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Evidence of self-affine multiplicity scaling of charged-particle multiplicity distribution in hadron–nucleus interaction

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Abstract. A self-affine analysis of charged-particle multiplicity distribution (protons + pions) in π^-–AgBr interaction at 350 GeV/c is performed according to the two-dimensional factorial moment methodology using the concept of Hurst exponent in $X_{\cos\theta}$–X_ϕ phase space. Comparing with the results obtained from self-similar analysis, the self-affine analysis shows a better power-law behaviour. Corresponding results are compared with shower multiplicity distribution (pions). Multifractal behaviour is observed for both types of distributions.

Keywords. Self-affine scaling; anomalous fractal dimension; multifractality.

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1. Introduction

The large density fluctuations in multiparticle production in small rapidity regions at high energies possess self-similar properties [1–3] as the resolution in phase space is increased up to the experimental or statistical limit. Bialas and Peschanski [4] have called such multiplicity fluctuations 'intermittency' which is based on the idea of analogous bursts of turbulence in the theory of chaos [5]. During recent years the subject of intermittency has gained significant interest in high-energy nuclear collisions [6–12]. It is the phenomenon of power-law behaviour of scaled factorial moment with decreasing bin size and is capable of extracting the non-statistical fluctuations after eliminating the statistical part.

Ever since the first reports of spiky events, first observed in cosmic ray studies [13] and later confirmed in the laboratory [14], non-statistical fluctuations in the density of produced particles have attracted a lot of attention. In the past few years many workers reported on large density fluctuations in different interacting systems [6–12]. Several theoretical interpretations of the origin of large fluctuations have been proposed. It has alternatively been suggested that the fluctuations are: an effect of hadronic Cerenkov radiation [15], a result of cascading or branching mechanism [4,16], consistent with jet models with scale invariant branching structure [17]. Another interpretation is based on the formation of globs of non-thermal QGP of sizeable parton number density but with very low mean energy per parton [18]. Understanding the origin of these large fluctuations may provide new insights into the underlying mechanisms responsible for the particle production. Therefore, analysis of the properties of fluctuations is important.

A large number of experiments have been performed to search for the power-law behaviour [10]

$$F_q(\delta\eta) \propto (\delta\eta)^{-}\phi_q \quad (\delta\eta \to 0).$$

The results have shown that for one-dimensional variables, the factorial moments tend to saturate at small phase space intervals. But the real process occurs in three dimensions. Ochs [19] indicated that one-dimensional analysis is not at all sufficient for extracting the proper fluctuation patterns of the real three-dimensional process. A restriction to one-dimension may yield quite misleading conclusions on the underlying dynamics of the multiparticle production. Therefore, multidimensional analysis is important for the interpretation of the fluctuation phenomena and fractal analysis. The non-linear dependence of $\ln F_q(\delta\eta)$ on $\ln(\delta\eta)$ can be explained as a projection effect of a three-dimensional phenomenon [19]. It is therefore expected that the scaling phenomenon can be observed in a higher dimensional analysis. In three-dimensional analyses, however, only approximate scaling is observed in all reactions. The $\ln F_q$ vs. $\ln M$ for 3D data is an upward bending curve [10,19], where M is the number of sub-cells in three-dimensional phase space. This means that the power-law behaviour does not hold exactly for these reactions. The reason for this is that the usual procedure for calculating the higher dimensional factorial moments is to divide phase space into sub-cells with the same M in each direction assuming that the phase spaces are isotropic in nature. This is called self-similar analysis. However, phase space in high-energy multiparticle production is anisotropic as indicated by Van Hove [20]. It is worthwhile to mention here that experimental observation regarding momentum values in different directions also support this notion. Thus, it is expected that the fluctuations or scaling properties if exist should also be anisotropic and the scaling behaviour at the same time should also be different in longitudinal and transverse directions (i.e. scaled anisotropically). Most of the present analyses were inadequate to some extent in a sense that they overlooked the anisotropy of phase space and consequently the possibility of having a self-affine fluctuation pattern (rather than self-similar pattern) consistent with the nature of the phase space structure. Only a few works have been reported so far where the evidence of self-affine multiparticle production is indicated by the data [21–27].

According to nuclear emulsion terminology [28], the particles emitted in high-energy interactions are classified as:

(a) *Black particles*: They are target fragments with ionization greater than or equal to $10I_0$, I_0 being the minimum ionization of a singly charged particle. Their ranges are less than 3 mm. Their velocity is less than $0.3c$ and their energy is less than 30 MeV, where c is the velocity of light in free space.
(b) *Grey particles*: They are mainly fast target recoil protons with energy up to 400 MeV. The ionization power of grey particles lies between $1.4I_0$ to $10I_0$. Their ranges are greater than 3 mm and they have velocities between $0.3c$ and $0.7c$.
(c) *Shower particles*: They are mainly pions with ionization $\leq 1.4I_0$. These particles are generally not confined within the emulsion pellicle.

Many attempts have been made to find the proper dynamics of the multiparticle production process which are mostly confined to the domain of investigation of the relativistic shower particles, e.g. [29–32] though the medium energy (30–400 MeV) knocked out protons, which manifest themselves as grey particles in nuclear emulsion, may also play an important role in this regard. It is generally believed that grey particles are supposed to carry relevant information about the hadronization mechanism, since the time-scale of emission of these particles is of the same order ($\approx 10^{-22}$ s) as that of the produced shower particles [28,33–35,38] and hence are expected to remember a part of the history of these reactions. Still only some elementary investigations have been done [36,37] and even the emission process of these protons has not yet been fully explained theoretically.

Therefore, if one combines the number of grey and shower particles per event in a collision as a new parameter, named 'compound multiplicity' ($C = N_\mathrm{g} + N_\mathrm{s}$), it could also play an important role in understanding the reaction dynamics in high-energy nuclear interactions. As per our knowledge, a pioneering study of compound multiplicity (charged-particle multiplicity) has been done by Jurak *et al* [38]. But so far only a few attempts have been made to work with this parameter [39–45]. In order to establish that the charged-particle multiplicity is also an important parameter in the study of the reaction mechanism, one has to investigate other kinds of effects with this parameter, already observed for the case of shower multiplicity.

Most of the self-affinity analysis was performed on shower particles in high-energy interactions. Recently, we have published a paper on self-affine scaling of compound multiplicity (charged-particle multiplicity) distribution [46]. But this is not enough to understand the characteristics of pions and protons. Therefore, in order to glean some more useful and important information we have carried out a detailed study of the nature of dynamical fluctuation of charged-particle multiplicity distribution for π^-–AgBr interaction at 350 GeV/c considering the anisotropy of phase space. The study has given the indication of self-affine behaviour in full bin range considered here. The same methodology is repeated for shower multiplicity also for comparison.

Study of intermittent behaviour in turbulent fluids has been done using the fractal dimension [47–49]. This has prompted exploration of intermittency concept in multiparticle production in terms of fractal formalism. The observed power-law behaviour of the normalized factorial moment has become indicative of fractal properties of multiparticle production process. A fractal object satisfies a power-law scaling which reflects the underlying dynamics. The word fractal was first coined by Mandelbrot [50]. He proposed that there is a fractal face to the geometry of nature. He opened a new window, namely fractal geometry, for looking into the world of apparent irregularities. Fractals fall into two categories: multifractals and monofractals. The fundamental characteristics of multifractality is that the scaling properties may be different for different regions of the system. Monofractals are those whose scaling properties are the same in different regions of the system.

Lipa and Buschbeck [51] have correlated the scaling behaviour of factorial moments to the physics of fractal objects. They have shown that the anomalous fractal dimension d_q, which is used for the description of fractal objects in other branches of physics, can be computed from the intermittency index α_q using the relation

$$d_q = \alpha_q/(q-1).$$

2. Experimental details

We study the hadron–nucleus interaction data of π^-–AgBr at 350 GeV/c. A stack of G5 nuclear emulsion plate was exposed horizontally to a π^- beam at CERN with 350 GeV/c.

The nuclear emulsion covers 4π geometry and provides very good accuracy, even less than 0.1 mrad, in angle measurements due to high spatial resolution and thus is suitable as a detector for the study of fluctuations in fine resolution intervals of the phase space. The emulsion plates were area scanned with a Leitz Metalloplan Microscope fitted with a semiautomatic scanning device, having a resolution of 1 μm along the X and Y axes while along the Z axis the resolution is 0.5 μm. A sample of 569 events of π^-–AgBr at 350 GeV/c was chosen, following the usual emulsion methodology for selection criteria of the events. The average multiplicity of the shower particles is 11.7 and compound multiplicity is 15.04. The details of scanning and measurement including selection of events are described in our earlier papers [52].

The emission angle (θ) and azimuthal angle (ϕ) are measured for each track by taking readings of the coordinates of the interaction point (X_0, Y_0, Z_0), coordinates (X_i, Y_i, Z_i) at the end of the linear portion of each secondary track and coordinates (X_1, Y_1, Z_1) of a point on the incident beam.

3. Method of analysis

The method of factorial moment is used here to analyse the intermittent type of fluctuations of emitted particles in two-dimensional space. The non-uniformity of particle spectra influences the scaling behaviour of factorial moments. Bialas and Gazdzicki [53] proposed a method to construct a set of variables which drastically reduces the distortion of intermittency due to non-uniformity of single-particle density distribution. According to them the new scaled variable X_z is related to the single-particle density distribution $\rho(z')$ as

$$X_z = \int_{z_1}^{z} \rho(z')\partial z' \bigg/ \int_{z_1}^{z_2} \rho(z')\partial z', \tag{1}$$

where z_1 and z_2 are the two extreme points of the distribution. The variable X_z varies between 0 and 1 keeping $\rho(X_z)$ almost constant.

Here intermittency analysis will be performed in two-dimensional phase space. Denoting the two phase space variables as x_1 and x_2, factorial moment of order q may be defined as [4]

$$F_q(\delta x_1, \delta x_2) = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m(n_m-1)\cdots(n_m-q+1)\rangle}{\langle n_m\rangle^q}, \tag{2}$$

where $\delta x_1 \delta x_2$ is the size of a two-dimensional cell, $\langle\rangle$ indicates the average over the whole ensemble of events and n_m is the multiplicity in the mth cell.

According to Bialas and Peschanski, the method of scaled factorial moments is free of statistical fluctuations caused by finite event and particle numbers and hence can be used to investigate the non-statistical (dynamical) fluctuations. It was shown [4] that if the experimental distribution is a convolution of the dynamical probability distribution and the statistical probability distribution represented by Poissonian distribution, the factorial moment of the experimental distribution is equivalent to C-moment of the dynamical distribution. This method actually filters out the statistical fluctuation if it is of Poissonian form and thus assures that experimental result is not marred by the Poissonian fluctuation.

Now the problem is how to fix δx_1, δx_2 and M. To solve this problem, let us fix a two-dimensional region $\Delta x_1 \Delta x_2$ and divide it into sub-cells of width $\delta x_1 = \Delta x_1/M_1$ and $\delta x_2 = \Delta x_2/M_2$, where M_1 is the number of bins along the x_1 direction and M_2 is the number of bins along the x_2 direction. Cell size dependence of factorial moment is studied by shrinking the bin widths in both directions. There are two ways of doing it; widths may be shrinked equally ($M_1 = M_2$) or unequally ($M_1 \neq M_2$) in the two dimensions. The shrinking ratios along x_1 and x_2 directions are characterized by a parameter $H = \ln M_1/\ln M_2$ ($0 < H \leq 1$) which is called the roughness or Hurst exponent [54]. If and only if the shrinking ratios along the two directions satisfy the above relation with a particular H value, the function $F_q(\delta x_1, \delta x_2)$ will have a well-defined scaling property. $H = 1$ signifies that the phase space is divided isotropically and consequently fluctuations are self-similar. When $H < 1$ it is clearly understood that the phase spaces along x_1 and x_2 directions are divided anisotropically and consequently the fluctuations are self-affine in nature.

The intermittent behaviour of multiplicity distribution manifests itself as a power-law dependence of factorial moment on the cell size as cell size $\to 0$.

$$\langle F_q\rangle \propto M^{\alpha_q}. \tag{3}$$

The exponent α_q is the slope characterizing the linear rise of $\ln\langle F_q\rangle$ with $\ln M$. The strength of intermittency is characterized by this exponent α_q. α_q can be obtained from a linear fit of the form

$$\ln\langle F_q\rangle = \alpha_q \ln(M) + a, \tag{4}$$

where a is a constant.

4. Results and discussions

In order to reduce the effect of non-flat average distribution, the cumulative variables $X_{\cos\theta}$ and X_ϕ are used instead of $\cos\theta$ and ϕ [55]. In the $X_{\cos\theta}$–X_ϕ space

Table 1. Values of intermittency exponent α_q, χ^2/DoF and confidence level of fittings at $H = 0.6$ and $H = 1$ for charged-particle multiplicity distribution.

H	q	α_q	χ^2/DoF	Confidence level of fittings
	2	0.657±0.011	1.014	44%
0.6	3	1.609±0.025	1.127	26%
	4	2.678±0.036	1.011	45%
	2	0.725±0.011	2.15	Almost 0%
1	3	1.731±0.046	5.701	Almost 0%
	4	2.789±0.085	6.616	Almost 0%

we divided the region [0,1] into $M_{\cos\theta}$–M_ϕ bins respectively. The partitioning was taken as $M_{\cos\theta} = M_\phi^H$, where we choose the partition number along ϕ direction as $M_\phi = 5, 6, 7, \ldots, 50$. We have not considered the first few data points $M_\phi = 2, 3, 4$ in order to reduce the effect of momentum conservation [56] which tends to spread the particles in opposite directions and thus reduce the value of the factorial moments. This effect becomes weaker as M increases. The $\cos\theta$–ϕ space is divided into $M = M_{\cos\theta} \times M_\phi$ bins and calculation is done in each bin independently.

To analyze the anisotropic nature of the $X_{\cos\theta}$–X_ϕ phase space, factorial moments of different orders with varying values of Hurst exponent starting from 0.3 to 1 in steps of 0.1 are calculated for charged-particle multiplicity distribution data set. The average factorial moment $\langle F_q\rangle$ is observed to depend linearly on the number of the two-dimensional cells M in a log–log plot. The dependence is more or less linear in all cases. In order to find the partition condition at which the anisotropic behaviour is best revealed, we have performed the linear best fits from which χ^2 per degree of freedom is calculated. We have also calculated the confidence level of fittings from the χ^2 values. The values of χ^2 per degree of freedom and the confidence level of fittings are tabulated in table 1. The minimum value of χ^2 per degree of freedom indicates the best linear behaviour. For charged-particle multiplicity distribution the best linear fit occurs at $H = 0.6$ which shows that the anisotropic behaviour is best revealed at $H = 0.6$.

For this H value, $\ln\langle F_q\rangle$ is plotted as a function of $\ln M$ in figure 1 for charged-particle multiplicity distribution. The best linear fit is shown in the figure. To compare the self-affine behaviour with the self-similar one, the variation of $\ln\langle F_q\rangle$ with $\ln M$ corresponding to $H = 1$ is shown in figure 2. For $H = 1$ also χ^2 per degree of freedom values are calculated and shown in table 1. From the table it is seen that χ^2 per degree of freedom values are significantly high and confidence level of fittings are very poor indicating that the scaling behaviour does not hold good at $H = 1$. χ^2 per degree of freedom values and confidence level of fittings at $H = 0.6$ are better than the corresponding values obtained at $H = 1$. Therefore, we may say that the dynamical fluctuation pattern of charged-particle multiplicity distribution in π^-–AgBr interaction at 350 GeV/c is not self-similar but self-affine.

The whole procedure is repeated for shower multiplicity distribution also. Corresponding values are calculated and tabulated in table 2. It is seen from the table

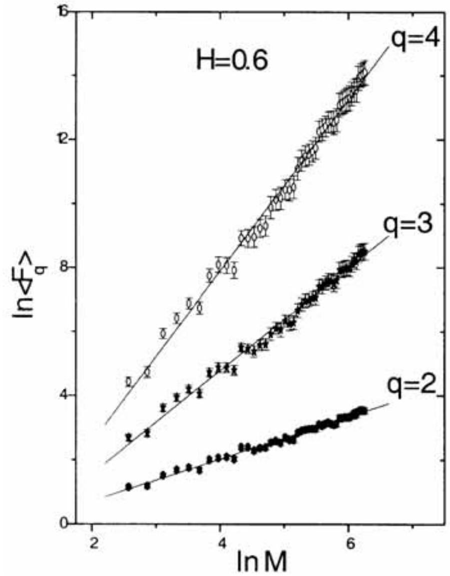

Figure 1. Plot of $\ln\langle F_q\rangle$ as a function of $\ln M$ at $H = 0.6$ for charged-particle multiplicity distribution.

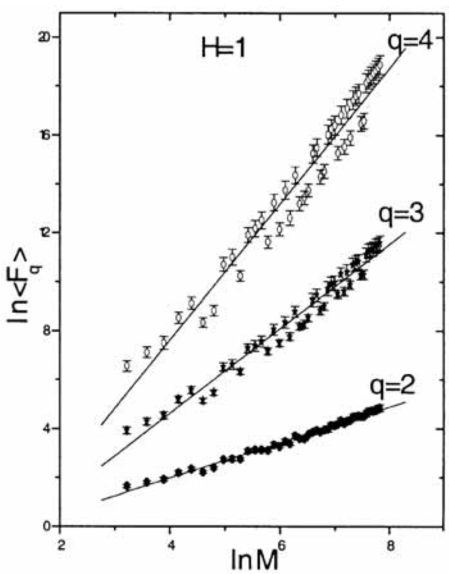

Figure 2. Plot of $\ln\langle F_q\rangle$ as a function of $\ln M$ at $H = 1$ for charged-particle multiplicity distribution.

that fluctuation pattern is self-affine rather than self-similar for shower multiplicity distribution also. $\ln\langle F_q\rangle$ vs. $\ln M$ graphs are shown in figures 3 and 4.

$\ln\langle F_q\rangle$ vs. $\ln M$ graphs are drawn and linear best fits are performed. From the linear best fits intermittency exponents are extracted. It is seen from tables 1 and 2 that intermittency index increases with the order of moment.

From tables 1 and 2 corresponding results of charged-particle multiplicity and shower multiplicity distribution are compared with each other. Minimum χ^2 per degree of freedom does not occur at the same H value for charged-particle multiplicity distribution as for shower multiplicity.

It is mentioned earlier that n_m is the number of particles in the mth cell. The typical value of $\langle n_m\rangle$ for compound multiplicity distribution is from 0 to 2.45 for $H = 0.6$ and 0 to 1.62 for $H = 1$. For shower multiplicity distribution it is 0 to 1.42 for $H = 0.7$ and 0 to 1.14 for $H = 1$. The cells where $\langle n_m\rangle$ are zero are excluded from the calculation.

It is worthwhile to mention that we have calculated the average factorial moment (more precisely horizontal average). It is not impossible that the factorial moment of the distribution in a number of bins may be zero. But due to clustering effect it is expected that the factorial moment in some bins will have non-zero values resulting in non-zero bin average of factorial moment. However, it is true that scaled factorial moment faces limitation due to statistical and systematic problems. The empty bin effect [57–59] is one of them. With the improvement of detection technique, it has become possible to increase the experimental resolution. There are strong indications, both experimental and theoretical, that deviation from

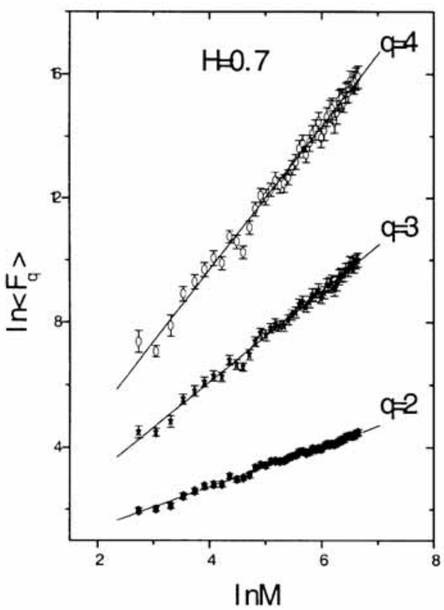

Figure 3. Plot of $\ln\langle F_q\rangle$ as a function of $\ln M$ at $H = 0.7$ for shower multiplicity distribution.

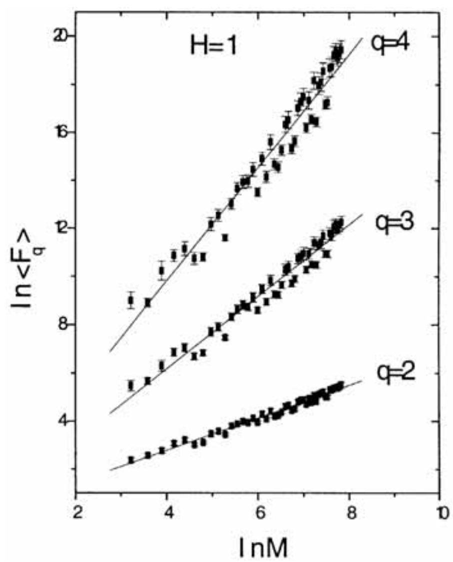

Figure 4. Plot of $\ln\langle F_q\rangle$ as a function of $\ln M$ at $H = 1$ for shower multiplicity distribution.

power-law behaviour will occur at such high resolutions. As multiplicities are finite, increase in experimental resolution has the obvious effect of reducing the statistics per bin. Such reduction causes inflection of the factorial moments F_q towards zero, in the region of the highest resolution attainable.

However, the method we have adopted in our analysis to calculate factorial moment attempts to reduce the so-called empty bin effect. This point has also been addressed in a paper by OPAL Collaboration Group [60] where the minimization of empty bin effect is done using a modified factorial moment. These moments are determined for a uniform single particle distribution. In order to account for non-uniformity, corrected modified factorial moments are used which is defined as

$$F_q^c = F_q/R_q,$$

where R_q is the correction factor and it is equal to unity for a uniform single particle distribution. Therefore, for uniform single particle distribution F_q^c is equal to F_q. In our analysis we have used cumulative variables, which also correspond to uniform particle distribution. Therefore, we can infer that two analyses are equivalent to each other. Further non-inflection of factorial moments towards zero (figures 1–4) at higher resolution implies that the effect is not predominant here.

To ensure that the observed behaviour of factorial moments is not a manifestation of mere statistics alone, the experimental data have to be compared with those calculated from Monte Carlo simulation. For obtaining the simulated events we have utilized the framework of independent emission hypothesis which includes the

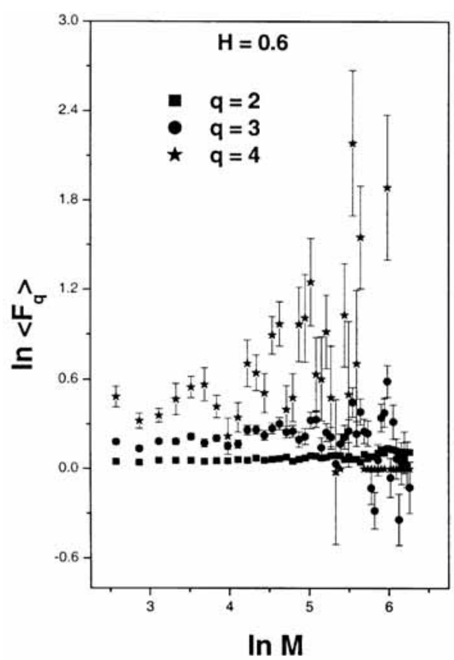

Figure 5. Plot of $\ln\langle F_q\rangle$ as a function of $\ln M$ at $H = 0.6$ for charged-particle multiplicity distribution in simulated events.

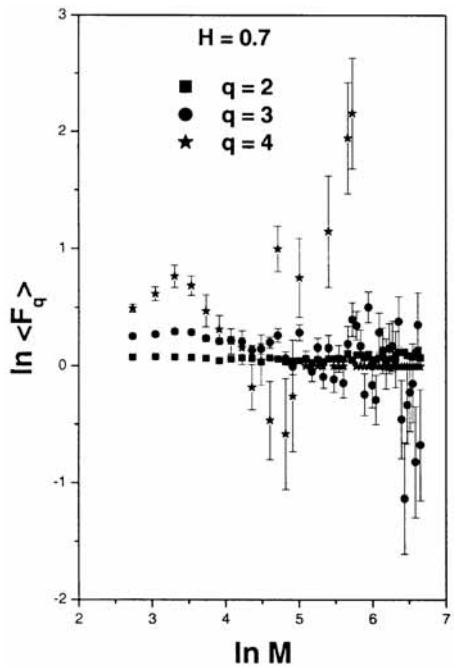

Figure 6. Plot of $\ln\langle F_q\rangle$ as a function of $\ln M$ at $H = 0.7$ for shower multiplicity distribution in simulated events.

following assumptions:

(i) The particles are emitted independently.

(ii) The multiplicity distribution of the Monte Carlo events is the same as the empirical multiplicity spectrum of the real ensemble.

(iii) The angular distribution of the simulated events reproduces the angular distribution of the experimental data in both the phase spaces for both shower and charged particle multiplicity.

Now the same self-affine analysis is repeated for this simulated data set. Non-intermittent behaviour is very much evident in all the cases. No scaling behaviour is observed for any value of Hurst exponent H. To compare the results of simulated data with that of experimental data we have shown the graphs of $\ln\langle F_q\rangle$ with $\ln M$ for those values of H for simulated data at which anisotropic scaling behaviour is best revealed for experimental data (figures 5 and 6). We have also shown plots of $\ln\langle F_q\rangle$ vs. $\ln M$ for $H = 1$ also in the case of simulated data (figures 7 and 8). Therefore, we can conclude that scaling behaviour is not observed for simulated data set which confirms that experimental data analysis reveals the true dynamical signal.

The self-affine scaling has been observed successfully in π^-–AgBr interaction at 350 GeV/c. But this is not enough for understanding the properties of the

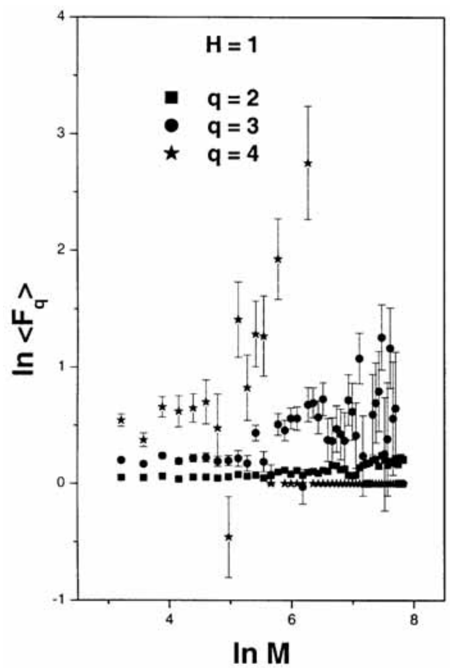

Figure 7. Plot of $\ln\langle F_q\rangle$ as a function of $\ln M$ at $H = 1$ for charged-particle multiplicity distribution in simulated events.

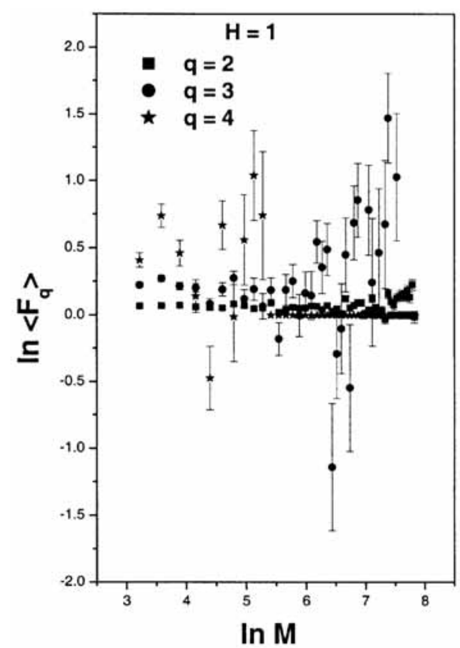

Figure 8. Plot of $\ln\langle F_q\rangle$ as a function of $\ln M$ at $H = 1$ for shower multiplicity distribution in simulated events.

dynamical fluctuations in high-energy multiparticle production. We should study the self-affine fractal nature also. In order to study the variation of anomalous fractal dimension d_q with the order of moment q, the d_q values are calculated at $H = 0.6$ for charged-particle multiplicity distribution (tabulated in table 3). The variation of d_q with order q is shown in figure 9. The values of d_q increase with order q. It reflects the multifractal geometry in charged-particle multiplicity distribution. For shower multiplicity distribution d_q values are calculated at $H = 0.7$ and listed in table 4. The variation of d_q with order q is shown in figure 10. d_q values increase with q values showing multifractality for shower multiplicity distribution also.

5. Conclusions

The following conclusions can be drawn from the present investigation:

1. Dynamical fluctuation pattern in π^-–AgBr interaction at 350 GeV/c of charged multiplicity distribution (pions+protons) is not self-similar but self-affine in nature. This is also true for shower multiplicity (pions).
2. The anisotropic behaviour is best revealed at $H = 0.6$ in full bin range for charged-particle multiplicity and at $H = 0.7$ for shower multiplicity.

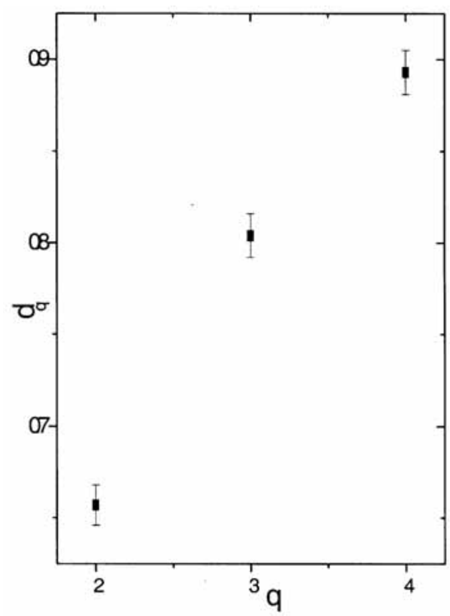

Figure 9. Plot of d_q as a function of q at $H = 0.6$ for charged-particle multiplicity distribution.

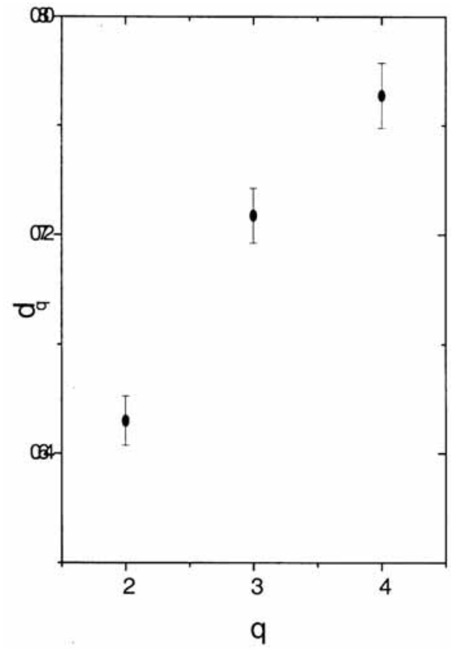

Figure 10. Plot of d_q as a function of q at $H = 0.7$ for shower multiplicity distribution.

Table 2. Values of intermittency exponent α_q, χ^2/DoF and confidence level of fittings at $H = 0.7$ and $H = 1$ for shower multiplicity distribution.

H	q	α_q	χ^2/DoF	Confidence level of fittings
	2	0.652±0.009	0.466	99.9 %
0.7	3	1.454±0.02	0.739	90 %
	4	2.313±0.037	1.159	22 %
	2	0.679±0.015	3.262	Almost 0%
1	3	1.496±0.045	6.364	Almost 0%
	4	2.361±0.079	8.654	Almost 0%

Table 3. Values of α_q and d_q at $H = 0.6$ for charged-particle multiplicity distribution.

H	q	α_q	d_q
	2	0.657±0.011	0.657±0.011
0.6	3	1.609±0.025	0.804±0.012
	4	2.678±0.036	0.893±0.012

Table 4. Values of α_q and d_q at $H = 0.7$ for shower multiplicity distribution.

H	q	α_q	d_q
	2	0.652±0.009	0.652±0.009
0.7	3	1.454±0.02	0.727±0.01
	4	2.313±0.037	0.771±0.012

3. d_q increases with increasing q showing that there is self-affine multifractal behaviour in multiparticle production at the 350 GeV/c π^-–AgBr interaction.

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