PRAMANA
— journal of
physics

Vol. 68, No. 6
June 2007
pp. 913–924

Collective modes of a quasi-two-dimensional Bose condensate in large gas parameter regime

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MS received 13 October 2006; revised 24 March 2007; accepted 11 April 2007

Abstract. We have theoretically studied the collective modes of a quasi-two-dimensional (Q2D) Bose condensate in the large gas parameter regime by using a formalism which treats the interaction energy beyond the mean-field approximation. The results show that incorporation of this higher order term leads to significant modifications in the mode frequencies.

Keywords. Bose–Einstein condensation; collective modes; scattering length.

PACS Nos 03.75.Kk; 03.75.Hh; 05.30.Jp

1. Introduction

Bose–Einstein condensation (BEC) of dilute atomic gases has been achieved in a variety of atomic samples using different magnetic and optical traps [1–5]. These advances have resulted in investigations of properties of these ultra-cold gases in different conditions. At present, there is considerable interest in studying the effects of reduced dimensionality on the properties of the condensates [6–32]. This is because the behaviour of a Bose gas in lower dimension is remarkably different from its behaviour in three dimensions. For example, a two-dimensional (2D) homogeneous Bose gas does not undergo Bose–Einstein condensation at finite temperature. Instead, there can be a phase transition called Berezinski–Kosterlitz–Thouless transition in which system becomes superfluid without having long-range ordering. For a homogeneous one-dimensional (1D) Bose gas, BEC does not exist even at absolute zero temperature. Nevertheless, in the inhomogeneous confining potentials, BEC can exist in the low-dimensional (1D and 2D) geometries because of the modified density of states [22].

In the last few years, several groups have experimentally investigated the BEC in lower dimensions including 1D and elongated 3D Bose gases [6–14] as well as 2D Bose condensates [15–19]. In some of the recent experiments [16,17], BECs in three-dimensional traps have been achieved which fall in the quasi-two-dimensional

(Q2D) regime. In this regime, the interaction energy between the atoms is of the order of or less than the harmonic oscillator level spacing along the direction of tight confinement. In refs [16,18] the cross-over to Q2D regime was observed by continuously removing atom from a highly anisotropic trap to reduce the interaction energy. On the other hand, in the experiment described in ref. [17] the Q2D cross-over is achieved for a fixed number of atoms by increasing the trap anisotropy from moderate to very large values.

It is important to note here that in Q2D regime, the scattering length characterizing the inter-atomic interaction is still smaller than the size of the condensate along the direction of tight confinement. As a result the particles obey 2D statistics but interact in the same way as in the three-dimensional system. Therefore, it is expected that, theoretically the properties of Q2D condensates can be described by mean-field Gross–Pitaevskii (GP) like equation with effective coupling strength characterized by 3D scattering length a. The mean-field equation for Q2D condensates can be derived from 3D GP equation by factorizing the wave function into two parts, one of them describing the zero-point oscillations along the direction of tight confinement and the other part representing the wave function of the condensate in the remaining two dimensions. This mean-field GP-like equation has been extensively used to study the static and the dynamic properties of Q2D condensates [23,25,27–33]. It is well-known that 3D GP equation is valid for dilute gases. Mathematically this is expressed as $\rho a^3 \ll 1$ (where ρ is the atomic density and a is the 3D s-wave scattering length of the inter-atomic potential, and the parameter ρa^3 is called the gas parameter). Recent studies have shown that for 3D condensates as long as $\rho a^3 \leq 10^{-3}$, the GP theory yields accurate results for both static and dynamic properties [34,35]. However, in a recent experiment, condensates with peak gas parameter of the order of 10^{-2} have been achieved by enhancing the scattering length a with the help of Feshbach resonance [36]. These kinds of condensates, then, naturally raise the question about the validity of the mean-field GP theory. For 3D condensates, the validity and the applicability of the GP equation have already been tested in the large gas parameter regime by employing beyond mean-field modified GP (MGP) equation which takes into account the effect of depletion of the condensate caused by the increased value of the inter-atomic interaction. As the mean-field equation for the Q2D condensates is derived from the 3D GP equation, it then becomes natural to test the validity of the mean-field equation for the Q2D condensate in the large gas parameter regime. The main aim of this paper is to check the validity of the mean-field GP-like equation and study the effect of going beyond mean-field theory, for Q2D condensates in the regime of large gas parameter. The calculations are carried out by employing Q2D MGP-like equation, which has been derived from the 3D MGP energy functional with the assumption that wave function can be factorized into two parts as mentioned above.

Collective oscillations of Bose condensates have been proved to be an important tool to study the interactions and many-body effects [37–39]. In this paper, we study the effect of going beyond mean-field theory on the frequencies of the collective oscillations of Q2D condensates in the large gas parameter regime. It is well-known that the frequencies of the collective oscillations can be measured with high precisions [40] and, therefore changes in the frequencies arising due to going beyond mean-field theory should be easily detectable in experiments. Before

proceeding further, in accordance with the experiment in ref. [17], we consider here a Q2D condensate confined in a trap potential having a more tight confinement along z-axis than the other two axes (x and y). But the oscillator length along the tightly confining z-axis ($a_z = (\hbar/m\omega_z)^{1/2}$, where m is the mass of the atom and ω_z is the trap frequency along z-axis) of the trap is chosen to be much larger than the scattering length a, so that the scattering process is still three-dimensional. On the other hand, for a truly two-dimensional (2D) condensate which is achieved by making $a_z \leq a$, the scattering process becomes purely two-dimensional (2D) and a different dependence of the interaction energy on density occurs as discussed in refs [21,24–27]. Consequently, one may expect different results for this case which is not considered in this paper.

The remaining paper is structured in the following manner. In §2, we present a brief derivation of the beyond mean-field equation for Q2D condensates and other theoretical methods applied in this paper. Section 3 is devoted to calculation of collective modes and a discussion of the results. The paper is concluded in §4.

2. Theory

We begin this section with a brief derivation of the energy functional and beyond mean-field equation for Q2D condensates starting from the expression of energy functional of a 3D condensate. The ground-state energy functional associated with a condensate of N bosons each with mass m confined in a three-dimensional trap potential $V_{\rm t}(\mathbf{r})$ can be written as

$$E[\psi] = \int {\rm d}^3\mathbf{r} \left[\frac{\hbar^2}{2m}|\nabla\psi|^2 + V_{\rm t}(\mathbf{r})|\psi|^2 + \varepsilon(\rho)|\psi|^2\right] \tag{1}$$

with the trapping potential

$$V_{\rm t}(\mathbf{r}) = \frac{1}{2}m(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2), \tag{2}$$

where ω_x, ω_y and ω_z denote trap frequencies along x-, y- and z-directions respectively. Moreover, in the above equations $\psi(\mathbf{r})$ is the condensate wave function (order parameter) and $\rho(\mathbf{r})$ represents the corresponding density and it is given by $\rho(\mathbf{r}) = |\psi(\mathbf{r})|^2$. The condensate wave function $\psi(\mathbf{r})$ can be determined by minimizing the above energy functional. In eq. (1), the first, second, and the third terms represent, respectively, the kinetic energy of bosons, the energy due to the trapping potential, and the inter-atomic interaction energy within the local density approximation (LDA). The LDA is applicable if the number of particles is sufficiently large and the energy per particle is larger than the level spacing of the harmonic oscillator [33]. In this paper all the parameters are chosen in such a way that the LDA condition is satisfied. To go beyond the GP theory, we make use of the perturbative expansion for $\varepsilon(\rho)$ in terms of the gas parameter ρa^3,

$$\varepsilon(\rho) = \frac{2\pi\hbar^2a\rho}{m}\left[1 + \frac{128}{15\sqrt{\pi}}\left(\rho a^3\right)^{1/2} + 8\left(\frac{4\pi}{3} - \sqrt{3}\right)(\rho a^3)\ln\left(\rho a^3\right) + \mathcal{O}\left(\rho a^3\right)\right]. \tag{3}$$

The first term in the above expansion, which corresponds to the energy of the homogeneous Bose gas within the mean-field theory as considered in the GP theory, was calculated by Bogoliubov [41]. The second term was obtained by Lee, Huang, and Yang (LHY) [42], while the third term was first calculated by Wu [43] using the hard-sphere model for the inter-atomic potential. In this paper our aim is to investigate the effect of the leading order correction term in the above expansion, i.e. the LHY term. Also, we note that the results of DMC simulation of ref. [44] match well with those obtained by the above expansion up to LHY term for the gas parameter of the order of 10^{-2} whereas inclusion of logarithmic term spoils the match. Therefore, we do not consider the logarithmic term in the expansion (3) for all the calculations in this study.

Now in order to achieve Q2D condensates we assume the trapping frequency $\omega_z \gg \omega_x$, ω_y such that the interaction energy of the condensate is small compared to the zero-point energy $\hbar\omega_z$. Under this condition the dynamics of the trapped gas in the z-direction are restricted to zero-point oscillations. For Q2D condensates, the condition of interaction energy being smaller than the zero-point energy results in an inequality $(Na/a_z) < (R_0/a_z)^2$, where R_0 is the size of the cloud in the x–y plane. Throughout the studies we have restricted N and a so that the above condition is satisfied. Under the above-mentioned assumption, the wave function for Q2D condensates ψ can be written in a factorized form as

$$\psi(\mathbf{r}) = \phi(z)\Psi(x, y), \tag{4}$$

with the normalized wave function along z-axis as

$$\phi(z) = \frac{1}{(\sqrt{\pi}a_z)^{1/2}} \exp\left[\frac{-z^2}{2a_z^2}\right]. \tag{5}$$

Substituting the above wave function in energy functional (1) and integrating over z leads to Q2D energy functional

$$E - \frac{\hbar\omega_z N}{2} = \int \mathrm{d}x\,\mathrm{d}y \times \left[\frac{\hbar^2}{2m}|\nabla_\perp \Psi|^2 + V_\mathrm{t}(x, y)|\Psi|^2 + \varepsilon_\mathrm{Q2D}(\rho_\mathrm{Q2D})|\Psi|^2\right], \tag{6}$$

where

$$\varepsilon_\mathrm{Q2D}(\rho_\mathrm{Q2D}) = \sqrt{2\pi}\left(\frac{\rho_\mathrm{Q2D}a^3}{a_z}\right) \times \left[1 + \frac{128}{15}\frac{2}{\sqrt{5}\pi^{3/4}}\left(\frac{\rho_\mathrm{Q2D}a^3}{a_z}\right)^{1/2}\right]\frac{\hbar^2}{ma^2}, \tag{7}$$

$\nabla_\perp$ is the gradient operator in x–y plane, $V_\mathrm{t}(x, y) = \frac{1}{2}m(\omega_x^2x^2 + \omega_y^2y^2)$, ρ_Q2D is the two-dimensional density, and it is given by $\rho_\mathrm{Q2D} = |\Psi(x, y)|^2$. It is worth mentioning that the above interaction energy, ε_Q2D, gives the chemical potential for Q2D case, which is similar to the one used in ref. [33]. The minimization of the

above functional with respect to $\Psi(x,y)$ with the constraint $\int |\Psi(x,y)|^2 \,\mathrm{d}x\,\mathrm{d}y = N$ (N is the total number of atoms) leads to MGP-like equation for Q2D condensate,

$$\left[-\frac{\hbar^2}{2m}\nabla_\perp^2 + V_{\mathrm{t}}(\mathbf{r}_\perp) + \frac{2\sqrt{2\pi}\hbar^2}{m}\left(\frac{a}{a_z}\right)|\Psi|^2\right]\Psi(\mathbf{r}_\perp) + \left[\frac{\sqrt{2\pi}\hbar^2}{m}\frac{128}{3\sqrt{5}\pi^{3/4}}a_z\left(\frac{a}{a_z}\right)^{5/2}|\Psi|^3\right]\Psi(\mathbf{r}_\perp) = \mu\Psi(\mathbf{r}_\perp), \tag{8}$$

where μ is the chemical potential arising from the above constraint condition and $\mathbf{r}_\perp = \{x, y\}$. The GP-like equation for the Q2D condensate can be obtained from eq. (8) by neglecting the interaction energy term proportional to $|\Psi|^3$ from the left-hand side of this equation. In the following, we apply eq. (6) to study the collective oscillations of Q2D condensates by employing time-dependent variational method. Before we proceed further it is important to note that the above MGP-like equation is valid for $a_z \gg a$. On the other hand, for $a_z \leq a$ the interaction energy term becomes proportional to $1/\log(a|\Psi|^2)$ [21,25,32] resulting in GP-like equation of nonpolynomial form in Ψ.

The dynamics of Q2D condensates can be studied by time-dependent generalization of the above equation. In order to calculate the frequencies of the collective oscillations we employ the time-dependent variational methods based on the action principle [45]. In accordance with this method we construct a Lagrangian for Q2D condensate using the energy functional (6) as

$$L - \frac{N\hbar\omega_z}{2} = \int\left[\frac{i\hbar}{2}\left(\Psi\frac{\partial\Psi^*}{\partial t} - \Psi^*\frac{\partial\Psi}{\partial t}\right) + \frac{\hbar^2}{2m}|\nabla_\perp\Psi|^2 + V_{\mathrm{t}}(x,y)|\Psi|^2\right]\mathrm{d}x\,\mathrm{d}y + \int[\varepsilon_{\mathrm{Q2D}}(\rho_{\mathrm{Q2D}})|\Psi|^2]\mathrm{d}x\,\mathrm{d}y, \tag{9}$$

where $*$ denotes complex conjugation. By minimizing action $S = \int L\,\mathrm{d}t$, one can get the time-dependent MGP-like equation describing the time evolution of Q2D condensates. To perform variational calculation we choose the general trial wave function in x–y coordinates as in ref. [28],

$$\Psi(x,y,t) = C(t)\exp\left[\left(-\frac{a_0^2}{2}[\alpha(t)X^2 + \beta(t)Y^2 + 2\gamma(t)XY]\right)\right], \tag{10}$$

where $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are time-dependent complex variational parameters. We have taken the trap to be anisotropic in the x–y plane with anisotropy parameter λ given by $\omega_x = \lambda\omega_y = \omega_0\sqrt{\lambda}$. In eq. (10), $X(= x/a_0)$ and $Y(= y/a_0)$ are dimensionless coordinate variables with $a_0 = (\hbar/m\omega_0)^{1/2}$. We can write the condensate parameters $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ such that

$$\alpha(t) = \alpha_1(t) + i\alpha_2(t), \quad \beta(t) = \beta_1(t) + i\beta_2(t)$$

and

$$\gamma(t) = \gamma_1(t) + i\gamma_2(t).$$

Here $\alpha_1(t)$ and $\beta_1(t)$ represent the inverse square of the widths along x- and y-axes respectively. The imaginary parts $\alpha_2(t)$ and $\beta_2(t)$ account for the time-dependent phases. The parameters $\gamma_1(t)$ and $\gamma_2(t)$ representing the cross term in the coordinates account for the scissors modes of collective oscillation [46,47]. From the normalization condition we can find that $|C(t)|^2 = N\sqrt{D}/\pi$, where $D = \alpha_1\beta_1 - \gamma_1^2$.

By substituting eq. (10) in eq. (9), we can evaluate Lagrangian as

$$L - \frac{N\hbar\omega_z}{2} = L_T + E_K + E_{\rm ho} + E_{\rm int}^{(1)} + E_{\rm int}^{(2)}, \tag{11}$$

where

$$L_T = N\hbar\omega_0 \frac{1}{4D}\left[-\frac{1}{\omega_0}(\alpha_1\dot{\beta}_2 + \beta_1\dot{\alpha}_2 - 2\gamma_1\dot{\gamma}_2)\right],$$

$$E_K = N\hbar\omega_0 \frac{a_0^2}{4D}[D(\alpha_1+\beta_1) + \alpha_1(\beta_2^2+\gamma_2^2) + \beta_1(\alpha_2^2+\gamma_2^2) - 2\gamma_1\gamma_2(\alpha_2+\beta_2)],$$

$$E_{\rm ho} = N\hbar\omega_0 \frac{1}{4D}\left[\frac{\alpha_1}{\lambda a_0^2} + \frac{\lambda\beta_1}{a_0^2}\right],$$

$$E_{\rm int}^{(1)} = N\hbar\omega_0 \frac{1}{4D}\frac{2\sqrt{2}}{\sqrt{\pi}}\left(\frac{a}{a_z}\right) a_0^2 N D^{3/2},$$

$$E_{\rm int}^{(2)} = N\hbar\omega_0 \left(\frac{128}{15\sqrt{\pi}}\right) a_0^2 (2\pi a)\left(\frac{2}{5\pi\sqrt{\pi}}\frac{a}{a_z}\right)^{3/2} N^{3/2} D^{3/4}.$$

To know the static properties of the condensate, we substitute the static values of different parameters as $\alpha_1 = \alpha_{10}$, $\beta_1 = \beta_{10}$, $\alpha_2 = \beta_2 = 0$, $\gamma_1 = \gamma_2 = 0$ in the Lagrangian in eq. (11). Then, from the Lagrange's equations of motion, we obtain the following coupled equations for α_{10} and β_{10}.

$$a_0^2\alpha_{10}^2 - \frac{\lambda}{a_0^2} + \frac{k_1}{2}\beta_{10}^{1/2}\alpha_{10}^{3/2} + \frac{3}{4}k_2\beta_{10}^{3/4}\alpha_{10}^{7/4} = 0, \tag{12}$$

$$a_0^2\beta_{10}^2 - \frac{1}{\lambda a_0^2} + \frac{k_1}{2}\alpha_{10}^{1/2}\beta_{10}^{3/2} + \frac{3}{4}k_2\alpha_{10}^{3/4}\beta_{10}^{7/4} = 0, \tag{13}$$

where

$$k_1 = \frac{4}{\sqrt{2\pi}}a_0^2\left(\frac{a}{a_z}\right)N,$$

and

$$k_2 = 4\frac{128}{15\sqrt{\pi}}a_0^2(2\pi a)\left(\frac{2}{5\pi\sqrt{\pi}}\frac{a}{a_z}\right)^{3/2} N^{3/2}.$$

These equations are coupled nonlinear equations. The terms with k_2 as coefficient in the above expressions appear due to the incorporation of beyond mean-field

corrections in the energy $\varepsilon(\rho)$. It can be noted here that for $k_2 = 0$, the above equations would lead to the same results as obtained earlier by Ghosh and Sinha [28]. For the known values of parameters (N, a, a_z, ω_0 and λ) of a condensate in a Q2D trap, the static size-parameters α_{10} and β_{10} along x- and y-axes can be evaluated by solving the above equations. We have solved these equations numerically. The values of α_{10} and β_{10} obtained thus have been used for the calculation of collective mode frequencies as described in the following section.

3. Collective modes

The low-energy excitations of the condensate correspond to the small oscillations in the cloud around the equilibrium configuration. Therefore, we can expand the time-dependent variational parameters around their static values in the following manner:

$$\begin{aligned}&\alpha_1 = \alpha_{10} + \delta\alpha_1(t), \quad \beta_1 = \beta_{10} + \delta\beta_1(t), \quad \gamma_1 = \delta\gamma_1(t) \quad (\gamma_{10} = 0),\\ &\alpha_2 = \delta\alpha_2(t), \quad \beta_2 = \delta\beta_2(t), \quad \gamma_2 = \delta\gamma_2(t).\end{aligned} \tag{14}$$

After substituting the above values in the Lagrangian in eq. (11) and using the Euler–Lagrange equations, the time evolution of the parameters $\delta\alpha_1$, $\delta\beta_1$ and $\delta\gamma_1$ can be obtained as

$$\begin{aligned}&\delta\ddot{\alpha}_1 + \omega_0^2 a_0^2 \left[\frac{4\lambda}{a_0^2} - \frac{k_1}{2}\alpha_{10}\sqrt{\alpha_{10}\beta_{10}} - \frac{3}{8}k_2\alpha_{10}(\alpha_{10}\beta_{10})^{3/4}\right]\delta\alpha_1\\ &+ \frac{\omega_0^2 a_0^2}{2}\frac{\alpha_{10}^2}{\beta_{10}}\left[k_1(\alpha_{10}\beta_{10})^{1/2} + \frac{9}{4}k_2(\alpha_{10}\beta_{10})^{3/4}\right]\delta\beta_1 = 0,\end{aligned} \tag{15}$$

$$\begin{aligned}&\delta\ddot{\beta}_1 + \omega_0^2 a_0^2 \left[\frac{4}{\lambda a_0^2} - \frac{k_1}{2}\beta_{10}\sqrt{\alpha_{10}\beta_{10}} - \frac{3}{8}k_2\beta_{10}(\alpha_{10}\beta_{10})^{3/4}\right]\delta\beta_1\\ &+ \frac{\omega_0^2 a_0^2}{2}\frac{\beta_{10}^2}{\alpha_{10}}\left[k_1(\alpha_{10}\beta_{10})^{1/2} + \frac{9}{4}k_2(\alpha_{10}\beta_{10})^{3/4}\right]\delta\alpha_1 = 0,\end{aligned} \tag{16}$$

$$\delta\ddot{\gamma}_1 + \omega_0^2\left[\left(\lambda + \frac{1}{\lambda}\right) + 2a_0^4\alpha_{10}\beta_{10}\right]\delta\gamma_1 = 0. \tag{17}$$

In the above eqs (15)–(17), the first two equations (i.e. (15) and (16)) are the coupled equations for $\delta\alpha_1$ and $\delta\beta_1$. Equation (17) for $\delta\gamma_1$ is not coupled to the other two. In order to calculate the mode frequencies, we take the harmonic time dependence of amplitudes as

$$\begin{aligned}&\delta\alpha_1 = \delta\alpha_1(\omega)\exp(-i\omega t), \quad \delta\beta_1 = \delta\beta_1(\omega)\exp(-i\omega t),\\ &\delta\gamma_1 = \delta\gamma_1(\omega)\exp(-i\omega t).\end{aligned} \tag{18}$$

This results in the solutions for mode frequencies as

$$\omega_\pm^2 = \left(\frac{a_1}{2} + \frac{b_2}{2}\right) \pm \left[\left(\frac{a_1}{2} - \frac{b_2}{2}\right)^2 + a_2\, b_1\right]^{1/2} \tag{19}$$

and

$$\frac{\omega_s^2}{\omega_0^2} = \left(\lambda + \frac{1}{\lambda}\right) + 2a_0^4(\alpha_{10}\beta_{10}), \tag{20}$$

where

$$a_1 = \omega_0^2 a_0^2 \left[\frac{4\lambda}{a_0^2} - \frac{k_1}{2}\alpha_{10}\sqrt{\alpha_{10}\beta_{10}} - \frac{3}{8}k_2\alpha_{10}(\alpha_{10}\beta_{10})^{3/4}\right],$$
$$b_1 = \frac{\omega_0^2 a_0^2}{2}\frac{\alpha_{10}^2}{\beta_{10}}\left[k_1(\alpha_{10}\beta_{10})^{1/2} + \frac{9}{4}k_2(\alpha_{10}\beta_{10})^{3/4}\right],$$
$$a_2 = \frac{\omega_0^2 a_0^2}{2}\frac{\beta_{10}^2}{\alpha_{10}}\left[k_1(\alpha_{10}\beta_{10})^{1/2} + \frac{9}{4}k_2(\alpha_{10}\beta_{10})^{3/4}\right],$$
$$b_2 = \omega_0^2 a_0^2 \left[\frac{4}{\lambda a_0^2} - \frac{k_1}{2}\beta_{10}\sqrt{\alpha_{10}\beta_{10}} - \frac{3}{8}k_2\beta_{10}(\alpha_{10}\beta_{10})^{3/4}\right].$$

Here it is worth mentioning that for $k_2 = 0$, eqs (19) and (20) correctly go to the corresponding results of ref. [28]. For the isotropic case ($\omega_x = \omega_y; \lambda = 1$) the modes with frequencies ω_+ and ω_- are excited by the operators $x^2 + y^2$ and $x^2 - y^2$ respectively. Comparing the operator of ω_+ with the three-dimensional monopole excitation operator $x^2 + y^2 + z^2$, we identify the ω_+ mode as the monopole mode frequency. For isotropic case within the mean-field approximation $\omega_+ = 2\omega_0$. On the other hand, the excitation operator for ω_- can be represented by the superposition $r_\perp^l e^{il\theta} + r_\perp^l e^{-il\theta}$ with $l = 2$. Therefore, we identify ω_- as the frequency of the quadrupolar mode in 2D. In the mean-field approximation, for large N, $\omega_- = \sqrt{2}\omega_0$. Analogous to the three-dimensional case we find that $\omega_+ > \omega_-$ and these two modes get coupled for the anisotropic trap case ($\lambda \neq 1$). The frequency ω_s corresponds to the oscillation by an excitation operator xy. This operator can also be written as superposition $r_\perp^l e^{il\theta} - r_\perp^l e^{-il\theta}$ with $l = 2$. Therefore, in an isotropic trap this mode corresponds to the quadrupole excitation and $\omega_s = \omega_-$. This excitation is also known as scissors mode [28,46,47]. In the above expressions for mode frequencies, the terms with k_1 represent the contribution of mean-field term (as studied by Ghosh and Sinha [28]) while terms with k_2 represent the contribution of the beyond mean-field correction term to the mode frequencies. It can be noted that monopole and quadrupole mode frequencies explicitly depend on interaction strength (k_1 and k_2), whereas scissors mode frequency ω_s does not explicitly depend on it. The dependence of ω_s on interaction strength occurs through parameters α_{10} and β_{10}.

We begin the discussion of the results by comparing the changes in the monopole frequency of an isotropic three-dimensional condensate with that of the corresponding Q2D condensate. It can be shown that shift in the monopole mode frequency (for $\lambda = 1$) due to beyond mean-field term for Q2D condensate is given by

$$\frac{\delta\omega_+}{\omega_+} = 0.216\left(\frac{Na}{a_z}\right)^{3/2}\left(\frac{a}{a_0}\right)\left(1 + \frac{2}{\sqrt{2\pi}}\frac{Na}{a_z}\right)^{-5/4} \tag{21}$$

which scales as $N^{1/4}$ and $a^{5/4}$ with number and scattering length respectively, in the limit $aN/a_z \gg 1$. In contrast to this, for three-dimensional case this shift scales as

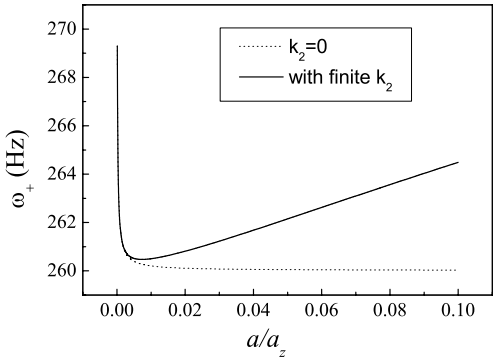

Figure 1. Variation of monopole mode frequency with scattering length. The other parameters are $N = 10^4$ (Na atoms), $\omega_z = 2\pi\times1000$ Hz, $\omega_0 = 2\pi\times20$ Hz, $a_z = 6.6 \times 10^{-7}$ m and $\lambda = 0.8$. The continuous curve shows the frequency values with a finite value of k_2 (i.e. after incorporating beyond mean-field term) whereas dotted curve shows those with $k_2 = 0$ (i.e. with mean-field term only as in ref. [28]).

$N^{1/5}$ and $a^{6/5}$ [48]. This difference in scaling is due to the reduced dimensionality in Q2D case. Next we estimate the order of changes of the frequencies of the collective oscillations associated with a Q2D condensate of Rb^{87} atoms reported recently in ref. [17]. For experimental parameters ($N = 8\times10^4$, $\omega_z = 2\pi\times1990$ Hz, $\omega_x = \omega_y = 2\pi \times 20$ Hz), we estimate the above frequency shift to be 0.36%, which is small to detect. The small correction obtained here is due to the low value of gas parameter ($\approx3.36 \times10^{-5}$) as well as small value of a/a_z (≈0.02). Nevertheless by increasing either N or scattering length a, the shift can be made experimentally detectable.

As scattering length can be conveniently varied with the applied magnetic field, we have calculated the dependence of monopole mode frequency ω_+ on scattering length (a) for finite k_2 as well as for $k_2 = 0$ (as shown in figure 1). Here, figure 1 shows the ratio a/a_z to compare the value of scattering length with oscillator length a_z. The maximum value of scattering length shown in this figure corresponds to about 24 times the scattering length in the absence of any magnetic field (i.e. 27.5 Å for Na atom), but it is still much smaller than a_z to insure the three-dimensional scattering in the trap. It is evident (figure 1) that with finite k_2, variation of monopole mode frequency with a/a_z is quantitatively as well as qualitatively different from that with $k_2 = 0$. With increasing a/a_z, after the initial fall, the frequency ω_+ becomes nearly constant if $k_2 = 0$. Contrasting this, with finite k_2, it increases monotonously after the initial fall. For large values of scattering length, we note that shift ($\delta\omega_+/\omega_+$) can be as large as few per cent, which is easily detectable with the kind of measurement accuracy reported in ref. [40].

As has been discussed earlier [28], the quadrupole and scissors modes in an anisotropic 2D trap get split. In addition to anisotropy parameter λ, the value of mode splitting ($\Delta = \omega_s - \omega_-$) depends upon interaction strength also. To assess the effect of modified interaction strength due to beyond mean-field term on splitting Δ, we have calculated Δ for different values of scattering length a. The calculation

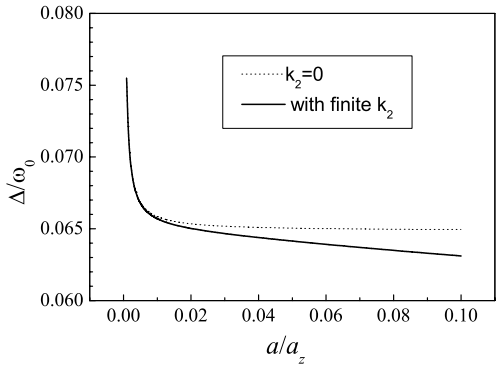

Figure 2. Variation of mode splitting ($\Delta = \omega_s - \omega_-$) with a/a_z. The other parameters are $N = 10^4$ (Na atoms), $\omega_z = 2\pi \times 1000$ Hz, $\omega_0 = 2\pi \times 20$ Hz, $a_z = 6.6 \times 10^{-7}$ m and $\lambda = 0.8$. The dotted curve shows Δ values calculated for $k_2 = 0$ (i.e. mean-field theory as in ref. [28]) whereas the continuous curve shows that for finite value of k_2 (i.e. incorporating the beyond mean-field term).

was done for finite k_2 as well as for $k_2 = 0$. For $k_2 = 0$, our results match to those reported in ref. [28]. The results of our calculation for anisotropy parameter $\lambda = 0.8$ are shown in figure 2. As is evident from the figure, the splitting Δ calculated with finite k_2 keeps on changing with a/a_z whereas it becomes nearly constant with a/a_z for $k_2 = 0$. Since scissors mode frequency ω_s has a negligible dependence on k_2, the difference in the values of Δ for two curves in figure 2 arises mainly due to k_2 dependence of quadrupole frequency (ω_-).

4. Conclusion

We have theoretically studied the collective mode frequencies of a quasi-two-dimensional Bose condensate in the large gas parameter regime. We find that, in this regime, the inclusion of beyond mean-field correction term in the interaction energy leads to quantitative as well as qualitative modifications in the mode frequencies. Such regime of parameters is experimentally accessible with the present advancements in this field and modifications in the mode frequencies are easily detectable.

Acknowledgments

We thank Dr S C Mehendale for a critical reading of the manuscript.

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