PRAMANA — journal of physics
© Indian Academy of Sciences
Vol. 68, No. 5
May 2007
pp. 735–748

On the relative energy associated with space-times of diagonal metrics

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MS received 5 October 2006; accepted 28 February 2007

Abstract. In order to evaluate the energy distribution (due to matter and fields including gravitation) associated with a space-time model of generalized diagonal metric, we consider the Einstein, Bergmann–Thomson and Landau–Lifshitz energy and/or momentum definitions both in Einstein's theory of general relativity and the teleparallel gravity (the tetrad theory of gravitation). We find same energy distribution using Einstein and Bergmann–Thomson formulations, but we also find that the energy–momentum prescription of Landau–Lifshitz disagree in general with these definitions. We also give eight different well-known space-time models as examples, and considering these models and using our results, we calculate the energy distributions associated with them. Furthermore, we show that for the Bianchi Type-I models all the formulations give the same result. This result agrees with the previous works of Cooperstock–Israelit, Rosen, Johri *et al*, Banerjee–Sen, Xulu, Vargas and Saltı *et al* and supports the viewpoints of Albrow and Tryon.

Keywords. Energy; diagonal space-times; teleparallel theory.

PACS Nos 04.40.-q; 04.20.Jb; 04.50.+h; 04.70.-s

1. Introduction

In the gravitation theories of general relativity and the tetrad theory of gravity (the teleparallel gravity), the formulation of energy and/or momentum distributions is one of the oldest, interesting and controversial problems. The first formulation was written by Einstein who proposed an expression for the energy–momentum of the gravitational field. After this pioneering work, there have been many attempts to resolve the energy–momentum problem; e.g. Tolman [1], Papapetrou [2], Bergmann–Thomson [3], Møller [4,5], Weinberg [6], Qadir–Sharif [7], Landau–Liftshitz [8] and the teleparallel gravity analogs of the Møller [9] and Einstein, Landau–Lifshitz, Bergmann–Thomson [10] prescriptions. Except for the Møller four-momentum definition, these complexes give meaningful results when we transform the line-element into the quasi-Cartesian coordinates. The energy and momentum complex of Møller gives the possibility to make the calculations in any coordinate system [11].

Virbhadra and his collaborators have considered many space-time models and showed that several energy–momentum complexes give same and acceptable results for a given space-time model [12–23]. Virbhadra [15], using the general relativity versions of energy and momentum complexes of Einstein, Landau–Lifshitz, Papapetrou and Weinberg's for a general non-static spherically symmetric metric of the Kerr–Schild class, showed that all of these energy–momentum formulations give the same energy distribution as in Penrose energy–momentum complex. And in teleparallel gravity, there have been some attempts to show that the teleparallel gravitational energy–momentum definitions give the same results as obtained by using the general relativistic ones [10,24–27]. In his recent paper, Vargas [10] using the Einstein and Landau–Lifshitz complexes, calculated the energy–momentum density of the Friedman–Robertson–Walker space-time and showed that the result is the same as obtained in general relativity.

We hope to find the same and acceptable energy distribution both in Einstein's theory of general relativity and the tetrad theory of gravity. The present paper is organized as follows: in the next section, we introduce our general space-time model to be considered and give eight special space-time models as examples for our model. Next, in §3, we give the energy and/or momentum definitions of Einstein, Bergmann–Thomson and Landau–Lifshitz both in general relativity and the tetrad theory of gravity. In §4, we calculate the energy distributions (due to matter and fields including gravitation) associated with our general model both in Einstein's theory of general relativity and the tetrad theory of gravity using the formulas which we give in §3. Section 5 gives us some special cases of our energy solutions. Finally, we summarize and discuss our results.

Throughout this paper the Greek indices take values from 0 to 3, Latin indices take values from 1 to 3 and $G = 1$, $c = 1$ units as a convention.

2. Space-time models to be considered

The general diagonal line-element can be given as

$$\begin{aligned} \mathrm{d}s^2 = &-A^2(t,x,y,z)\mathrm{d}t^2 + B^2(t,x,y,z)\mathrm{d}x^2 + C^2(t,x,y,z)\mathrm{d}y^2 \\ &+D^2(t,x,y,z)\mathrm{d}z^2. \end{aligned} \tag{1}$$

This space-time model can be reduced to some well-known space-times under special choices of A, B, C and D. One can easily find lots of models which are special cases of our general line-element. Here we give a few of them as examples.

(A) The de Sitter C-space-time: The massive charged de Sitter C-metric has been found by Plebanski and Demianski [28], and its gravitational field can be written by the conditions (see e.g. [29])

$$A^2(x,y) = \frac{\sqrt{K}}{M(x+y)}, \quad B^2(x,y) = \frac{1}{M(x+y)\sqrt{W}}, \tag{2}$$

$$C^2(x,y) = \frac{1}{M(x+y)\sqrt{K}}, \quad D^2(x,y) = \frac{\sqrt{W}}{M(x+y)}, \tag{3}$$

where

$$K(y) = -\frac{\Lambda + 3M^2}{3M^2} + y^2 - 2mMy^3 + q^2M^2y^4, \tag{4}$$

$$W(x) = 1 - x^2 - 2mMx^3 - q^2M^2x^4. \tag{5}$$

This solution depends on four parameters, namely, the cosmological constant Λ, $M > 0$ which is the acceleration of the black holes, and m and q which are interpreted as electromagnetic charge of the non-accelerated black holes.

(B) The spatially self-similar locally rotationally symmetric models: These models can be written under the following definitions [30]:

$$A^2(t,x,y,z) = 1, \quad B^2(t,x,y,z) = D_1^2(t), \tag{6}$$

$$C^2(t,x,y,z) = D_2^2(t)\mathrm{e}^{-2ax}, \quad D^2(t,x,y,z) = D_2^2(t)k^{-1}\sin\sqrt{ky}, \tag{7}$$

where a and k are parameters describing symmetry groups of various models.

(C) The general Bianchi Type-I space-times: This model is given by the following conditions [24]:

$$A^2(t,x,y,z) = 1, \quad B^2(t,x,y,z) = B^2(t), \tag{8}$$

$$C^2(t,x,y,z) = C^2(t), \quad D^2(t,x,y,z) = D^2(t). \tag{9}$$

Also, under special choices of $A(t)$, $B(t)$ and $C(t)$ functions, one can define some well-known closed Universes as given below.

1. Defining $A = B = C = R(t)$ and transforming the line-element (1) to t, x, y, z coordinates according to

$$\begin{aligned} x &= r\sin\theta\cos\phi, \\ y &= r\sin\theta\sin\phi, \\ z &= r\cos\theta, \end{aligned} \tag{10}$$

gives

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + R^2(t)[\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)], \tag{11}$$

which describes the well-known spatially flat Friedmann–Robertson–Walker space-time. The Friedmann–Robertson–Walker cosmological model has attracted considerable attention in the relativistic cosmology literature. One of the most important properties of this model may be, as predicted by inflation [31], the flatness, which agrees with the observed cosmic microwave background radiation.

In the early Universe the sorts of the matter fields are uncertain. The existence of anisotropy at early times is a very natural phenomenon to investigate, as an attempt to clarify among other things, the local anisotropies that we observe today in galaxies, clusters and superclusters. So, at the early time, it appears appropriate to suppose a geometry that is more general than just the isotropic and homogenous Friedmann–Robertson–Walker geometry. Even though the Universe, on a large scale, appears homogenous and isotropic at the present time, there are no observational data that guarantee this in an epoch prior to the recombination. The anisotropies defined above have many possible sources; they could be associated with cosmological magnetic or electric fields, long-wavelength gravitational waves, Yang–Mills fields [32].

2. Defining $A = \mathrm{e}^{l(t)}$, $B = \mathrm{e}^{m(t)}$ and $C = \mathrm{e}^{n(t)}$, then the line-element describes the well-known Bianchi Type-I metric.

3. Writing $A = t^{p_1}$, $B = t^{p_2}$ and $C = t^{p_3}$ (where p_1, p_2 and p_3 are constants), then we obtain the well-known viscous Kasner-type model.

4. When the functions $A(t)$, $B(t)$ and $C(t)$ are given by the following equations [33]

$$A(t) = \left(\frac{N}{H_0^2}\right)^{1/3} \left(1 + \frac{3H_0 t_0}{2}\right)^{k_1-(1/3)} \left(\sinh\frac{3H_0 t}{2}\right)^{k_1} \times \left(\sinh\frac{3H_0 t}{2} + \frac{3H_0 t_0}{2}\cosh\frac{3H_0 t}{2}\right)^{(2/3)-k_1}, \tag{12}$$

$$B(t) = \left(\frac{N}{H_0^2}\right)^{1/3} \left(1 + \frac{3H_0 t_0}{2}\right)^{k_2-(1/3)} \left(\sinh\frac{3H_0 t}{2}\right)^{k_2} \times \left(\sinh\frac{3H_0 t}{2} + \frac{3H_0 t_0}{2}\cosh\frac{3H_0 t}{2}\right)^{(2/3)-k_2}, \tag{13}$$

$$C(t) = \left(\frac{N}{H_0^2}\right)^{1/3} \left(1 + \frac{3H_0 t_0}{2}\right)^{k_3-(1/3)} \left(\sinh\frac{3H_0 t}{2}\right)^{k_3} \times \left(\sinh\frac{3H_0 t}{2} + \frac{3H_0 t_0}{2}\cosh\frac{3H_0 t}{2}\right)^{(2/3)-k_3}, \tag{14}$$

then the line-element describes the Heckmann–Schucking solution of the Bianchi Type-I space-time. Here $H_0 = \sqrt{\Lambda}$ is nothing but a Hubble parameter for the de Sitter Universe with a cosmological constant Λ, while the constant N characterizes the quantity of the dust in the Universe.

(D) The Kantowski–Sachs model [24]: We can write this space-time by choosing

$$A^2(t,x,y,z) = B^2(t,x,y,z) = C^2(t,x,y,z) = 1, \quad D^2(t,x,y,z) = \sin y. \tag{15}$$

(E) The Bianchi Type-V space-time [24] is defined under the case given below.

$$A^2(t,x,y,z) = C^2(t,x,y,z) = 1, \quad B^2(t,x,y,z) = D^2(t,x,y,z) = \mathrm{e}^{2y}. \tag{16}$$

3. Gravitational energy and momentum

3.1 *In general relativity*

In this section, we give the Einstein, Bergmann–Thomson and Landau–Lifshitz energy–momentum definitions.

Einstein's formulation: The energy and momentum formulation of Einstein [4,5] in general relativity is given by

$$\Theta_\mu^\nu = \frac{1}{16\pi} H_{\mu,\alpha}^{\nu\alpha}, \tag{17}$$

where

$$H_\mu^{\nu\alpha} = \frac{g_{\mu\beta}}{\sqrt{-g}} \left[-g(g^{\nu\beta} g^{\alpha\xi} - g^{\alpha\beta} g^{\nu\xi})\right]_{,\xi}. \tag{18}$$

Θ_0^0 is the energy density, Θ_α^0 are the momentum density components, and Θ_0^α are the components of energy current density. The Einstein energy and momentum densities satisfy the local conservation laws

$$\frac{\partial \Theta_\mu^\nu}{\partial x^\nu} = 0. \tag{19}$$

and energy–momentum components are given by

$$P_\mu = \int\int\int \Theta_\mu^0 \mathrm{d}x\mathrm{d}y\mathrm{d}z. \tag{20}$$

Bergmann–Thomson's formulation: The energy–momentum prescription of Bergmann–Thomson [3] is given by

$$\Xi^{\mu\nu} = \frac{1}{16\pi} \Pi_{,\alpha}^{\mu\nu\alpha}, \tag{21}$$

where

$$\Pi^{\mu\nu\alpha} = g^{\mu\beta} V_\beta^{\nu\alpha} \tag{22}$$

with

$$V_\beta^{\nu\alpha} = -V_\beta^{\alpha\nu} = \frac{g_{\beta\xi}}{\sqrt{-g}} \left[-g \left(g^{\nu\xi} g^{\alpha\rho} - g^{\alpha\xi} g^{\nu\rho}\right)\right]_{,\rho}. \tag{23}$$

The Bergmann–Thomson's energy–momentum prescription satisfies the following local conservation law

$$\frac{\partial \Xi^{\mu\nu}}{\partial x^{\nu}} = 0 \tag{24}$$

in any coordinate system. The energy and momentum components are defined by

$$P^{\mu} = \int\int\int \Xi^{\mu 0} \mathrm{d}x\mathrm{d}y\mathrm{d}z. \tag{25}$$

The energy and momentum (energy current) density components are represented by Ξ^{00} and Ξ^{a0}, respectively.

Landau–Lifshitz's formulation: The energy–momentum prescription of Landau–Lifshitz [8] is

$$\Upsilon^{\mu\nu} = \frac{1}{16\pi} S^{\mu\nu\alpha\beta}_{,\alpha\beta}, \tag{26}$$

where

$$S^{\mu\nu\alpha\beta} = -g(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta}). \tag{27}$$

$\Upsilon^{\mu\nu}$ is symmetric in its indices. Υ^{00} is the energy density and Υ^{a0} are the momentum (energy current) density components. $S^{\mu\nu\alpha\beta}$ has symmetries of Riemann curvature tensor. The energy–momentum of Landau and Lifshitz satisfies the local conservation laws:

$$\frac{\partial \Upsilon^{\mu\nu}}{\partial x^{\nu}} = 0 \tag{28}$$

with

$$\Upsilon^{\mu\nu} = -g(T^{\mu\nu} + t^{\mu\nu}). \tag{29}$$

Here $T^{\mu\nu}$ is the energy–momentum tensor of the matter and all non-gravitational fields, and $t^{\mu\nu}$ is known as Landau–Lifshitz energy–momentum pseudotensor. Thus the locally conserved quantity $\Upsilon^{\mu\nu}$ contains contributions from the matter, non-gravitational fields and gravitational fields. The energy–momentum component is given by

$$P^{\mu} = \int\int\int \Upsilon^{\mu 0} \mathrm{d}x\mathrm{d}y\mathrm{d}z. \tag{30}$$

3.2 *In the teleparallel gravity*

The teleparallel theory of gravity (the tetrad theory of gravitation) is an alternative approach to gravitation and corresponds to a gauge theory for the translation group based on Weitzenböck geometry [34]. In the theory of teleparallel gravity, gravitation is attributed to torsion [35], which plays the role of a force [36], and the curvature tensor vanishes identically. The essential field is acted by a non-trivial tetrad field, which gives rise to the metric as a by-product. The translational gauge potentials appear as the non-trivial item of the tetrad field, and so induce on space-time a teleparallel structure which is directly related to the presence of the gravitational field. The interesting quality of teleparallel theory is that, due to its gauge structure, it can reveal a more appropriate approach for some specific problems. This is the situation, for example, in the energy and momentum problem, which becomes more transparent [10].

Møller modified general relativity by constructing a new field theory in teleparallel space. The aim of this theory was to overcome the problem of the energy–momentum complex that appears in Riemannian Space [37]. The field equations in this new theory were derived from a Lagrangian which is not invariant under local tetrad rotation. Saez [38] generalized Møller theory into a scalar tetrad theory of gravitation. Meyer [39] showed that Møller theory is a special case of Poincaré gauge theory [40,41].

The energy–momentum complexes of Einstein, Bergmann–Thomson and Landau–Lifshitz in the teleparallel gravity [10] are given by the following equations, respectively:

$$hE_\nu^\mu = \frac{1}{4\pi}\partial_\lambda(U_\nu^{\mu\lambda}), \tag{31}$$

$$hB^{\mu\nu} = \frac{1}{4\pi}\partial_\lambda(g^{\mu\beta}U_\beta^{\nu\lambda}), \tag{32}$$

$$hL^{\mu\nu} = \frac{1}{4\pi}\partial_\lambda(hg^{\mu\beta}U_\beta^{\nu\lambda}), \tag{33}$$

where $h = \det(h_\mu^a)$ and $U_\beta^{\nu\lambda}$ is the Freud's super-potential, which is given by

$$U_\beta^{\nu\lambda} = hS_\beta^{\nu\lambda}. \tag{34}$$

Here $S^{\mu\nu\lambda}$ is the tensor where

$$S^{\mu\nu\lambda} = m_1T^{\mu\nu\lambda} + \frac{m_2}{2}(T^{\nu\mu\lambda} - T^{\lambda\mu\nu}) + \frac{m_3}{2}(g^{\mu\lambda}T_\beta^{\beta\nu} - g^{\nu\mu}T_\beta^{\beta\lambda}) \tag{35}$$

with m_1, m_2 and m_3 the three dimensionless coupling constants of teleparallel gravity [40]. For the teleparallel equivalent of general relativity the specific choice of these three constants are

$$m_1 = \frac{1}{4}, \quad m_2 = \frac{1}{2}, \quad m_3 = -1. \tag{36}$$

To calculate this tensor firstly we must calculate Weitzenböck connection:

$$\Gamma_{\mu\nu}^\alpha = h_a^\alpha\partial_\nu h_\mu^a \tag{37}$$

and after this calculation we get torsion of the Weitzenböck connection:

$$T^{\mu}_{\nu\lambda} = \Gamma^{\mu}_{\lambda\nu} - \Gamma^{\mu}_{\nu\lambda}. \tag{38}$$

For the Einstein, Bergmann–Thomson and Landau–Lifshitz complexes, we have the relations

$$P^{E}_{\mu} = \int_{\Sigma} hE^{0}_{\mu}\mathrm{d}x\mathrm{d}y\mathrm{d}z, \tag{39}$$

$$P^{B}_{\mu} = \int_{\Sigma} hB^{0}_{\mu}\mathrm{d}x\mathrm{d}y\mathrm{d}z, \tag{40}$$

$$P^{L}_{\mu} = \int_{\Sigma} hL^{0}_{\mu}\mathrm{d}x\mathrm{d}y\mathrm{d}z, \tag{41}$$

where P_i give momentum components P_1, P_2, P_3 while P_0 gives the energy and the integration hyper-surface Σ is described by $x^0 = t =$constant.

4. Calculations

4.1 *In general relativity*

The matrix form of the metric tensor $g_{\mu\nu}$ for the line-element (1) is defined by

$$\begin{pmatrix} -A^2(t,x,y,z) & 0 & 0 & 0 \\ 0 & B^2(t,x,y,z) & 0 & 0 \\ 0 & 0 & C^2(t,x,y,z) & 0 \\ 0 & 0 & 0 & D^2(t,x,y,z) \end{pmatrix}, \tag{42}$$

and its inverse matrix $g^{\mu\nu}$ is

$$\begin{pmatrix} -A^{-2}(t,x,y,z) & 0 & 0 & 0 \\ 0 & B^{-2}(t,x,y,z) & 0 & 0 \\ 0 & 0 & C^{-2}(t,x,y,z) & 0 \\ 0 & 0 & 0 & D^{-2}(t,x,y,z) \end{pmatrix}, \tag{43}$$

and the required components of $H^{\nu\alpha}_{\mu}$, $\Pi^{\mu\nu\alpha}$ and $S^{\mu\nu\alpha\beta}$ are

$$H^{01}_{0} = -\Pi^{001} = \frac{A}{B}(CD)_x, \tag{44}$$

$$H^{02}_{0} = -\Pi^{002} = \frac{A}{C}(BD)_y, \tag{45}$$

$$H^{02}_{0} = -\Pi^{003} = \frac{A}{D}(BC)_z, \tag{46}$$

$$S^{0011} = C^2 D^2, \tag{47}$$

$$S^{0022} = B^2 D^2, \tag{48}$$

$$S^{0033} = B^2 C^2. \tag{49}$$

Here the sub-index defines derivative with respect to that coordinate. Next, using these results we obtain the following energy distributions in the general relativity versions of Einstein, Bergmann–Thomson and Landau–Lifshitz complexes, respectively.

$$8\pi\Theta_0^0 = \left[\left(\frac{A}{B}\right)_x (CD)_x + \frac{A}{B}(CD)_{xx} + \left(\frac{A}{C}\right)_y (BD)_y + \frac{A}{C}(BD)_{yy} + \left(\frac{A}{D}\right)_z (BC)_z + \frac{A}{D}(BC)_{zz}\right], \tag{50}$$

$$-8\pi\Xi^{00} = \left[\frac{1}{AB}(CD)_x\right]_x + \left[\frac{1}{AC}(BD)_y\right]_y + \left[\frac{1}{AD}(BC)_z\right]_z, \tag{51}$$

$$8\pi\Upsilon^{00} = \left[\left(C^2D^2\right)_{xx} + \left(B^2D^2\right)_{yy} + \left(B^2C^2\right)_{zz}\right]. \tag{52}$$

4.2 *In the teleparallel gravity*

In this section, using the teleparallel gravity analogs of the Einstein, Bergmann–Thomson and Landau–Lifshitz energy and/or momentum formulations, we evaluate the energy density.

The tetrad components of h^a_μ for the line-element (1) are given by

$$h^a_\mu = A\delta^a_0\delta^0_\mu + B\delta^a_1\delta^1_\mu + C\delta^a_2\delta^2_\mu + D\delta^a_3\delta^3_\mu \tag{53}$$

and for h^μ_a are

$$h^\mu_a = A^{-1}\delta^0_a\delta^\mu_0 + B^{-1}\delta^1_a\delta^\mu_1 + C^{-1}\delta^2_a\delta^\mu_2 + D^{-1}\delta^3_a\delta^\mu_3. \tag{54}$$

Using eqs (37), (53) and (54), we obtain the following non-vanishing components of the Weitzenböck connection

$$\Gamma^0_{00} = \frac{A_t}{A}, \quad \Gamma^0_{01} = \frac{A_x}{A}, \quad \Gamma^0_{02} = \frac{A_y}{A}, \quad \Gamma^0_{03} = \frac{A_z}{A} \tag{55}$$

$$\Gamma^1_{10} = \frac{B_t}{B}, \quad \Gamma^1_{11} = \frac{B_x}{B}, \quad \Gamma^1_{12} = \frac{B_y}{B}, \quad \Gamma^1_{13} = \frac{B_z}{B} \tag{56}$$

$$\Gamma^2_{20} = \frac{C_t}{C}, \quad \Gamma^2_{21} = \frac{C_x}{C}, \quad \Gamma^2_{22} = \frac{C_y}{C}, \quad \Gamma^2_{23} = \frac{C_z}{C} \tag{57}$$

$$\Gamma^3_{30} = \frac{D_t}{D}, \quad \Gamma^3_{31} = \frac{D_x}{D}, \quad \Gamma^3_{32} = \frac{D_y}{D}, \quad \Gamma^3_{33} = \frac{D_z}{D}. \tag{58}$$

The corresponding non-vanishing torsion components are found to be

$$T^0_{01} = -\frac{A_x}{A}, \quad T^0_{02} = -\frac{A_y}{A}, \quad T^0_{03} = -\frac{A_z}{A} \tag{59}$$

$$T^1_{10} = -\frac{B_t}{B}, \quad T^1_{12} = -\frac{B_y}{B}, \quad T^1_{13} = -\frac{B_z}{B} \tag{60}$$

$$T^2_{20} = -\frac{C_t}{C}, \quad T^2_{21} = -\frac{C_x}{C}, \quad T^2_{23} = -\frac{C_z}{C} \tag{61}$$

$$T^3_{30} = -\frac{D_t}{D}, \quad T^3_{31} = -\frac{D_x}{D}, \quad T^3_{32} = -\frac{D_y}{D}. \tag{62}$$

Taking these results into eq. (35), the non-zero energy components of the tensor $S_\mu^{\nu\lambda}$ are found to be

$$S^{001} = -\frac{1}{2\,(AB)^2}\frac{(CD)_x}{(CD)}, \tag{63}$$

$$S^{002} = -\frac{1}{2\,(AC)^2}\frac{(BD)_y}{(BD)}, \tag{64}$$

$$S^{003} = -\frac{1}{2\,(AD)^2}\frac{(BC)_z}{(BC)}. \tag{65}$$

Next, the non-vanishing components of Freud's super-potential are

$$U_0^{01} = \frac{1}{2}\left(\frac{A}{B}\right)(CD)_x, \tag{66}$$

$$U_0^{02} = \frac{1}{2}\left(\frac{A}{C}\right)(BD)_y, \tag{67}$$

$$U_0^{03} = \frac{1}{2}\left(\frac{A}{D}\right)(BC)_z. \tag{68}$$

Using eqs (31)–(33), the Einstein, Bergmann–Thomson and Landau–Lifshitz's energy densities are found:

$$hE_0^0 = \frac{1}{8\pi}\left[\left(\frac{A}{B}\right)_x(CD)_x + \frac{A}{B}(CD)_{xx} + \left(\frac{A}{C}\right)_y(BD)_y + \frac{A}{C}(BD)_{yy} + \left(\frac{A}{D}\right)_z(BC)_z + \frac{A}{D}(BC)_{zz}\right], \tag{69}$$

$$hB^{00} = -\frac{1}{8\pi}\left\{\left[\frac{1}{AB}(CD)_x\right]_x + \left[\frac{1}{AC}(BD)_y\right]_y + \left[\frac{1}{AD}(BC)_z\right]_z\right\}, \tag{70}$$

$$hL^{00} = \frac{1}{8\pi}\left[\left(C^2D^2\right)_{xx} + \left(B^2D^2\right)_{yy} + \left(B^2C^2\right)_{zz}\right]. \tag{71}$$

We obtain that for the Einstein, Bergmann–Thomson and Landau–Lifshitz energy complexes, each complex's value is the same evaluated either in general relativity or in teleparallel gravity.

5. Energy distributions for the special cases of the general metric

1. The de Sitter C-space-time solutions: One can easily find the following result for the given metric,

$$\begin{aligned}\Theta^0_0 &= hE^0_0 = \Xi^0_0 = hB^0_0\\ &= \frac{(x+y)^{-2}}{16\pi M^2}\left[\frac{W_{,xx}}{2} + \frac{(x+y-2)K_{,y} - 3W}{x+y}\right.\\ &\qquad\left. + \frac{6W - 2K(x+y-3)}{(x+y)^2}\right]\\ &= \frac{(x+y)^{-4}}{48\pi M^4}[3y^2M^2 - 6\Lambda^2 - 6q^2M^4y^4 - 45mM^3x^3\\ &\quad -36q^2M^4x^4 - 6mM^3y^4 + 6q^2M^4y^5 + 6xy^2\\ &\quad -18M^2xy + 12M^3mx^4 + 6M^4q^2x^5 - 21x^2M^2\\ &\quad +2(x+y)\Lambda^2 + 6M^2x^3 - 24mxM^3y^3 + 18xq^2M^4y^4\\ &\quad -18myx^2M^3 + 27mxy^2M^3 - 36q^2M^4x^3y - 18q^2M^4x^2y^2\\ &\quad -24M^4xy^3q^2 + 12M^2yx^2 - 18M^3x^2my^2 + 12M^4x^2q^2y^3\\ &\quad +12M^3mx^3y + 6M^4q^2x^4y]\end{aligned} \tag{72}$$

and

$$\begin{aligned}\Upsilon^{00} &= hL^{00}\\ &= \frac{\partial^2}{\partial x^2}\left(\frac{1}{M^2(x+y)^2}\sqrt{\frac{1-x^2-2mMx^3-q^2M^2x^4}{-\frac{\Lambda+3M^2}{3M^2}+y^2-2mMy^3+q^2M^2y^4}}\right)\\ &\quad + \frac{\partial^2}{\partial y^2}\left(\frac{1}{M^3(x+y)^2}\right).\end{aligned} \tag{73}$$

Here, for the energy in Landau–Lifshitz complex, since the intermediary mathematical exposition is lengthy, we give the energy in the form as given above.

2. The solutions for spatially self-similar locally rotationally symmetric space-times:

$$\begin{aligned}\Theta^0_0 &= hE^0_0 = \Xi^0_0 = hB^0_0\\ &= \frac{k^2D_1^2(t)[\cos(2ky)-3] + 32a^2\mathrm{e}^{-2ax}D_2^2(t)\sin^2(ky)}{64\pi k^{1/2}D_1(t)\sin^{3/2}(ky)}\end{aligned} \tag{74}$$

and

$$\Upsilon^{00} = hL^{00} = \frac{1}{8\pi k}\mathrm{e}^{-2ax}\sin(ky)[16a^2\mathrm{e}^{-2ax}D_2^2(t) - k^2D_1^2(t)]. \tag{75}$$

3. The general Bianchi Type-I solutions:

$$\Theta_0^0 = hE_0^0 = \Xi_0^0 = hB_0^0 = \Upsilon^{00} = hL^{00} = 0. \tag{76}$$

Albrow [42] and Tryon [43] assumed that the net energy of the Universe may be equal to zero. The subject of the energy–momentum distributions of closed and open Universes was initiated by an interesting work of Cooperstock and Israelit [44]. They found zero value of energy for any homogenous isotropic Universe described by a Friedmann–Robertson–Walker metric in the context of general relativity. This interesting result influenced some general relativists [45].

4. The Kantowski–Sachs solutions:

$$\Theta_0^0 = hE_0^0 = \Xi_0^0 = hB_0^0 = \frac{\sin(y)}{8\pi}, \quad \Upsilon^{00} = hL^{00} = \frac{\cos(2y)}{4\pi}. \tag{77}$$

5. The Bianchi Type-V solutions: We obtain the energy distributions as

$$\Theta_0^0 = hE_0^0 = \Xi_0^0 = hB_0^0 = \frac{2}{\pi}\mathrm{e}^{4y}, \quad \Upsilon^{00} = hL^{00} = \frac{8}{\pi}\mathrm{e}^{8y}. \tag{78}$$

6. Summary and final remarks

The main goal of the present paper is to show that it is possible to calculate the energy distribution by using the energy–momentum formulations in not only general relativity but also the teleparallel gravity. To compute the energy density (due to matter and fields including gravitation), we considered two different approaches of the Einstein, Bergmann–Thomson and Landau–Lifshitz energy–momentum definitions such as the general relativity and the teleparallel gravity analogs of them.

We found that the teleparallel gravity analog of the formulations considered give the same result as its general relativity versions. Using these results, we also calculated the energy distributions for some special space-times and showed that the Einstein and Bergmann–Thomson formulations give the same results in these space-times, but the Landau–Lifshitz formulation does not.

In Bianchi Type-I models all the formulations give the same zero value energy. This result is the same as obtained by Cooperstock–Israelit, Rosen, Johri *et al.*, Banerjee–Sen, Xulu, Vargas and Saltı *et al* and supports the viewpoints of Albrow and Tryon. Furthermore, if the calculations are performed for the Bianchi Type-I model again, one can easily show that the energy is also independent of the teleparallel dimensionless coupling constants, which means that it is valid not only in the teleparallel equivalent of general relativity, but also in any teleparallel model.

Acknowledgement

We would like to thank Dr Elias C Vagenas for his help.

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