminate between these two. Hence, we conclude that these two potentials are equivalent. The results for Urbana and Myint, Shinmura and Akaishi [9] $\Lambda\Lambda$ potentials together with Isle $\Lambda\alpha$ potential are also given in the table. The $B_{\Lambda\Lambda}$ for Urbana potential depth $V_0^{\Lambda\Lambda} = 5.65$ MeV is close to the experimental one and for the $V^e_{\Lambda\Lambda}$ and $V^{e1}_{\Lambda\Lambda}$ lies within the experimental error. Our calculation demonstrates that Urbana and NAGSIM potentials are equivalent as far as the energy of ${}^{6}_{\Lambda\Lambda}$He is concerned. The $B_{\Lambda\Lambda}$ of the hypernucleus for $V_{\Lambda\Lambda} = 0$ and $f_{\Lambda\Lambda} = 1$ yields a value slightly larger than twice the B_Λ value of ${}^{5}_{\Lambda}$He, which is used to check consistency of our calculations.

B. ${}^{5}_{\Lambda\Lambda}$*He and* ${}^{5}_{\Lambda\Lambda}$*H*

Both the hypernuclei ${}^{5}_{\Lambda\Lambda}$He and ${}^{5}_{\Lambda\Lambda}$H for the aforesaid ΛΛ potentials combined with $\Lambda - x$ potentials [5,9] are predicted to be stable. We have also calculated $B_{\Lambda\Lambda}$ for $A = 5$ for $V^{e2}_{\Lambda\Lambda}$ potential [9] which simulates the effect of $\Lambda\Lambda \longrightarrow \Xi N$ coupling. The $\Delta B_{\Lambda\Lambda}(=B_{\Lambda\Lambda} - 2B_\Lambda)$ for these systems compares fairly well with those of Lanskoy and Yamamoto [20] who have also explicitly included, in an approximate way, the coupling of $\Lambda\Lambda \longrightarrow \Xi N$ channel. Thus $V^{e2}_{\Lambda\Lambda}$ of Myint, Shinmura and Akaishi [9] appears to simulate the coupling of the channel despite the questionable approximations, as discussed in ref. [8], incorporated in the construction of the potential.

The overall agreement of $B_{\Lambda\Lambda}$ with the non-VMC calculations for all s-shell hypernuclei is satisfactory. It is worth mentioning that numerical accuracy achieved in the minima of the energies of the systems included in the VMC for $V^e_{\Lambda\Lambda}$ and $V^{e1}_{\Lambda\Lambda}$ is very close to that in the Faddeev method where the contributions of higher partial waves are explicitly incorporated. This is not unexpected as the two-body correlations, evaluated using Urbana procedure for realistic hadron–hadron potentials, are peaked away from the origin and higher partial waves are inherently built in the wave function. Consequently, contributions to the potential energy from higher angular momentum states to a large extent is being taken care off in the present work. The marginal differences in energies between the VMC and Faddeev methods may be attributed to the difficulty in searching for the variational parameters near the energy minima which seem to be shallow. On the other hand, a slightly larger difference for the purely attractive $\Lambda\alpha$ potentials [16,17] originates from the higher partial waves which are not being simulated in the VMC wave function used here but are explicitly included in the Faddeev method [10] for these potentials.

The depth of Urbana potential found here is about 10% less than that in ref. [4] where ${}^{6}_{\Lambda\Lambda}$He was treated as a six-body system. This reflects the inadequacy of cluster model wave function where the correlation factors of the many hadron–hadron pairs are being averaged out to a few ones. Despite this, $\Delta B_{\Lambda\Lambda}$ for $A = 5$ and 6 hypernuclei is consistent with the microscopic calculation of ref. [4] for Urbana-type potential.

In conclusion, we may remark that results of VMC calculation of s-shell hypernuclei in the three-cluster model closely agree with the non-VMC approaches [5,9]. This convergence of results reinforces the confidence in the methodology of all the three approaches and may be termed as equivalent. In table 3, the r.m.s. radii for pairs of hadrons are shown for all the hypernuclei under study for the NAGSIM and $V^{e2}_{\Lambda\Lambda}$ combined with Isle [5] and Myint, Shinmura, and Akaishi [9] $\Lambda\alpha$ potentials. These radii form an isosceles triangle with the longer side $R_{\Lambda\Lambda}$ as the base. This is consistent with the property of central potentials. The r.m.s. distance increases with decrease of $B_{\Lambda\Lambda}$ as we go down the table from the ${}^{6}_{\Lambda\Lambda}$He to ${}^{5}_{\Lambda\Lambda}$H.

Table 3. The r.m.s distances (in fm) for hypernuclei listed in the first column and $\Lambda\Lambda$ potentials in the second column. $\mathbf{R_{\Lambda\Lambda}}$ is the r.m.s. distance between $\Lambda\Lambda$, $\mathbf{R_{(\Lambda\Lambda)x}}$ is the distance between the c.m. of two Λ hyperons and core nucleus x and $\mathbf{R_{\Lambda x}}$ is the distance between Λ hyperon and x. Distances within square brackets are for MSA $\Lambda - x$ potential and those outside bracket are for Isle type.

Hypernucleus	$\Lambda\Lambda$ potential	$\mathbf{R_{\Lambda\Lambda}}$	$\mathbf{R_{(\Lambda\Lambda)x}}$	$\mathbf{R_{\Lambda x}}$
${}^{6}_{\Lambda\Lambda}$He	NAGSIM	3.82[3.77]	2.26[2.22]	2.95[2.89]
${}^{5}_{\Lambda\Lambda}$He	$V^{e2}_{\Lambda\Lambda}$	4.23[4.29]	2.70[2.81]	3.42[3.53]
${}^{5}_{\Lambda\Lambda}$H	$V^{e2}_{\Lambda\Lambda}$	4.51[4.21]	2.98[2.74]	3.72[3.44]

Table 4. ${}^{10}_{\Lambda\Lambda}$Be and ${}^{13}_{\Lambda\Lambda}$B in $\Lambda\Lambda C$ model: $B_{\Lambda\Lambda}$ for ΛC Woods–Saxon potential and $\Lambda\Lambda$ potentials listed in the first column. The results within square brackets are from ref. [5]. The other quantities are the same as in the preceding table. Experimental values [5,14,24,25] $B_{\Lambda}({}^{9}_{\Lambda}\mathrm{Be}) = 6.71 \pm 0.04$ MeV, $B_{\Lambda\Lambda}({}^{10}_{\Lambda\Lambda}\mathrm{Be}) = 17.6 \pm 0.4$ MeV (14.5 ± 0.4 MeV assuming ${}^{10}_{\Lambda\Lambda}\mathrm{Be} \rightarrow \pi^- + p + {}^{9}_{\Lambda}\mathrm{Be}^*$), $B_{\Lambda}({}^{12}_{\Lambda}\mathrm{B}) = 11.37 \pm 0.06$ MeV and $B_{\Lambda\Lambda}({}^{13}_{\Lambda\Lambda}\mathrm{B}) = 27.6 \pm 0.4$ MeV.

$\Lambda\Lambda$ Potential	${}^{10}_{\Lambda\Lambda}$Be		${}^{13}_{\Lambda\Lambda}$B	
	$B_{\Lambda\Lambda}$	$\mathbf{R_{\Lambda\Lambda}}$	$B_{\Lambda\Lambda}$	$\mathbf{R_{\Lambda\Lambda}}$
ND	16.07[16.0]	3.22[3.4]	26.86	2.95
NAGSIM	14.28	3.65	24.62	3.24
Urbana	14.30	3.60	24.57	3.21
$V^{e1}_{\Lambda\Lambda}$	14.29	3.64	24.61	3.28
$V_{\Lambda\Lambda} = 0$	13.24[13.3]	3.92[3.8]	22.90	3.87

p-shell: We have calculated $B_{\Lambda\Lambda}$ for ${}^{10}_{\Lambda\Lambda}$Be in the three-body $\Lambda\Lambda C$ model and compared our results with those of the Faddeev method. The ΛC W–S potential consistent with the energy of ΛC is used. Following the spirit of ref. [5] we also include ${}^{13}_{\Lambda\Lambda}$B in the present work although such simplified treatment of p-shell system seems unrealistic. Nevertheless, this may be helpful in understanding the nature of $\Lambda\Lambda$ potential. In the calculation the ND, NAGSIM, Urbana, and $V^{e1}_{\Lambda\Lambda}$ $\Lambda\Lambda$ potentials have been used. The binding energies of these hypernuclei have also been calculated for $V_{\Lambda\Lambda} = 0$. The results are shown in table 4 and the values of binding energies and $\mathbf{R_{\Lambda\Lambda}}$, r.m.s. distance between $\Lambda\Lambda$ for Be hypernucleus taken from ref. [5] are put in square bracket. Our results for ${}^{10}_{\Lambda\Lambda}$Be in the case of relevant $\Lambda\Lambda$ potential are very close to the Faddeev calculation. In this simple model the $B_{\Lambda\Lambda}$ values for the NAGSIM, Urbana and $V^{e1}_{\Lambda\Lambda}$ potentials for both the hypernuclei fall short by about 3.0 MeV from experimental value. The $B_{\Lambda\Lambda}$ of ${}^{10}_{\Lambda\Lambda}$Be in this simple-minded picture is consistent with the value 14.5 MeV which is speculated on the assumption that a γ-ray must have escaped the identification of the decay products of the event ${}^{10}_{\Lambda\Lambda}\mathrm{Be} \longrightarrow \pi^- + p + {}^{9}_{\Lambda}\mathrm{Be}^*$) in emulsion [14]. A similar situation [1] is not ruled out for ${}^{13}_{\Lambda\Lambda}$B case also.

4.2 *p-Shell hypernucleus* ${}^{10}_{\Lambda\Lambda}$*Be in* $\Lambda\Lambda\alpha\alpha$ *model*

A detailed study in the $\Lambda\alpha\alpha$ model of ${}^{9}_{\Lambda}$Be is a pre-requisite because this forms a subsystem in the $B_{\Lambda\Lambda}$ calculation of ${}^{10}_{\Lambda\Lambda}$Be in the $\Lambda\Lambda\alpha\alpha$ model. Therefore, we first calculate B_Λ in the cluster model for the two types of s-state $\Lambda\alpha$ potentials. We show, in table 5, the results for the s-state $\alpha\alpha$ potentials (from [18,19]). The energy is corrected by adding 0.1 MeV, the resonance energy of ^{8}Be*. The energies from the earlier analysis [10] for the purpose of comparison with the present work are also shown in table 5 for appropriate combinations of hadron–hadron potentials. The agreement between the B_Λ found here and the one obtained in the Faddeev method [10] including explicitly the higher partial waves up to $l_{\max}$ is reasonably good for the soft repulsive inner core $\Lambda\alpha$ potentials. On the other hand, for purely attractive [16,17] $\Lambda\alpha$ potentials, the VMC energies which we do not quote here are higher by about 3–7% from the Faddeev results [10]. This is a consequence of the failure of purely attractive potential to simulate in the variational wave function higher partial waves which are being explicitly included in the Faddeev method. Since the purely attractive potentials [16,17] are not considered to be realistic we retain only the $\Lambda\alpha$ potentials [5,9] in our detailed study of p-shell systems. The insensitivity of the energy to the choice of $\alpha\alpha$ potentials is not unexpected as these potentials belong to the same family. Therefore, we retain the $\alpha\alpha$ potential of [18] for the calculation of ${}^{10}_{\Lambda\Lambda}$Be. It is important to note that the binding energy of Be hypernucleus for the $\Lambda\alpha$ potentials exceeds the experimental value by about 1.5 MeV which is related to classic problem of $A = 5$ anomaly, i.e. inability to account for dispersive energy between Λ and two α clusters. Therefore, a phenomenological dispersive three-body force, $V_{\Lambda\alpha\alpha}$ of Yukawa shape eq. (3) is adjusted to reproduce the B_Λ of ${}^{9}_{\Lambda}$Be. The results are shown in table 6. The $\alpha\alpha$ r.m.s. distance is almost the same as calculated microscopically [12]. Our results, as expected, reiterate the appropriateness of VMC method in explaining B_Λ of few-body systems. Further, we pursue four-body VMC calculations of the $B_{\Lambda\Lambda}$ of ${}^{10}_{\Lambda\Lambda}$Be including $V_{\Lambda\alpha\alpha}$ potential

Table 5. ${}^{9}_{\Lambda}$Be in $\alpha\alpha\Lambda$ model: The energy B_Λ (MeV) is given in the third column for a combination of $\alpha\Lambda$ and $\alpha\alpha$ potentials ([18] and [19]) listed in the first and the second column, respectively. $\mathbf{R}_{\alpha\alpha}$ is the r.m.s. distance between α clusters, $\mathbf{R}_{(\alpha\alpha)\boldsymbol{\Lambda}}$ is the distance between the c.m. of two alphas and Λ hyperon (all in fm). Results from earlier work within round brackets are quoted in the third column. Other quantities are the same as in the preceding table.

$\Lambda\alpha$ Potential	$\alpha\alpha$ Potential	$B_\Lambda^\dagger$	$\mathbf{R}_{\alpha\alpha}$	$\mathbf{R}_{\alpha\boldsymbol{\Lambda}}$	$\mathbf{R}_{(\alpha\alpha)\boldsymbol{\Lambda}}$
Isle	[18]	8.245(8.142[a] for $l_{\max} = 5$)	3.65	3.07	2.47
	[19]	8.238(8.266[a] for $l_{\max} = 4$)	3.62	3.11	2.52
MSA	[18]	8.094(7.953[a] for $l_{\max} = 5$)	3.61	3.10	2.54
	[19]	8.006(8.079[a] for $l_{\max} = 4$)	3.71	3.18	2.57

† $\alpha\alpha$ resonance energy of 0.1 MeV to be added in the energy of the present work.
[a] Ref. [10].

Table 6. ${}^{9}_{\Lambda}$Be in $\alpha\alpha\Lambda$ model: In the second column the strength W_0 (in MeV) and range a (in fm) and in the third column the contribution of the dispersive three-body $\langle V_{\Lambda\alpha\alpha}\rangle$ (in MeV) for $\alpha\Lambda$ potentials listed in the first column are given. The results within square brackets are for ref. [19] and those outside for ref. [18]. Other quantities are the same as in the preceding table.

$\alpha\Lambda$ Potential	$\Lambda\alpha\alpha$ Potential				
	W_0, a	$\langle V_{\Lambda\alpha\alpha}\rangle$	$B_{\Lambda}^{\dagger}$	$\mathbf{R}_{\alpha\alpha}$	$\mathbf{R}_{(\alpha\alpha)\Lambda}$
Isle	17.0, 0.5	1.43[1.40]	6.72[6.63]	3.81[3.80]	2.66[2.57]
MSA	13.5, 0.5	1.18[1.25]	6.70[6.65]	3.91[3.86]	2.76[2.78]

${}^{\dagger}\alpha\alpha$ Resonance energy of 0.1 MeV to be added in the energy.

whose parameters are fixed as discussed above. The $B_{\Lambda\Lambda}$ values for a range of simulated Nijmegen, Urbana type and $V^{e1}_{\Lambda\Lambda}$ $\Lambda\Lambda$ and two types of $\Lambda\alpha$ potentials are shown in table 7 for Yukawa shape dispersive force. We also show our results without three-body force (in square brackets) and those from ref. [5] (in round brackets) in the table. Before comparing the numerical results with other work we would like to point out that the $\Lambda\alpha - \Lambda\alpha$, $\Lambda\Lambda\alpha - \alpha$, and $\alpha\alpha\Lambda - \Lambda$ channels which make predominant contributions to the energy of Be hypernucleus in Faddeev–Yakubovsky method [5] are also entering the wave function, eq. (9), through the variational parameters κ_{xy} ($xy = \alpha\alpha$, $\Lambda\alpha$ and $\Lambda\Lambda$) related to the separation energies of these channels. Thus cluster model wave function describes the behaviour of Be hypernucleus in the VMC as accurately as in the Faddeev–Yakubovsky method. The channel $\Lambda\Lambda - \alpha\alpha$ which makes a small contribution in the Faddeev–Yakubovsky method is not entering in our wave function but might have been simulated during optimization of energy in the multidimensional parameter space. The $B_{\Lambda\Lambda}$ in the present work for $V_{\Lambda\alpha\alpha}$=0 is about 20% more than that given by Filikhin and Gal [5]. The source of large discrepancy in the two is mainly because ref. [5] uses s-wave in Faddeev–Yakubovsky method while in the present work the contribution of higher partial waves is simulated in the correlated wave function.

Here we also note down the equivalence of NAGSIM, Urbana and $V^{e1}_{\Lambda\Lambda}$ potentials just as found in $\Lambda\Lambda C$ model in §4.1. The $B_{\Lambda\Lambda}$ in the case of NAGSIM, Urbana and $V^{e1}_{\Lambda\Lambda}$ potential is about 12% lower than the experimental value and therefore, the recent NAGARA event is not consistent with ${}^{10}_{\Lambda\Lambda}$Be, in accord with earlier observation [5,9,14]. An inspection of table 7 shows that effective dispersive force $V_{\Lambda\alpha\alpha}$, lowers the system energy by about 25% and therefore, highlights the importance of the inclusion of dispersive force in the calculation of $B_{\Lambda\Lambda}$. Further the choice of the Yukawa shape seems to be justified as the contribution of the phenomenological dispersive three-body force to the energy turns out to be not too far away from the one calculated microscopically in cluster model work [12]. Thus we have proposed an alternative but simpler way to simulate dispersive energy reasonably well without doing a detailed microscopic calculation. Further, a difference of only about 1% in the $B_{\Lambda\Lambda}$ for two $\Lambda\alpha$ potentials [5,9] shows that these potentials appear to be dynamically almost equivalent as has been found in the study s-shell hypernuclei. $R_{\alpha\alpha}$, the r.m.s. distance between two alphas in the ${}^{10}_{\Lambda\Lambda}$Be is 10% less than in ${}^{9}_{\Lambda}$Be

Table 7. ${}^{10}_{\Lambda\Lambda}$Be in $\alpha\alpha\Lambda\Lambda$ model: The $B_{\Lambda\Lambda}$ values are listed in the third column for the ΛΛ potentials in the first column. The results for combinations of $\Lambda\alpha + \alpha\alpha$ potentials are marked using abbreviations and references corresponding to these. $\mathbf{R}_{(\alpha\alpha)\Lambda\Lambda}$ is the r.m.s. distance between the c.m. of two alphas and two Λ hyperons. Results from ref. [5] are given within round brackets and square brackets have our results without three-body force. The other quantities are the same as in the preceding tables.

ΛΛ Potential	$\langle V_{\Lambda\alpha\alpha}\rangle$	$B_{\Lambda\Lambda}$	$\mathbf{R}_{\alpha\alpha}$	$\mathbf{R}_{\Lambda\Lambda}$	$\mathbf{R}_{(\alpha\alpha)\Lambda}$	$\mathbf{R}_{(\alpha\alpha)\Lambda\Lambda}$
			Isle + [18]			
NSC97b	3.14	14.75	3.50	3.52	2.59	1.91
		[18.35]	[3.22]	[3.18]	[2.37]	[1.74]
NSC97e	3.18	15.03	3.49	3.50	2.62	1.94
		[18.70](15.4)	[3.26](3.5)	[3.21](4.2)	[2.33](3.17)	[1.70](2.4)
NAGSIM	3.23	15.48	3.50	3.31	2.55	1.93
		[19.40]	[3.24]	[3.06]	[2.33]	[1.67]
ND	3.84	17.88	3.40	3.00	2.38	1.84
		[22.18](17.7)	[3.20](3.4)	[2.72](3.9)	[2.13](3.0)	[1.64](2.3)
Urbana	3.33	15.43	3.40	3.26	2.43	1.82
$V_{\Lambda\Lambda}=0$	3.09	14.32	3.42	3.67	2.59(3.26)	1.82(2.4)
		[18.08](14.8)	[3.22](3.6)	[3.28](4.3)	[3.0]	
$V^{e1}_{\Lambda\Lambda}$	3.29	15.58	3.45	3.36	2.52	1.87
			MSA + [18]			
NSC97b	2.98	14.79	3.49	3.63	2.67	1.95
NSC97e	3.02	15.11	3.42	3.47	2.56	1.88
NAGSIM	3.15	15.58	3.47	3.43	2.60	1.94
ND	3.60	18.03	3.44	2.95	2.30	1.77
Urbana	3.21	15.60	3.46	3.29	2.51	1.90
$V^{e1}_{\Lambda\Lambda}$	3.12	15.76	3.42	3.30	2.47	1.84

for NAGSIM, Urbana, and $V^{e1}_{\Lambda\Lambda}$ potentials which is a reflection of polarization of the core of ${}^{9}_{\Lambda}$Be by the addition of one more Λ. The reduction in the $\alpha\alpha$ separation agrees with that found in ref. [12]. The calculated $B_{\Lambda\Lambda}$ for $V_{\Lambda\Lambda}$=0 and $f_{\Lambda\Lambda}=1$ is more than twice the B_Λ of ${}^{9}_{\Lambda}$Be ignoring $V_{\Lambda\alpha\alpha}$, as expected. From the tables we find that $R_{\Lambda\Lambda}$, the r.m.s. distance between two hyperons is sensitive to the choice of ΛΛ potential, the deeper the well the shorter the distance. A similar result is also found for s-shell systems.

The $B_{\Lambda\Lambda}$ for ${}^{10}_{\Lambda\Lambda}$Be in the $\Lambda\Lambda\alpha\alpha$ model is greater than for the $\Lambda\Lambda C$ but both are considerably lower than the currently accepted experimental $B_{\Lambda\Lambda}$ value (table 4). However, $R_{\Lambda\Lambda}$ in the four-body model is considerably smaller than that for the three-body model. This shrinkage in the size is a reflection of the enhancement of the binding of ${}^{10}_{\Lambda\Lambda}$Be in the four-body model over that of the three-body which is a result of increased flexibility of the wave function in the former model compared to the latter. We may conclude that VMC energy of ${}^{10}_{\Lambda\Lambda}$Be for dispersive Yukawa shape is fairly close to the value (-14.5 ± 0.4 MeV) deduced on the speculation that a γ-ray must have escaped undetected in the identification of ${}^{10}_{\Lambda\Lambda}$Be through the decay of ${}^{9}_{\Lambda}$Be*, as discussed in refs [14,24]. Essentially similar result was obtained by Hiyama *et al* [14] but they used $l = 1$ $\Lambda\alpha$ potential adjusted to the B_Λ of ${}^{9}_{\Lambda}$Be. Thus explicit use of dispersive forces was ruled out. However, we feel these were being simulated in l-dependent potential. On the contrary, in our calculation it appears that dispersive force alone is sufficient to explain Be hypernuclei but this may be simulating the l-dependent potential. It may be worth recalling here that in earlier analyses [21–23] the role of both dispersive ΛNN and space-exchange ΛN force has been emphasized in the binding energies analyses of hypernuclei. Further space-exchange ΛN force is equivalent to l-dependent force in the case of hypernuclei. In our work [22] we have investigated that extraction of a unique combination of dispersive and space-exchange forces is impossible as the determination of one masks the other. Thus our cluster model calculation for the energy of ${}^{10}_{\Lambda\Lambda}$Be raises the question of whether a dispersive or the space-exchange force or a combination of both is needed for the description of the binding of Be hypernuclei. Further, revising the existing $B_{\Lambda\Lambda}$ datum of ${}^{10}_{\Lambda\Lambda}$Be may help in deciding between the three- and four-cluster model of $\Lambda\Lambda$-hypernuclei. With reference to ${}^{13}_{\Lambda\Lambda}$B calculation we may remark that a situation [1,24] where a γ-ray must have escaped undetected in the identification of boron hypernucleus in emulsion, is not ruled out and hence the binding energy may be much lower than what is experimentally reported.

4.3 $\Delta B_{\Lambda\Lambda}$ *for the s- and p-shell systems*

In table 8 we show $\Delta B_{\Lambda\Lambda}$ for all the $S = -2$ s- and p-shell hypernuclei treated in the $\Lambda\Lambda C$ model using NAGSIM $\Lambda\Lambda$ combined with Isle or Woods–Saxon ΛC potential. Our calculation predicts the incremental values $\Delta B_{\Lambda\Lambda}$ for $A = 5$ and 10 systems close to the experimental value extracted from NAGARA event and for $A = 13$ it is almost twice the experimental value. The $\Delta B_{\Lambda\Lambda}$ value for the Be hypernucleus in the $\Lambda\Lambda\alpha\alpha$ model is also about twice the experimental value. Unless new experiments are planned to measure the $B_{\Lambda\Lambda}$ of ${}^{10}_{\Lambda\Lambda}$Be and ${}^{13}_{\Lambda\Lambda}$B more accurately, reconciling theoretically the $\Delta B_{\Lambda\Lambda}$ values for the s- and p-shell systems remains an open problem.

5. Conclusions

We have made the three- and four-body cluster model variational Monte Carlo calculations for s- and p-shell hypernuclei using a variety of single channel effective $\Lambda\Lambda$ potentials in combination with $\Lambda\alpha$ or *Core*, and $\alpha\alpha$ potentials. For s-shell this is the first cluster model VMC calculation reported by us. The $\Lambda\Lambda C$ model of s-shell hypernuclei in the present work gives very good fit to the NAGARA event for a variety of potentials and predicts particle stable ${}_{\Lambda\Lambda}^{5}$H and ${}_{\Lambda\Lambda}^{5}$He. The $(2J+1)$ spin averaged $\Lambda Core$ potentials for these two latter systems yield binding energies which are in close agreement with the non VMC results. The calculated $\Delta B_{\Lambda\Lambda}$ values of s-shell and ${}^{10}_{\Lambda\Lambda}$Be in the three-cluster model is consistent with the NAGARA event. The binding of boron hypernucleus in the three-body model is found to be much lower than the experimental value and $\Delta B_{\Lambda\Lambda}$ is almost twice that of the NAGARA

Table 8. The $\Delta B_{\Lambda\Lambda}$ (in MeV) for each hypernucleus listed in the first column is shown in the last column for baryon–baryon potentials discussed in the text.

Hypernucleus	ΛΛ potential	ΛC potential	$\alpha\alpha$ potential	$\Delta B_{\Lambda\Lambda}$
		$\Lambda\Lambda C$ model		
${}^{6}_{\Lambda\Lambda}$He	NAGSIM	Isle	–	1.09
${}^{5}_{\Lambda\Lambda}$He		Isle	–	0.78
${}^{5}_{\Lambda\Lambda}$H		Isle	–	0.71
${}^{10}_{\Lambda\Lambda}$Be		W–S	–	0.86
${}^{13}_{\Lambda\Lambda}$B		W–S	–	1.78
		$\Lambda\Lambda\alpha\alpha$ model		
${}^{10}_{\Lambda\Lambda}$Be	NAGSIM	Isle	[18]	2.19

event. A phenomenological dispersive three-body $\Lambda\alpha\alpha$ potential of Yukawa shape, constrained to fit the existing B_Λ of the ${}^{9}_{\Lambda}$Be in the three-body model, yields $B_{\Lambda\Lambda}$ of ${}^{10}_{\Lambda\Lambda}$Be in the four-body model lower than the experimental value. Further, the small difference in the binding of ${}^{10}_{\Lambda\Lambda}$Be for the two-type of realistic $\Lambda\alpha$ potentials does not help in discriminating between these. The $B_{\Lambda\Lambda}$ of ${}^{10}_{\Lambda\Lambda}$Be in the $\Lambda\Lambda\alpha\alpha$ model exceeds the $\Lambda\Lambda C$ one by about 1.0 MeV. Therefore, our calculation in the absence of revised experimental values of binding, is unable to decide whether a three- or four-body model is appropriate for studying ${}^{10}_{\Lambda\Lambda}$Be. The result of non-VMC calculation of $B_{\Lambda\Lambda}$ for this system using $l = 1$ $\Lambda\alpha$ potential agrees with our results where a dispersive $\Lambda\alpha\alpha$ force is employed. We may remark that present work raises the ambiguity whether effective dispersive or l-dependent force alone or an appropriate combination of the these two is needed to explain the $B_{\Lambda\Lambda}$ of ${}^{10}_{\Lambda\Lambda}$Be.

The present work suggests that there is a need not only to accurately measure $B_{\Lambda\Lambda}$ values of the existing $S = -2$ p-shell hypernuclei but also to add many more new species in the existing list to resolve the inconsistency in the $\Delta B_{\Lambda\Lambda}$ and to settle the ambiguity found here.

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