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Bianchi Type-II inflationary models with constant deceleration parameter in general relativity

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Abstract. Einstein's field equations are considered for a locally rotationally symmetric Bianchi Type-II space–time in the presence of a massless scalar field with a scalar potential. Exact solutions of scale factors and other physical parameters are obtained by using a special law of variation for Hubble's parameter that yields a constant value of deceleration parameter. To get inflationary solutions, a flat region is considered in which the scalar potential is constant. Power-law and exponential cases are studied and in both solutions there is an anisotropic expansion of the cosmic fluid, but the fluid has vanishing vorticity. A detailed study of geometrical and kinematical properties of solutions has been carried out.

Keywords. Cosmology; Bianchi space–time; deceleration parameter; inflationary models.

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1. Introduction

Inflation, the stage of accelerated expansion of the Universe, first proposed in the beginning of the 1980s, nowadays receives a great deal of attention. Guth [1] proposed inflationary model in the context of grand unified theory (GUT), which has been accepted soon as the model of the early Universe. So far there are two basic types of inflationary cosmological models. One is due to appearance of a flat potential e.g. in GUT'S phase transitions, and the other is due to a scalar curvature-squared term. Barrow and Turner [2] showed that the large anisotropy prevents transition into an inflationary era according to Guth's original inflationary scenario. Several versions of inflationary scenario exist as investigated by Linde [3,4], Abrecht and Steinhardt [5], Abbott and Wise [6], Mathiazhagan and Johri [7], Mataresse and Luechin [8], Mijic *et al* [9] and La and Steinhardt [10]. Rothman and Ellis [11] discovered that one can have a solution of the isotropy problem if one work with anisotropic metrics and they showed that these can be isotropized and inflated under very general circumstances. The notion of inflation is so far one of the best mechanism at the early stages of evolution to explain the flat, homogeneous and isotropic nature of the present day Universe. In these models, the Universe undergoes a phase transition characterized by the evolution of a Higg's field ϕ. The inflation will take place if the potential $V(\phi)$ has a 'flat' region and in this region the ϕ field evolves slowly but the Universe expands in an exponential way due to vacuum field energy (see Stein-Schabes [12]). The flat part of the potential is naturally associated with a vacuum energy that can be identified as an effective cosmological constant Λ and that makes the Universe to enter an inflationary period. Grøn [13,14] studied inflationary cosmological models of Bianchi Type-I with shear, bulk and non-linear viscosity. Chakraborty [15] studied inflationary solutions for Bianchi-IX space–time in the presence of massless scalar field with flat potential. Banerjee *et al* [16] studied exponential inflation and power function inflation in general scalar tensor theory where the coupling parameter ω is a function of the scalar field. Shri Ram and Singh [17] studied the inflationary solutions in totally anisotropic Bianchi Type-II space–time in the presence of massless scalar field with flat potential. Most of the inflationary models have been constructed by assuming the background space–time metric as homogeneous, either the Bianchi models or the Kantowski–Sachs models.

Bianchi Type-II space–time has a fundamental role in constructing cosmological models suitable for describing the early stages of evolution of Universe. Asseo and Sol [18] emphasized the importance of Bianchi Type-II Universe. Locally rotationally symmetric (LRS) Bianchi Type-II space–times have already been considered by a number of authors. Lorenz [19] presented the exact solutions for LRS Bianchi-II space–time stiff matter models in an electromagnetic field theory. Hajj-Boutros [20,21] studied Bianchi Type-II space–time with perfect fluid by a generating technique and also constructed LRS Bianchi Type-II perfect fluid models with an equation of state, which is a function of time. Shanthi and Rao [22] studied the same model in Barber's self-creation theory of gravitation. Venkateswarlu and Reddy [23] found cosmological solutions for Bianchi-II stiff fluid models in electromagnetic field theory. Coley and Wainwright [24] studied LRS Bianchi Type-II in two fluid Bianchi cosmologies.

In a recent paper [25], we have formulated a special law of variation of Hubble's parameter for anisotropic and homogeneous LRS Bianchi Type-II perfect fluid models in general relativity that yields a constant value of deceleration parameter. In the case of perfect fluid, the law transforms the Einstein's field equations together with the conservation equation into an autonomous system of differential equations, which is important in applications to cosmology.

In this paper we extend our work to anisotropic and homogeneous LRS Bianchi Type-II space–time in the presence of massless scalar field with a flat potential in general relativity. We use the special law of variation of Hubble's parameter as proposed in our previous work [25] to solve the Einstein's field equations. Three types of cosmologies have been discussed. The evolution of anisotropy during the inflationary era is investigated. A detailed study of geometrical and kinematical properties of solutions has been carried out. The nature of singularities has been clarified in each case.

2. Model and field equations

In an orthonormal frame, the metric for Bianchi Type-II space–time in the LRS case is given by (see ref. [19])

$$ds^2 = g_{\mu\nu}\theta^\mu\theta^\nu, \quad g_{\mu\nu} = \mathrm{diag.}(-1,1,1,1), \tag{1}$$

where the cartan bases θ^μ are given by

$$\theta^0 = \mathrm{d}t, \quad \theta^1 = S(t)\omega^1, \quad \theta^2 = R(t)\omega^2, \quad \theta^3 = R(t)\omega^3. \tag{2}$$

Here $R(t)$ and $S(t)$ are the metric functions. The time-independent differential one forms ω^μ are given by

$$\omega^1 = \mathrm{d}y + x\mathrm{d}z, \quad \omega^2 = \mathrm{d}z, \quad \omega^3 = \mathrm{d}x. \tag{3}$$

The spatial average scale factor a is given by

$$a = (R^2S)^{1/3}. \tag{4}$$

A 'volume scale factor' will also be useful, and is given as

$$V = a^3. \tag{5}$$

Also we define the average Hubble's factor as

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{6}$$

where $H_1 = \dot{R}/R$, $H_2 = H_3 = \dot{S}/S$ are directional Hubble's factors in the directions of x, y and z respectively.

From eqs (4)–(6), the average Hubble's parameter may be generalized in anisotropic cosmological models as

$$H = \frac{1}{3}(\ln\dot{V}) = (\ln\dot{a}) = \frac{1}{3}\left(\frac{2\dot{R}}{R} + \frac{\dot{S}}{S}\right), \tag{7}$$

where an overdot denotes derivative with respect to cosmic time t.

The action of the gravitational field minimally coupled to a scalar field with potential $V(\phi)$ (see ref. [12]) is

$$L = \int\sqrt{-g}\left[R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]\mathrm{d}^4x, \tag{8}$$

where $g = \det(g_{\mu\nu})$ and R is the Ricci scalar. Units are taken such that $8\pi G = c = 1$.

Now from variation of action L with respect to dynamical fields, the Einstein's field equations, in the case of massless scalar field ϕ with a potential $V(\phi)$, are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -T_{\mu\nu} \tag{9}$$

with

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \left[\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi + V(\phi)\right] g_{\mu\nu}. \tag{10}$$

The conservation relation is of the form

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\partial^\mu\phi\right) = -\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi}. \tag{11}$$

For LRS Bianchi Type-II space–time (1), Einstein's field equations (9) reduce, in case of eq. (10), to three non-linear differential equations:

$$\frac{2\dot{R}\dot{S}}{RS} + \frac{\dot{R}^2}{R^2} - \frac{1}{4}\frac{S^2}{R^4} = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{12}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{3}{4}\frac{S^2}{R^4} = -\frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{13}$$

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} + \frac{1}{4}\frac{S^2}{R^4} = -\frac{1}{2}\dot{\phi}^2 + V(\phi). \tag{14}$$

The conservation relation (11) gives

$$\ddot{\phi} + \left(\frac{2\dot{R}}{R} + \frac{\dot{S}}{S}\right)\dot{\phi} = -\frac{\mathrm{d}V(\phi)}{\mathrm{d}\phi}. \tag{15}$$

3. Solution of field equations

In a perfect fluid interpretation we define the equivalent pressure and energy-density as

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \tag{16}$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi). \tag{17}$$

As to the effect of scalar field on the space–time geometry, the scalar field acts as a combination of a vacuum fluid with $\rho_v = -p_v = V(\phi)$ (or in the case of flat potential; a cosmological constant $\Lambda = \rho_v$), and a Zel'dovich fluid with $\rho = p = \frac{1}{2}\dot{\phi}^2$ (or a minimally coupled massless scalar field). During the vacuum dominated era, the scalar field is practically constant and $V(\phi)$ dominates. However, at the end of this era, $V(\phi)$ approaches zero while the scalar field fluctuates violently due to $\frac{1}{2}\dot{\phi}^2$-term. Space–time may then be dominated by the Zel'dovich fluid.

Now we consider the flat region of the potential where the scalar potential is approximated as constant $V(\phi) \sim \Lambda$ (see ref. [12]). The field equations (12) to (15) with this assumption now reduce to

$$\frac{2\dot{R}\dot{S}}{RS} + \frac{\dot{R}^2}{R^2} - \frac{1}{4}\frac{S^2}{R^4} = \frac{1}{2}\dot{\phi}^2 + \Lambda, \tag{18}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{3}{4}\frac{S^2}{R^4} = -\frac{1}{2}\dot{\phi}^2 + \Lambda, \tag{19}$$

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} + \frac{1}{4}\frac{S^2}{R^4} = -\frac{1}{2}\dot{\phi}^2 + \Lambda, \tag{20}$$

$$\ddot{\phi} + \left(\frac{2\dot{R}}{R} + \frac{\dot{S}}{S}\right)\dot{\phi} = 0. \tag{21}$$

The first integral of eq. (21) is

$$\dot{\phi} = \frac{l}{R^2 S}, \tag{22}$$

where l is a constant of integration.

The Einstein's field equations (18)–(21) are a coupled system of highly non-linear differential equations and we seek physical solutions to the field equations for applications in cosmology and astrophysics. In order to solve the field equations, we normally assume a form for the matter content or relation between metric coefficients. But solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter, which was initially proposed by Berman [26] for Friedmann–Robertson–Walker (FRW) metric and that yields a constant value of deceleration parameter. The variation of Hubble's parameter as assumed by Berman is not inconsistent with observation. Berman and Gomide [27], Johri and Desikan [28], Singh and Desikan [29], Pradhan and Vishwakarma [30] and others have studied cosmological models with constant deceleration parameter.

Most of the work has been done on spatially homogeneous and isotropic FRW models using Berman's law. Recently we have proposed a special law of variation for Hubble's parameter in spatially homogeneous and anisotropic LRS Bianchi Type-II space–time that also yields a constant value of deceleration parameter (see ref. [25]). According to the proposed law, the variation of Hubble's parameter is given by

$$H = \frac{D}{3}\left(R^2 S\right)^{-n/3}, \tag{23}$$

where $D(>0)$ and $n(\geq 0)$ are constants. From eqs (7) and (23), we get

$$R^2 S = \left(\frac{n}{3}Dt + C_1\right)^{3/n}, \quad n \neq 0 \tag{24}$$

and

$$R^2S = C_2\mathrm{e}^{Dt}, \quad n = 0, \tag{25}$$

where C_1 and C_2 are constants of integration.

The deceleration parameter q is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{26}$$

Using eq. (24) into (26), we get

$$q = n - 1. \tag{27}$$

This shows that the law (23) gives a constant value of deceleration parameter.

Equations (24) and (25) equivalently in terms of deceleration parameter can be written as

$$R^2S = \left(\frac{1+q}{3}Dt + C_1\right)^{3/(1+q)}, \quad q \neq -1, \tag{28}$$

$$R^2S = C_2\mathrm{e}^{Dt}, \quad q = -1. \tag{29}$$

It is pointed out that the above law (23) refers to LRS Bianchi Type-II space–time in any context, i.e. in any theory based on LRS Bianchi Type-II space–time. Our intention in this paper is to analyze the law (23) in solving the Einstein's field equations (18)–(21) in the case of massless scalar field with flat potential by using (24) and (25).

Adding eqs (18) and (20), we get

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{3\dot{R}\dot{S}}{RS} + \frac{\dot{R}^2}{R^2} = 2\Lambda. \tag{30}$$

Integrating eq. (30), we get

$$R^2\dot{S} + R\dot{R}S = \int 2\Lambda(R^2S)\mathrm{d}t + h, \tag{31}$$

where h is a constant of integration. Now we solve eq. (31) for scale factors $R(t)$ and $S(t)$ using eqs (24) and (25) for all possible values of n in the following sections.

4. Cosmology for $n \neq 0$

Using eq. (24) into (31) and simplifying, we get

$$R^2\dot{S} + R\dot{R}S = \frac{6\Lambda}{(n+3)D}\left(\frac{n}{3}Dt + C_1\right)^{(n+3)/n} + h. \tag{32}$$

Dividing eq. (32) by R^2S and integrating, we get

$$RS = m_1 \exp\left[\frac{9\Lambda}{n(n+3)D^2}\left(\frac{n}{3}Dt + C_1\right)^2 + \frac{3h}{(n-3)D}\left(\frac{n}{3}Dt + C_1\right)^{(n-3)/n}\right], \tag{33}$$

where $m_1(>0)$ is a constant of integration and $n \neq 3$.

Solving eqs (24) and (33), we get

$$R(t) = \frac{1}{m_1}\left(\frac{n}{3}Dt + C_1\right)^{3/n} \times \exp\left[-\frac{9\Lambda}{n(n+3)D^2}\left(\frac{n}{3}Dt + C_1\right)^2 - \frac{3h}{(n-3)D}\left(\frac{n}{3}Dt + C_1\right)^{(n-3)/n}\right], \tag{34}$$

$$S(t) = m_1^2\left(\frac{n}{3}Dt + C_1\right)^{-3/n} \times \exp\left[\frac{18\Lambda}{n(n+3)D^2}\left(\frac{n}{3}Dt + C_1\right)^2 + \frac{6h}{(n-3)D}\left(\frac{n}{3}Dt + C_1\right)^{(n-3)/n}\right]. \tag{35}$$

Using solutions (34) and (35) into (22) and integrating, we get

$$\phi = \frac{3l}{(n-3)D}\left(\frac{n}{3}Dt + C_1\right)^{(n-3)/n} + \phi_0, \tag{36}$$

where ϕ_0 is a constant of integration. The above solutions are valid for all possible values of n except for $n = 3$ and $n = 0$. We shall obtain solutions for $n = 3$ and $n = 0$ separately in the next sections. Now we discuss the geometrical and kinematical properties of the above solutions obtained when $n \neq 0$.

The directional Hubble's factors H_1, H_2 and H_3 in the directions of x, y and z are given by

$$H_1 = \frac{12\Lambda}{(n+3)D}\left(\frac{n}{3}Dt + C_1\right) + 2h\left(\frac{n}{3}Dt + C_1\right)^{-3/n} - D\left(\frac{n}{3}Dt + C_1\right)^{-1}, \tag{37}$$

$$H_2 = H_3 = \frac{-6\Lambda}{(n+3)D}\left(\frac{n}{3}Dt + C_1\right) - h\left(\frac{n}{3}Dt + C_1\right)^{-3/n} + D\left(\frac{n}{3}Dt + C_1\right)^{-1} \tag{38}$$

with

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{D}{3}\left(\frac{n}{3}Dt + C_1\right)^{-1}. \tag{39}$$

The anisotropy parameter A is defined as

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{40}$$

where $\Delta H_i = H_i - H$ $(i = 1, 2, 3)$.

Using eqs (37)–(39) in (40), we get

$$A = 2\left[\frac{18\Lambda}{(n+3)D^2}\left(\frac{n}{3}Dt + C_1\right)^2 + \frac{3h}{D}\left(\frac{n}{3}Dt + C_1\right)^{(n-3)/n} - 2\right]^2. \tag{41}$$

The expansion scalar Θ is given by

$$\Theta = 3H = D\left(\frac{n}{3}Dt + C_1\right)^{-1}. \tag{42}$$

The shear scalar σ is defined as

$$\sigma^2 = \frac{1}{2}\sigma^{\alpha\beta}\sigma_{\alpha\beta} = \frac{1}{2}\sum_{i=1}^{3}(H_i^2 - 3H^2) = \frac{3}{2}AH^2 = \frac{1}{6}A\Theta^2. \tag{43}$$

Using (41) and (42) in (43), we get

$$\sigma^2 = \frac{1}{3}\left[2D\left(\frac{n}{3}Dt + C_1\right)^{-1} - \frac{18\Lambda}{(n+3)D}\left(\frac{n}{3}Dt + C_1\right) - 3h\left(\frac{n}{3}Dt + C_1\right)^{-3/n}\right]^2. \tag{44}$$

The Ray Chaudhary equation is

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 - 2\sigma^2 - \left[\dot{\phi}^2 - V(\phi)\right]. \tag{45}$$

Or in terms of $a = (R^2S)^{1/3}$,

$$\frac{3\ddot{a}}{a} = -2\sigma^2 - \left[\dot{\phi}^2 - V(\phi)\right]. \tag{46}$$

We see that the potential $V(\phi)$ acts as a cosmological constant and in order to have positive volume expansion of acceleration, necessary for inflation, we need to have $V(\phi) > 2\sigma^2 + \dot{\phi}^2$.

It is observed that the spatial volume is zero at $t = t_0$, where $t_0 = -3C_1/nD$. This shows that the Universe starts evolving with zero volume at t_0 and expands with time t. For this model the condition of expansion is $n > 0$. One of the scale factors vanishes while the other one diverges at t_0. The model has a cigar-type singularity at t_0. The scalar field has a constant value ϕ_0 at t_0 provided $n > 3$. The solutions indicate that the Universe is expanding exponentially fast making the scalar field ϕ to have a constant value ϕ_0. The expansion scalar and shear scalar diverge at t_0. It has been pointed by Collins and Wainwright [31] that the shear tensor σ_{ij} plays an important role in general relativistic cosmology. The shear tensor arises in the decomposition of four-vector velocity of the fluid. The shear scalar is non-zero for $t > 0$ and becomes infinitely large as $t \to \infty$. Thus the Universe is not shear free at infinite time. Also it is observed that the ratio σ/Θ does not tend to zero as $t \to \infty$, which shows that our model does not approach isotropy for large values of t. At the time of evolution t_0, the anisotropy of the Universe is constant. The expansion scalar $\Theta \to 0$ as $t \to \infty$, indicates that the Universe is expanding with increase of time and the rate of expansion decreases with increase of time.

The solution of the scale factors has a combination of power-law term $\left(\frac{n}{3}Dt + C_1\right)^{3/n}$ and the exponential term in the product form. We observe that the solutions are inflationary in nature. So initially the exponential term may be more significant and it is possible to have inflationary scenario during the evolution of the Universe. Thus the space–time may be dominated by vacuum fluid. The interesting feature of the present solution is that it is possible to exit from exponential inflationary scenario as $V(\phi) = \Lambda = 0$ and $h = 0$. So, if after some inflation time the power-law begins to dominate the dynamics, the Universe will continue to expand with power-law inflation. Thus it is possible to have expansion without any cosmological term and that is only due to a minimally coupled massless scalar field. Thus space–time may then be dominated by the Zel'dovich fluid. Thus, if we assume that the Universe is homogeneous, $T_{\mu\nu}$ takes the perfect fluid form with pressure and energy–density given by eqs (16) and (17). We find that during the vacuum-dominated era the scalar field is practically constant and $V(\phi)$ dominates ($\rho_\nu = -p_\nu = V(\phi)$). Then the solutions (34) and (35) match with the solutions (4.8) and (4.9) (putting $\gamma = -1$ and taking $8\pi G = 1$) as obtained in our previous work [25]. However, at the end of this era $V(\phi)$ is approaching zero, while the scalar field fluctuates violently due to $\frac{1}{2}\dot{\phi}^2$-term. The space–time may then be dominated by the Zel'dovich fluid ($\rho = p = \frac{1}{2}\dot{\phi}^2$). Then the solutions (34) and (35) match with the solutions (4.14) and (4.15) of our previous work.

5. Cosmology for $n = 3$

From eq. (24) we get

$$R^2 S = Dt + C_1. \tag{47}$$

Using eq. (47) into eq. (31) and simplifying, we find

$$RS = m_2 (Dt + C_1)^{h/D} \exp\left[\frac{\Lambda}{2D^2}(Dt + C_1)^2\right], \tag{48}$$

where $m_2(>0)$ is a constant of integration.

From eqs (47) and (48), the scale factors have the following expressions respectively:

$$R(t) = \frac{1}{m_2}(Dt + C_1)^{(D-h)/D} \exp\left[-\frac{\Lambda}{2D^2}(Dt + C_1)^2\right], \tag{49}$$

$$S(t) = m_2^2(Dt + C_1)^{(2h-D)/D} \exp\left[\frac{\Lambda}{D^2}(Dt + C_1)^2\right]. \tag{50}$$

Using eqs (49) and (50) into eq. (22), we get

$$\phi = \log(Dt + C_1)^{\frac{l}{D}} + \phi_1. \tag{51}$$

(Here ϕ_1 is a constant of integration).

The anisotropy parameter is given by

$$A = 2\left[\frac{3\Lambda}{D^2}(Dt + C_1) + \frac{3h}{D} - 2\right]^2. \tag{52}$$

The volume expansion Θ and shear scalar σ are given by

$$\Theta = D\,(Dt + C_1)^{-1}, \tag{53}$$

$$\sigma^2 = \frac{1}{3}\left[(2D - 3h)(Dt + C_1)^{-1} - \frac{3\Lambda}{D}(Dt + C_1)\right]^2. \tag{54}$$

From the above results it is observed that the spatial volume shows the linear expansion of the Universe with time. At $t = -C_1/D$, the spatial volume is zero. One of the scale factors diverges while the other one vanishes at $t = -C_1/D$. The model has a singularity at $t = -C_1/D$. It is cigar-type or pancake-type according as $h < D$ or $h > D$. As $t \to \infty$, the shear scalar does not vanish but the expansion ceases. Since $\lim_{t\to\infty} \sigma/\Theta \neq 0$, the model does not approach isotropy for large values of t. The solutions for scale factors again have a combination of power-law term $(Dt + C_1)^{(D-h)/D}$ and the exponential term in the product form. The physical behavior will be same as we have discussed for $n \neq 0$. It is observed that the solution is deflationary in the scale factor $R(t)$ and inflationary in $S(t)$ provided $D < h$. These inflationary or deflationary solutions will continue and there is no mechanism to halt this process. If after some inflation time the power-law begins to dominate the dynamics, then only the inflationary behavior will slow down but it will continue with power-law inflation. The interesting feature of the present solutions is that exit of inflationary scenario is possible as $V(\phi) = \Lambda = 0$. Thus it is possible to have expansion without any cosmological term and that is only due to a minimally coupled massless scalar field. Thus space–time may then be dominated by the Zel'dovich fluid.

6. Cosmology for $n = 0$

Using eq. (25) into (31) and simplifying, we get

$$R^2\dot{S} + R\dot{R}S = \frac{2\Lambda C_2}{D}\mathrm{e}^{Dt} + h. \tag{55}$$

Dividing eq. (55) by R^2S and integrating, we get

$$RS = m_3 \exp\left(\frac{2\Lambda}{D}t - \frac{h}{C_2 D}\mathrm{e}^{-Dt}\right), \tag{56}$$

where $m_3(> 0)$ is a constant of integration.

Solving eqs (25) and (56), we get

$$R(t) = \frac{C_2}{m_3}\exp\left[\left(D - \frac{2\Lambda}{D}\right)t + \frac{h}{C_2 D}\mathrm{e}^{-Dt}\right], \tag{57}$$

$$S(t) = \frac{m_3^2}{C_2}\exp\left[\left(\frac{4\Lambda}{D} - D\right)t - \frac{2h}{C_2 D}\mathrm{e}^{-Dt}\right]. \tag{58}$$

The Higg's field ϕ is given by

$$\phi = -\frac{l}{DC_2}\mathrm{e}^{-Dt} + \phi_2. \tag{59}$$

(Here ϕ_2 is a constant of integration).

The directional Hubble's factors H_1, H_2 and H_3 in the directions of x, y and z are given by

$$H_1 = \frac{4\Lambda}{D} - D + \frac{2h}{C_2}\mathrm{e}^{-Dt}, \tag{60}$$

$$H_2 = H_3 = D - \frac{2\Lambda}{D} - \frac{h}{C_2}\mathrm{e}^{-Dt}. \tag{61}$$

The average Hubble's parameter and anisotropy parameter are given by

$$H = \frac{D}{3}, \tag{62}$$

$$A = 2\left[\left(\frac{6\Lambda}{D^2} - 2\right) + \frac{3h}{DC_2}\mathrm{e}^{-Dt}\right]^2. \tag{63}$$

The shear and expansion scalars have the following expressions:

$$\sigma^2 = \frac{1}{3}\left[2\left(D - \frac{3\Lambda}{D}\right) - \frac{3h}{C_2}\mathrm{e}^{-Dt}\right]^2. \tag{64}$$

$$\theta = D. \tag{65}$$

For $n = 0$, we get the deceleration parameter $q = -1$, which is consistent with the recent observations of Supernovae Ia requiring the present Universe to be accelerating [32,33]. From the above solutions it is observed that the geometrical and kinematical quantities such as the spatial volume, Hubble's factors, scale factors, shear scalar, scalar field all are constants at $t = 0$. This shows that the Universe starts evolving with a constant volume and expands with exponential rate and anisotropy. The expansion scalar is constant for all values of t. Thus the model represents uniform expansion and the volume grows exponentially with time when the potential is in the flat region. The shear scalar decreases as time increases but does not vanish completely even for large time t. Since $\lim_{t\to\infty} \sigma/\Theta \neq 0$, the model does not approach isotropy. As $t \to \infty$, it is found that the scalar field ϕ becomes constant. It is also observed that one of the scale factors diverges while the other one vanishes as $t \to \infty$. The solutions have cigar-type or pancake-type singularity according as $D^2 < 2\Lambda$ or $D^2 > 2\Lambda$. The solutions are deflationary in the scale factor $R(t)$ and inflationary in $S(t)$ provided $D^2 < 2\Lambda$. The situation is reversed if $D^2 > 4\Lambda$. Interesting feature of these solutions is that the model has always exponential inflation.

7. Conclusion

In this paper we have obtained exact solutions of Einstein's field equations for LRS Bianchi Type-II space–time in the presence of massless scalar field with flat potential. The solutions are obtained using a special law of variation of Hubble's parameter for anisotropic models that yields a constant value of deceleration parameter. The law in eq. (23) provides an alternative and easy approach to get the exact solutions of the highly non-linear Einstein's field equations for Bianchi type models in a very simple manner. We have discussed geometrical and kinematical properties of different parameters in §4, 5, and 6 in detail for all possible values of n. The condition for the expanding Universe is $n \geq 0$ and all the physically viable models of the expanding Universe exist only for this condition. The nature of singularities of the model has been clarified and explicit forms of scale factors have been obtained in each case. It has been observed that the Universe starts expanding at $t = -3C_1/nD$ for $n \neq 0$ and at $t = 0$ for $n = 0$. Thus the model of the Universe has a singular origin for $n \neq 0$ and a non-singular origin for $n = 0$. Also for $n = 0$, $q = -1$; incidentally this value of deceleration parameter leads to $\mathrm{d}H/\mathrm{d}t = 0$, which implies the greatest value of Hubble's parameter and the fastest rate of expansion of the Universe. This class of solutions is consistent with the recent observations of Supernovae Ia [32,33] that require the present Universe to be accelerating.

All the kinematical and geometrical parameters start with singularity for $n \neq 0$. The rate of expansion slows down with the increase of time and finally drops to zero as $t \to \infty$. It is also observed that the ratio of the shear and expansion is non-zero for all values of t. Thus the Universe remains anisotropic throughout the evolution and the scalar field is constant for large values of t. Thus the model represents a shearing, non-rotating and expanding Universe. The purpose of this paper has been to investigate the evolution of LRS Bianchi Type-II space–time in inflationary era taking into account the massless scalar field with flat potential. During the inflationary era the dominant contributions to the cosmic energy density came from a vacuum energy and a scalar field. This has been designed as a two-component fluid, consisting of a vacuum fluid with $\rho_\nu = -p_\nu = V(\phi)$ and a Zel'dovich fluid with $\rho = p = \frac{1}{2}\dot{\phi}^2$. At the end of inflationary expansion, there is a transition from vacuum fluid to Zel'dovich fluid. At the time of evolution of Universe, the anisotropy is constant and during the inflation it is still homogeneous and anisotropic. Thus the results obtained in this paper can be naturally interpreted in the framework of perfect fluid scenario. During an initial period, the Universe is assumed to be dominated by constant potential term $V(\phi) = \Lambda$ of a scalar field (with energy density $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and pressure $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$), giving rise to exponential growth. Hence an inflationary scenario has been observed for LRS Bianchi Type-II space–time in the presence of massless scalar field with flat potential. The solutions presented in this paper are new and may be useful for better understanding of inflationary scenario by using the special law of variation for Hubble's parameter.

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