PRAMANA — journal of physics | © Indian Academy of Sciences | Vol. 68, No. 5 May 2007 pp. 721–734

Bianchi Type-V universe with a viscous fluid and Λ-term

T SINGH and R CHAUBEY
Department of Applied Mathematics, Institute of Technology, Banaras Hindu University, Varanasi 221 005, India
E-mail: drtrilokisingh@yahoo.co.in

MS received 10 August 2006; revised 11 January 2007; accepted 28 February 2007

Abstract. We have studied the evolution of a homogeneous, anisotropic universe given by a Bianchi Type-V cosmological model filled with viscous fluid, in the presence of cosmological constant Λ. The role of viscous fluid and Λ-term in the Bianchi Type-V universe has been studied.

Keywords. Bianchi type universe; viscous fluid; cosmological term; bulk and shear viscocity.

PACS Nos 98.80.Jk; 04.20.Jb

1. Introduction

The investigation of relativistic cosmological models usually has the energy–momentum tensor of matter generated by a perfect fluid. To consider more realistic models the viscosity has attracted the attention of many workers. Misner [1,2] suggested that the dissipation due to neutrino viscosity may considerably reduce the anisotropy of the black body radiation. The viscosity mechanism in cosmology can explain the anomalously high entropy per baryon in the present universe [3,4]. Bulk viscosity associated with the grand unified theory phase transition [5] may lead to an inflationary scenario [6–8].

A homogeneous and isotropic cosmological model filled with a fluid with pressure and bulk viscosity was investigated by Murphy [9]. His solution leads to the conclusion that the Big Bang singularity occurs in the infinite past. The solutions for open, closed and flat universes have been found by Santos *et al* [10] under the assumption that the bulk viscosity (ξ) is a power function of energy density (ρ).

The nature of cosmological solutions for homogeneous Bianchi Type-I model was investigated by Belinski and Khalatnikov [11] by taking into account dissipative process due to viscosity. They showed that viscosity cannot remove cosmological singularity but can cause a qualitatively new behavior of the solutions near singularity. Bianchi Type-I solutions in the case of stiff matter with shear viscosity being the power function of energy density were obtained by Banerjee *et al* [12], whereas models with bulk viscosity as a power function of energy density and stiff matter were studied by Huang [13]. The effect of bulk viscosity, with a time varying bulk viscous coefficient, on the evolution of isotropic FRW models was investigated in the context of open thermodynamic system by Desikan [14]. This study was further extended by Krori and Mukherjee [15] for anisotropic Bianchi models. Cosmological solutions with nonlinear bulk viscosity were obtained by Chimento *et al* [16]. Models with both shear and bulk viscosities have been studied by Gavrilov *et al* [17]. Grøn [18] has reviewed the research on viscous cosmological models. Inflationary cosmological models of Bianchi Type-I with shear and nonlinear bulk viscosities have been studied.

Although Murphy [9] claimed that the introduction of bulk viscosity can avoid the initial singularity at finite past, the results obtained by Barrow [19] showed that it is, in general, not valid, since for some cases Big Bang singularity occurs in infinite past. Recently, Saha [20] has considered a nonlinear spinor field in a Bianchi Type-I universe with viscous fluid. Saha [21] has studied the influence of viscous fluid and Λ-term in the evolution of the Bianchi Type-I universe. In the present work we have studied the influence of viscous fluid and Λ-term in the evolution of the Bianchi Type-V universe.

2. Basic equations

In this section, using the variational principle, one can derive the fundamental equations for the gravitational field from the action

$$S(g,\rho) = \int L\sqrt{-g}\ \mathrm{d}\Omega \tag{2.1}$$

with

$$L = L_{\text{grav.}} + L_{\text{vf}}. \tag{2.2}$$

The gravitational part of the Lagrangian $L_{\text{grav.}}$ is given by a Bianchi Type-V metric whereas the term L_{vf} describes a viscous fluid.

The gravitational part of the Lagrangian (2.2) is of the form

$$L_{\text{g}} = \frac{R}{2\kappa}, \tag{2.3}$$

where R is the scalar curvature and $\kappa = 8\pi G$ is the Einstein's gravitational constant.

The influence of the viscous fluid in the evolution of the Universe is introduced by means of its energy–momentum tensor, which acts as the source of the corresponding gravitational field. The reason for writing L_{vf} in (2.2) is to underline the fact that we are dealing with a self-consistent system. The energy–momentum tensor of a viscous field has the form

$$T_{i(m)}^{j} = (\rho + p')u_i u^j - p'\delta_i^j + \eta g^{j\beta}[u_{i;\beta} + u_{\beta;i} - u_i u^\alpha u_{\beta;\alpha} - u_\beta u^\alpha u_{i;\alpha}], \tag{2.4}$$

where

$$p' = p - \left(\xi - \frac{2}{3}\eta\right) u^i_{;i}. \tag{2.5}$$

Here ρ is the energy density, p is the pressure, η and ξ are the coefficients of shear and bulk viscosity, respectively. Note that the bulk and shear viscosities, η and ξ, are both positive, i.e.,

$$\eta > 0, \quad \xi > 0. \tag{2.6}$$

We have taken them as either constant or function of time or energy, such as

$$\eta = |A|\rho^{\alpha}, \quad \xi = |B|\rho^{\beta}, \tag{2.7}$$

where A and B are constants.

The pressure p is connected to the energy density by means of an equation of state. In this paper we consider the one describing a perfect fluid

$$p = \gamma\rho, \quad \gamma \in (0, 1]. \tag{2.8}$$

We use comoving system of reference, then $u^i = (1, 0, 0, 0)$. We introduce the dynamical scalars such as the expansion and the shear scalar as usual.

$$\theta = u^i_{;i}, \quad \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}, \tag{2.9}$$

where

$$\sigma_{ij} = \frac{1}{2}(u_{i;\alpha}P^{\alpha}_j + u_{j;\alpha}P^{\alpha}_i) - \frac{1}{3}\theta P_{ij}. \tag{2.10}$$

Here P_{ij} is the projection tensor. For the space-time with signature $(+, -, -, -)$ it has the form

$$P_{ij} = g_{ij} - u_i u_j, \quad P^i_j = \delta^i_j - u^i u_j. \tag{2.11}$$

For the Bianchi Type-V metric the dynamical scalar has the form

$$\theta = \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} = \frac{\dot{V}}{V} \tag{2.12}$$

and

$$2\sigma^2 = \frac{\dot{a}_1^2}{a_1^2} + \frac{\dot{a}_2^2}{a_2^2} + \frac{\dot{a}_3^2}{a_3^2} - \frac{1}{3}\theta^2. \tag{2.13}$$

Variation of (2.1) with respect to metric tensor g_{ij} gives the Einstein's field equation. On account of the Λ-term we have

$$G^j_i = R^j_i - \frac{1}{2}\delta^j_i R = \kappa T^j_i - \delta^j_i \Lambda, \tag{2.14}$$

where T^j_i is the energy–momentum tensor of a viscous fluid.

3. Bianchi Type-V universe

We take Bianchi Type-V metric in the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a_1^2\,\mathrm{d}x^2 - a_2^2 \mathrm{e}^{-2mx}\,\mathrm{d}y^2 - a_3^2 \mathrm{e}^{-2mx}\,\mathrm{d}z^2. \tag{3.1}$$

where the metric functions a_1, a_2, a_3 are the functions of t only and m is a constant.

In view of eqs (2.4) and (2.5) for the Bianchi Type-V space-time (3.1) the Einstein's field equations lead to

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} - \frac{m^2}{a_1^2} = \kappa\left(-p' + 2\eta\frac{\dot{a}_1}{a_1}\right) - \Lambda. \tag{3.2a}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1 a_3} - \frac{m^2}{a_1^2} = \kappa\left(-p' + 2\eta\frac{\dot{a}_2}{a_2}\right) - \Lambda. \tag{3.2b}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1 a_2} - \frac{m^2}{a_1^2} = \kappa\left(-p' + 2\eta\frac{\dot{a}_3}{a_3}\right) - \Lambda. \tag{3.2c}$$

$$\frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3 a_1} - \frac{3m^2}{a_1^2} = \kappa\rho - \Lambda. \tag{3.2d}$$

$$\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} = \frac{2\dot{a}_1}{a_1}. \tag{3.2e}$$

From eq. (3.2e) we have

$$a_2 a_3 = a_1^2. \tag{3.3}$$

To write the metric functions explicitly, we define a new dependent function $V(t)$

$$V = a_1 a_2 a_3. \tag{3.4}$$

Let us now solve the Einstein's equations (3.2a)–(3.2e). Subtracting eq. (3.2a) from (3.2b), one finds the following relation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right) + \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) = -2\kappa\eta\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right). \tag{3.5}$$

Then from eqs (3.4) and (3.5) we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right) + \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)\frac{\dot{V}}{V} = -2\kappa\eta\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right). \tag{3.6}$$

Integrating the above equation, we get

$$\frac{a_1}{a_2} = d_1 \exp\left(x_1 \int \frac{\mathrm{e}^{-2\kappa\int\eta\mathrm{d}t}}{V}\mathrm{d}t\right), \quad d_1 = \text{constant}, x_1 = \text{constant}. \tag{3.7}$$

By subtracting eq. (3.2c) from (3.2a) and eq. (3.2a) from (3.2b), we obtain

$$\frac{a_1}{a_3} = d_2 \exp\left(x_2 \int \frac{\mathrm{e}^{-2\kappa \int \eta \mathrm{d}t}}{V} \mathrm{d}t\right),$$
$$d_2 = \text{constant}, x_2 = \text{constant}, \tag{3.8a}$$
$$\frac{a_2}{a_3} = d_3 \exp\left(x_3 \int \frac{\mathrm{e}^{-2\kappa \int \eta \mathrm{d}t}}{V} \mathrm{d}t\right),$$
$$d_3 = \text{constant}, x_3 = \text{constant}, \tag{3.8b}$$

where d_2, d_3, x_2, x_3 are integration constants.

In view of the relations $V = a_1 a_2 a_3$ we find the following relation between the constants $d_1, d_2, d_3, x_1, x_2, x_3$:

$$d_2 = d_1 d_3, \quad x_2 = x_1 + x_3.$$

Finally from eqs (3.7) and (3.8), we write $a_1(t), a_2(t)$, and $a_3(t)$ in the explicit form.

$$a_1(t) = D_1 V^{1/3} \exp\left(X_1 \int \frac{\mathrm{e}^{-2\kappa \int \eta \mathrm{d}t}}{V(t)} \mathrm{d}t\right), \tag{3.9a}$$

$$a_2(t) = D_2 V^{1/3} \exp\left(X_2 \int \frac{\mathrm{e}^{-2\kappa \int \eta \mathrm{d}t}}{V(t)} \mathrm{d}t\right), \tag{3.9b}$$

$$a_3(t) = D_3 V^{1/3} \exp\left(X_3 \int \frac{\mathrm{e}^{-2\kappa \int \eta \mathrm{d}t}}{V(t)} \mathrm{d}t\right), \tag{3.9c}$$

where D_i $(i = 1,2,3)$ and X_i $(i = 1,2,3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

From eqs (3.3) and (3.9) we obtain $X_1 = 0$, $X_2 = X_3 = -X$, $D_1 = 1$, $D_2 = D_3^{-1} = D$, where X and D are constants. Then eq. (3.9) can be written as

$$a_1(t) = V^{1/3}, \tag{3.10a}$$
$$a_2(t) = D V^{1/3} \exp\left(X \int \frac{\mathrm{e}^{-2\kappa \int \eta \mathrm{d}t}}{V(t)} \mathrm{d}t\right), \tag{3.10b}$$
$$a_3(t) = D^{-1} V^{1/3} \exp\left(-X \int \frac{\mathrm{e}^{-2\kappa \int \eta \mathrm{d}t}}{V(t)} \mathrm{d}t\right), \tag{3.10c}$$

where X and D are constants.

Thus, the metric functions are found explicitly in terms of V and viscosity. By adding eqs (3.2a), (3.2b), (3.2c) and three times eq. (3.2d), we get

$$\left(\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_3}{a_3}\right)+2\left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\dot{a}_3\dot{a}_1}{a_3a_1}\right)-\frac{6m^2}{a_1^2}$$
$$=\frac{3\kappa}{2}(\rho-p')+2\kappa\eta\left(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)-3\Lambda. \tag{3.11}$$

From eqs (3.2e), (3.4), (2.5) and eq. (3.11) we obtain

$$\ddot{V}-\frac{3}{2}\kappa\xi\dot{V}=\frac{3\kappa}{2}(\rho-p)V-3\Lambda V+6m^2V^{1/3}. \tag{3.12}$$

The energy–momentum to be conserved, i.e.,

$$T^{\nu}_{\mu};_{\nu}=T^{\nu}_{\mu},_{\nu}+\Gamma^{\nu}_{\rho\nu}T^{\rho}_{\mu}-\Gamma^{\rho}_{\mu\nu}T^{\nu}_{\rho}=0 \tag{3.13}$$

which gives

$$\frac{1}{V}\left[\frac{\mathrm{d}}{\mathrm{d}t}(VT^0_0)\right]-\frac{\dot{a}_1}{a_1}T^1_1-\frac{\dot{a}_2}{a_2}T^2_2-\frac{\dot{a}_3}{a_3}T^3_3=0. \tag{3.14}$$

After a little manipulation from eq. (3.14) we obtain

$$\dot{\rho}+\frac{\dot{V}}{V}\omega-\left(\xi+\frac{4}{3}\eta\right)\frac{\dot{V}^2}{V^2}+4\eta\left(\frac{3m^2}{V^{2/3}}+\kappa\rho-\Lambda\right)=0, \tag{3.15}$$

where

$$\omega=\rho+p \tag{3.16}$$

is the thermal function.

Let us introduce a generalised Hubble constant H:

$$\frac{\dot{V}}{V}=\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}=3H. \tag{3.17}$$

Then eqs (3.12) and (3.15) can be rewritten as

$$\dot{H}=\frac{\kappa}{2}(3\xi H-\omega)-(3H^2-\kappa\rho+\Lambda)+\frac{2m^2}{V^{2/3}}, \tag{3.18a}$$

$$\dot{\rho}=3H(3\xi H-\omega)+4\eta\left(3H^2-\kappa\rho+\Lambda-\frac{3m^2}{V^{2/3}}\right). \tag{3.18b}$$

From eq. (3.10), we can also rewrite shear (2.14) as

$$\sigma^2=\frac{X^2}{3V^2}\mathrm{e}^{-4\kappa\int\eta\mathrm{d}t} \tag{3.19}$$

From eq. (3.2d) we obtain

$$\frac{1}{3}\theta^2 - \sigma^2 = \kappa\rho - \Lambda + \frac{3m^2}{V^{2/3}}. \tag{3.20}$$

Equations (3.18) now can be written in terms of θ and σ as follows:

$$\dot{\theta} = \frac{3\kappa}{2}(\xi\theta - \omega) - 3\sigma^2 - \frac{3m^2}{V^{2/3}}, \tag{3.21a}$$

$$\dot{\rho} = \theta(\xi\theta - \omega) + 4\eta\sigma^2 \tag{3.21b}$$

The presence of Λ reduces the value of $\dot{H}$ but it increases the value of $\dot{\rho}$ in eqs (3.18a) and (3.18b).

4. Some special solution

We solve the system of equations for V, H and ρ. It is convenient to rewrite eqs (3.17) and (3.18) as a single system of equations.

$$\dot{V} = 3HV \tag{4.1a}$$

$$\dot{H} = \frac{\kappa}{2}(3\xi H - \omega) - \left(3H^2 - \kappa\rho + \Lambda - \frac{2m^2}{V^{2/3}}\right) \tag{4.1b}$$

$$\dot{\rho} = 3H(3\xi H - \omega) + 4\eta\left(3H^2 - \kappa\rho + \Lambda - \frac{3m^2}{V^{2/3}}\right). \tag{4.1c}$$

On account of eqs (3.16), (2.7) and (2.8), eq. (4.1) can now be rewritten as

$$\dot{V} = 3HV \tag{4.2a}$$

$$\dot{H} = \frac{\kappa}{2}(3B\rho^\beta H - (1+\gamma)\rho) - \left(3H^2 - \kappa\rho + \Lambda - \frac{2m^2}{V^{2/3}}\right) \tag{4.2b}$$

$$\dot{\rho} = 3H(3B\rho^\beta H - (1+\gamma)\rho) + 4A\rho^\alpha\left(3H^2 - \kappa\rho + \Lambda - \frac{3m^2}{V^{2/3}}\right). \tag{4.2c}$$

Now, we consider the system of eqs (4.1a)–(4.1c) for some special choices of the viscosity parameters.

Case I. Case with bulk viscosity

Let us first consider the case when the fluid possesses the bulk viscosity only. The corresponding system of equations can then be obtained by putting $\eta = 0$ in eqs (4.1a)–(4.1c) or $A = 0$ in eqs (4.1a)–(4.1c). In this case eqs (4.1a) and (4.1b) remain unaltered, while (4.1c) takes the form

$$\dot{\rho} = 3H(3\xi H - \omega) \tag{4.3}$$

From eqs (4.3) and (4.1b), we get

$$V^2\left(\kappa\rho - 3H^2 - \Lambda + \frac{3H^2}{V^{2/3}}\right) = C_1, \quad C_1 = \text{Constant}. \tag{4.4}$$

The relation (4.4) can be interpreted as follows. At the initial stage of evolution the volume scale V tends to zero and the energy density ρ tends to infinity.

Let us now consider the case when the bulk viscosity is inversely proportional to expansion, i.e.

$$\xi\theta = C_2, \quad C_2 = \text{Constant}. \tag{4.5}$$

This means that with the expansion of the universe the effect of bulk viscosity diminishes and as expansion becomes very large the effect of bulk viscosity becomes negligible.

Now from eqs (2.8), (2.13) and (4.4), eq. (4.3) can be written as

$$\frac{\dot{\rho}}{C_2 - (1+\gamma)\rho} = \frac{\dot{V}}{V}. \tag{4.6}$$

From eq. (4.5), we get

$$\rho = \frac{1}{1+\gamma}[C_2 + C_3 V^{-(1+\gamma)}] \tag{4.7}$$

with C_3 being some arbitrary constant. Further, inserting ρ from (4.7) into (3.12), one finds the expansion for V explicitly.

Taking into account the equation of state (2.8) in view of (4.5) and (4.7), eq. (3.12) gives

$$\ddot{V} = \left[\frac{3\kappa C_2}{1+\gamma} - 3\Lambda\right] V + \frac{3\kappa}{2}\left(\frac{1-\gamma}{1+\gamma}\right) C_3 V^{-\gamma} + 6m^2 V^{1/3}. \tag{4.8}$$

Equation (4.8) gives the following solution in quadrature form:

$$\int \frac{\mathrm{d}V}{\sqrt{C_2^2 + C_0^0 V^2 + C_1^1 V^{1-\gamma} + 9m^2 V^{4/3}}} = t + t_0, \tag{4.9}$$

where C_2^2 and t_0 are some constants. Further, we set $t_0 = 0$. Here, $C_0^0 = [3\kappa C_2/(1+\gamma)] - 3\Lambda$ and $C_1^1 = [3\kappa C_3/(1+\gamma)]$. As one can see, C_0^0 is negative for

$$\Lambda > \frac{\kappa C_2}{1+\gamma}. \tag{4.10}$$

It means that for a positive Λ obeying eq. (4.10) (we assume that the constant C_2 is a positive quantity) V should be found from above as well. Let us now rewrite eq. (4.9) in the form

$$\dot{V} = \sqrt{2[\zeta - \upsilon(V)]}, \tag{4.11}$$

where $\zeta = C_2^2/2$ can be viewed as energy and $\upsilon(V) = -\frac{1}{2}(C_0^0 V^2 + C_1^1 V^{1-\gamma} + 9m^2 V^{4/3})$ is the potential.

When $\gamma = 1$, eq. (4.9) becomes

$$\int \frac{\mathrm{d}V}{\sqrt{C_3^3 + C_0^0 V^2 + 9m^2 V^{4/3}}} = t, \tag{4.12}$$

where $C_3^3 = C_2^2 + C_1^1$.

Case II. Case with shear and bulk viscosities

Let us now cosider the general case with the shear viscosity η being proportional to the expansion, i.e.

$$\eta \propto \theta = 3H. \tag{4.13}$$

This means that shear viscosity is small in the beginning of the model and with the expansion of the universe, the effect of shear viscosity increases.

We will consider the case when

$$\eta = -\frac{3H}{2\kappa}. \tag{4.14}$$

In this case from (4.1b) and (4.1c) we obtain

$$3H^2 = \kappa\rho + \frac{3m^2}{V^{2/3}} + C_4, \quad C_4 = \text{Constant}. \tag{4.15}$$

From eq. (4.15) it follows that at the initial stage of expansion, when ρ is large, the Hubble constant is also large and with the expansion of the universe H decreases as does ρ. Inserting the relation (4.15) into eq. (4.1b) we obtain

$$\dot{H} = -\frac{3}{2}(1+\gamma)H^2 + \frac{3\kappa\xi}{2}H + \frac{1}{2}C_4(\gamma-1) - \Lambda + \left(\frac{1+3\gamma}{2}\right)\frac{m^2}{V^{2/3}}. \tag{4.16}$$

5. A particular case

Case I. $\gamma = 1/3$ (Radiation)

For $C_2^2 = 0$, eq. (4.9) reduces to

$$\int \frac{\mathrm{d}V}{\sqrt{C_0^0 V^2 + C_1^1 V^{2/3} + 9m^2 V^{4/3}}} = t, \tag{5.1}$$

where

$$C_0^0 = \frac{9\kappa C_2}{4} - 3\Lambda \quad \text{and} \quad C_1^1 = \frac{9\kappa C_3}{4}$$

when $\Lambda < 3\kappa C_2/4$, eq. (5.1) gives

$$V = \left[\left(\frac{\sqrt{3\kappa C_3(3\kappa C_2 - 4\Lambda) - 36m^4}}{3\kappa C_2 - 4\Lambda}\right) \times \sinh\left(\sqrt{\frac{3\kappa C_2 - 4\Lambda}{3}}t\right) - \frac{6m^2}{3\kappa C_2 - 4\Lambda}\right]^{3/2}. \tag{5.2}$$

For small t (i.e. near $t = 0$), we have

$$\sinh\left(\sqrt{\frac{3\kappa C_2 - 4\Lambda}{3}}t\right) \approx \sqrt{\frac{3\kappa C_2 - 4\Lambda}{3}}t. \tag{5.3}$$

Then from eqs (5.2) and (5.3), we get

$$V = \left[\sqrt{\frac{\kappa C_3(3\kappa C_2 - 4\Lambda) - 12m^4}{3\kappa C_2 - 4\Lambda}}t - \frac{6m^2}{3\kappa C_2 - 4\Lambda}\right]^{3/2}. \tag{5.4}$$

At $t = 0$, V becomes imaginary. For reality of the model, t must satisfy $t > \frac{6m^2}{\sqrt{(3\kappa C_2 - 4\Lambda)[\kappa C_3(3\kappa C_2 - 4\Lambda) - 12m^4]}}$. From eqs (5.4) and (3.10), we get

$$a_1(t) = (C_4 t - C_5)^{1/2}, \tag{5.5a}$$

$$a_2(t) = D(C_4 t - C_5)^{1/2} \exp\left[-\frac{2X}{C_4\sqrt{C_4 t - C_5}}\right], \tag{5.5b}$$

$$a_3(t) = D^{-1}(C_4 t - C_5)^{1/2} \exp\left[\frac{2X}{C_4\sqrt{C_4 t - C_5}}\right], \tag{5.5c}$$

where

$$C_4 = \sqrt{\frac{\kappa C_3(3\kappa C_2 - 4\Lambda) - 12m^4}{3\kappa C_2 - 4\Lambda}} \quad \text{and} \quad C_5 = \frac{6m^2}{3\kappa C_2 - 4\Lambda}$$

where X and D are constants. At $C_4t - C_5 = 0$, a_1 and a_2 tend to zero but a_3 becomes indeterminate.

The physical quantities of observational interest in cosmology are the expansion scalar θ, the mean anisotropy parameter A, the shear scalar σ^2 and the deceleration parameter q. They are defined as

$$\theta = 3H. \tag{5.6}$$

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2. \tag{5.7}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) = \frac{3}{2}AH^2. \tag{5.8}$$

$$q = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{H}\right) - 1. \tag{5.9}$$

Using eqs (5.6)–(5.9) we can express the physical quantities as

$$\theta = \frac{3\sqrt{\frac{\kappa C_3(3\kappa C_2-4\Lambda)-12m^4}{3\kappa C_2-4\Lambda}}}{2\left[\sqrt{\frac{\kappa C_3(3\kappa C_2-4\Lambda)-12m^4}{3\kappa C_2-4\Lambda}}t - \frac{6m^2}{3\kappa C_2-4\Lambda}\right]}, \tag{5.10}$$

$$A = \frac{8X^2(3\kappa C_2 - 4\Lambda)}{3[\kappa C_3(3\kappa C_2 - 4\Lambda) - 12m^4]} \times \left[\sqrt{\frac{\kappa C_3(3\kappa C_2 - 4\Lambda) - 12m^4}{3\kappa C_2 - 4\Lambda}}t - \frac{6m^2}{3\kappa C_2 - 4\Lambda}\right]^3, \tag{5.11}$$

$$\sigma^2 = X^2\left[\sqrt{\frac{\kappa C_3(3\kappa C_2 - 4\Lambda) - 12m^4}{3\kappa C_2 - 4\Lambda}}t - \frac{6m^2}{3\kappa C_2 - 4\Lambda}\right], \tag{5.12}$$

$$q = 1, \tag{5.13}$$

where X is constant. As $t \to \infty$, the anisotropy increases indefinitely and the model approaches singularity.

6. Models with constant deceleration parameter

Case I. Power-law

Here we take

$$V = at^b, \tag{6.1}$$

where a and b are constants.

From eqs (6.1) and (3.10), we get

$$a_1(t) = a^{1/3}t^{b/3}, \tag{6.2a}$$

$$a_2(t) = Da^{1/3}t^{b/3}\exp\left[\frac{X}{a}\int\left(\frac{\mathrm{e}^{-2\kappa\int\eta\mathrm{d}t}}{t^b}\right)\mathrm{d}t\right], \tag{6.2b}$$

$$a_3(t) = D^{-1}a^{1/3}t^{b/3}\exp\left[-\frac{X}{a}\int\left(\frac{\mathrm{e}^{-2\kappa\int\eta\mathrm{d}t}}{t^b}\right)\mathrm{d}t\right], \tag{6.2c}$$

where X and D are constants.

Using eqs (5.6)–(5.9) we can express the physical quantities as

$$\theta = \frac{b}{t}, \tag{6.3}$$

$$A = \frac{6X^2}{a^2b^2}\frac{\mathrm{e}^{-4\kappa\int\eta\mathrm{d}t}}{t^{2(b-1)}}, \tag{6.4}$$

$$\sigma^2 = \frac{X^2}{a^2}\frac{\mathrm{e}^{-4\kappa\int\eta\mathrm{d}t}}{t^{2b}} \tag{6.5}$$

$$q = \frac{3}{b} - 1, \tag{6.6}$$

where X is a constant.

For large t, the model tends to be isotropic for $b > 1$. The expansion becomes zero and we have a static universe.

Case II. Exponential type

Here we take

$$V = \alpha' \mathrm{e}^{\beta' t}, \tag{6.7}$$

where α' and β' are constants.

From eqs (6.7) and (3.10), we get

$$a_1(t) = \alpha'^{1/3}\exp\left(\frac{\beta' t}{3}\right), \tag{6.8a}$$

$$a_2(t) = D\alpha'^{1/3}\exp\left(\frac{\beta' t}{3} + \frac{X}{\alpha'}\int e^{[-2\kappa\int\eta dt + \beta' t]}dt\right), \tag{6.8b}$$

$$a_3(t) = D^{-1}\alpha'^{1/3}\exp\left(\frac{\beta' t}{3} - \frac{X}{\alpha'}\int e^{[-2\kappa\int\eta dt + \beta' t]}dt\right), \tag{6.8c}$$

where X and D are constants.

By using eqs (5.6)–(5.9) we can express the physical quantities as

$$\theta = \beta', \tag{6.9}$$

$$A = \frac{6X^2}{\alpha'^2\beta'^2}e^{-[2\beta' t + 4\kappa\int\eta dt]}, \tag{6.10}$$

$$\sigma^2 = \frac{X^2}{\alpha'^2}e^{-[2\beta' t + 4\kappa\int\eta dt]}, \tag{6.11}$$

$$q = -1, \tag{6.12}$$

where X is a constant.

For large t, the model tends to be isotropic.

7. Conclusion

The solution for Bianchi Type-V universe with viscous fluid and cosmological constant has been obtained in quadrature form. The particular case $\gamma = 1/3$ (radiation) has been studied in detail. The solutions for constant deceleration parameter have been discussed in both power-law and exponential forms. Some of the models tend to be isotropic for large t.

Acknowledgements

The authors express their gratitude to Prof. A Dolgov for valuable suggestions. The authors wish to place on record their sincere thanks to the referee whose valuable comments have helped in improving the quality of the paper.

References

[1] C W Misner, *Nature (London)* **214**, 40 (1967)
[2] C W Misner, *Astrophys. J.* **151**, 441 (1968)
[3] S Weinberg, *Astrophys. J.* **168**, 175 (1972)
[4] S Weinberg, *Gravitation and cosmology* (Wiley, New York, 1972)
[5] P Langacker, *Phys. Rep.* **72**, 185 (1981)
[6] I Waga, R C Falcan and R Chanda, *Phys. Rev.* **D33**, 1839 (1986)

[7] T Pacher, J A Stein-Schabes and M S Turmer, *Phys. Rev.* **D36**, 1603 (1987)
[8] A Guth, *Phys. Rev.* **D23**, 347 (1981)
[9] G L Murphy, *Phys. Rev.* **D8**, 4231 (1973)
[10] N O Santos, R S Dias and A Banerjee, *J. Math. Phys.* **26**, 876 (1985)
[11] V A Belinski and I M Khalatnikov, *JETP* **69**, 401 (1975)
[12] A Banerjee, S B Duttachoudhary and A K Sanyal, *J. Math. Phys.* **26**, 3010 (1985)
[13] W Huang, *J. Math. Phys.* **31**, 1456 (1990)
[14] K Desikan, *Gen. Relativ. Gravit.* **29**, 435 (1997)
[15] K D Krori and A Mukherjee, *Gen. Relativ. Gravit.* **32**, 1429 (2000)
[16] L P Chimento, A S Jacubi, V Mendez and R Marten, *Class. Quant. Grav. (U.K.)* **14**, 3363 (1997)
[17] V R Gavrilov, V N Melnikov and R Triay, *Class. Quant. Grav. (U.K.)* **14**, 2203 (1997)
[18] Ø Grøn, *Astrophys. Space Sci.* **173**, 191 (1990)
[19] J D Barrow, *Nucl. Phys.* **B310**, 743 (1988)
[20] B Saha, *Nonlinear spinor field in Bianchi Type-I cosmology*: *Accelerated regions*, gr-qc/0608047 (2006)
[21] B Saha, *Bianchi Type-I universe with viscous fluid*, gr-qc/0409104 (13 october 2004) (preprint) (2004); *Mod. Phys. Lett.* **A20**, 2127 (2005)